Uncertainty shocks and monetary policy rules in a small open economy

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December 9, 2019

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Motivation

1. Uncertainty and Macro models: how does uncertainty affects real variables ?

- Recently macro literature has introduced uncertainty shocks as a shock to second moment of the variable. It has been shown that these shocks to second order moments effects the levels of macro real variables
  - **Advanced Economies:** Bloom (2009), Gourio et al. (2013) , Basu and Bundick (2017) , Bloom et al. (2018) , Ravn and Sterk (2017) .
  - **Emerging Market Economies:** Fernandez-Villaverde et al.(2011) – Adverse effects of uncertainty in real interest rate on the levels of macroeconomic variables in emerging markets.
  - Swallow & Céspedes (2013) –Empirically show that uncertainty shocks affect EMEs more adversely.
  - Chatterjee (2018) role of trade openness
- No literature on exchange rate channel and role of monetary policy in amplifying/ stabilizing effects these shocks.

Outline of the presentation

1. **Empirical facts**
   - Reproduce facts from the data
   - Why role of monetary policy is important?
2. **The Model:** Small open economy new-Keynesian dynamic stochastic model with second moment shocks to global demand
   - To understand the transmission of uncertainty shocks and match the data
3. **Counterfactual experiments:** Monetary policy rules that reduce welfare losses
4. **Conclusion**

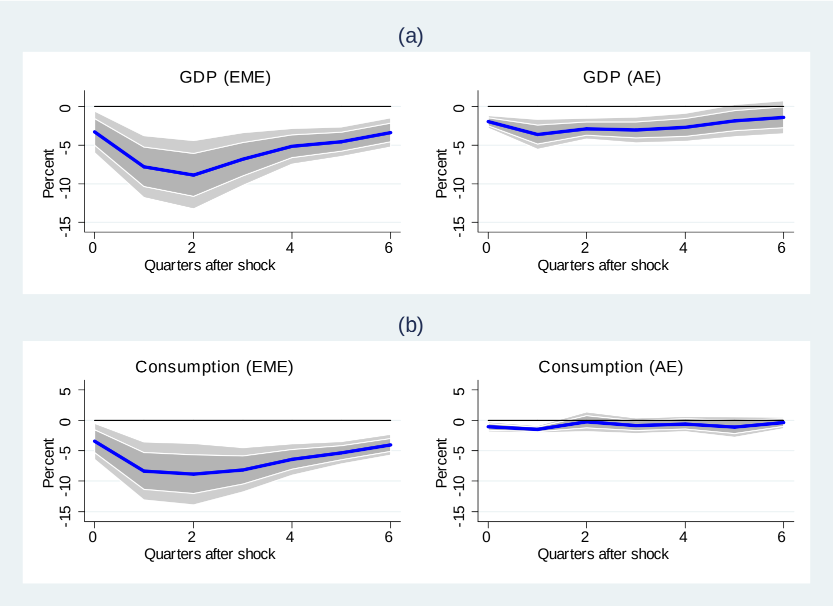
Preview of the results

- The global uncertainty shocks affect real economy in EMEs **adversely**
- **Depreciating exchange rates** can amplify the adverse real effects
- Monetary policy rule using **interest rates as instrument** (Taylor rule) fails to stabilize the economy
  - Policymakers observe *trade-off* in inflation and output gap stabilization
- This happens because **UIP fails** and risk premiums are positive with interest rate rules
- Monetary policy rule using **nominal exchange rates as instrument** (Exchange rate rule) reduces welfare losses by 20%
  - Lower hedging motive and thus risk premiums with ERRs
- The second order moments from the model show that the **variability** of nominal exchange rates, output and CPI **is reduced** by 85%, 36% and 45% respectively with ERRs

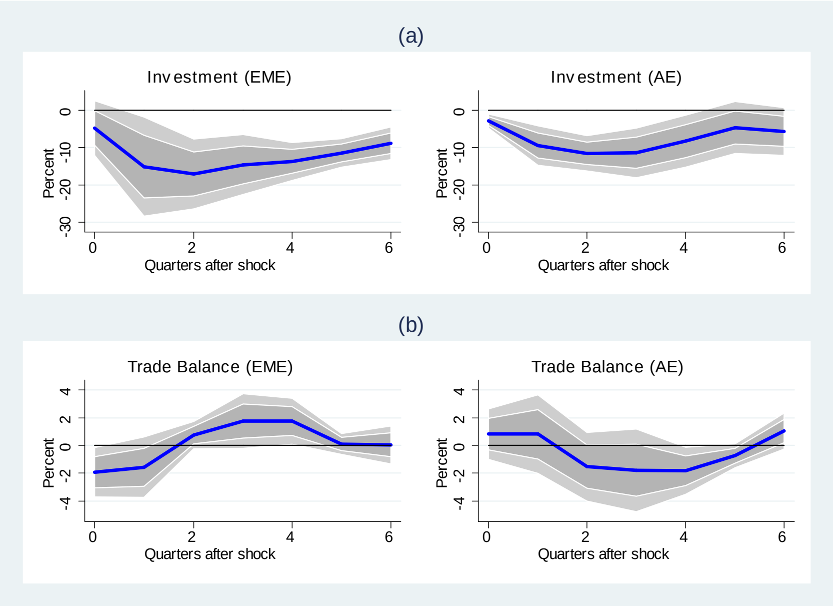
Empirical Strategy

- We use Panel **Local Projections** to estimate impulse response functions.
  - Impulse variable (measure of uncertainty): **VXO** series
  - Response variables: Macroeconomic variables
- Data description:
  - Quarterly data for 12 economies from 1996:Q1 to 2018:Q4
    - Source: Quarterly National Accounts data of OECD and IMF's International Financial Statistics
  - Six AEs (US, UK, Canada, Japan, Australia and South Korea) and Six EMEs (Brazil, Indonesia, India, Mexico, Russia and South Africa)

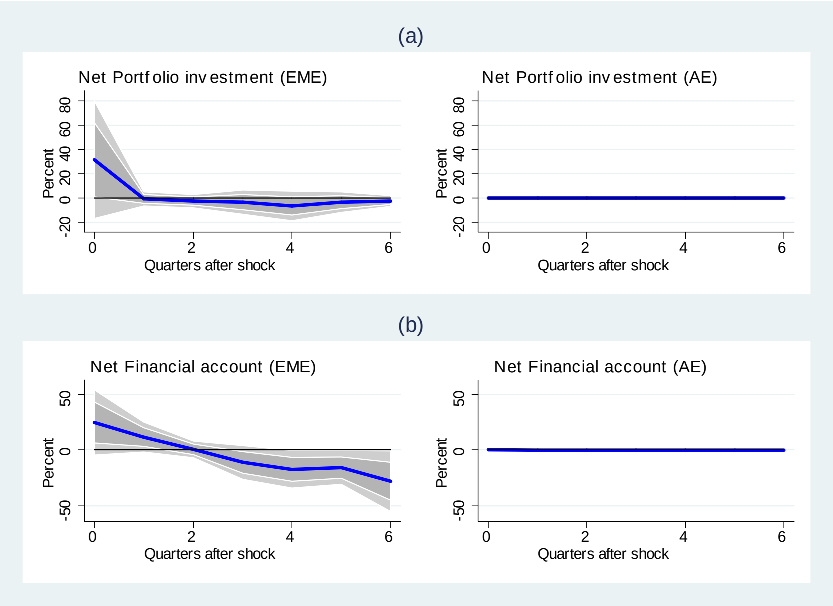
Local projection responses on GDP & consumption



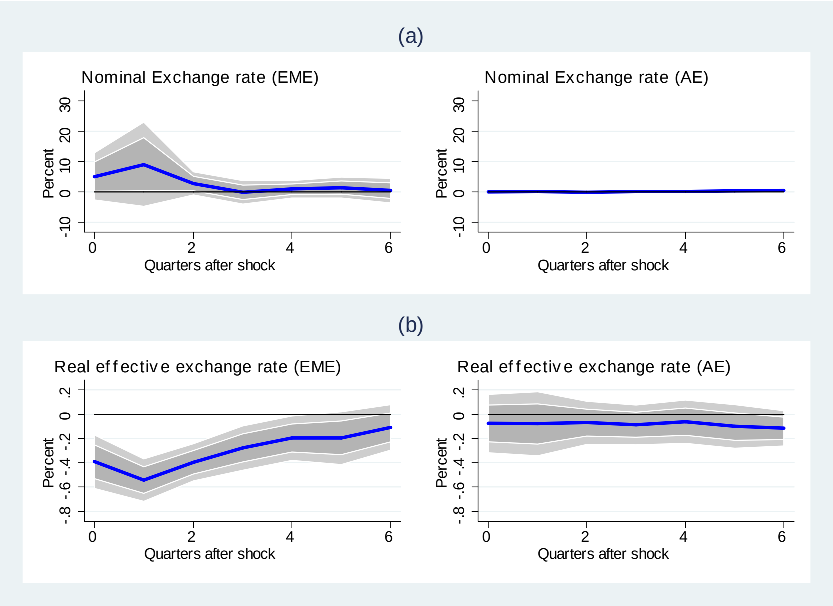
Local projection responses to Investment & Trade B.



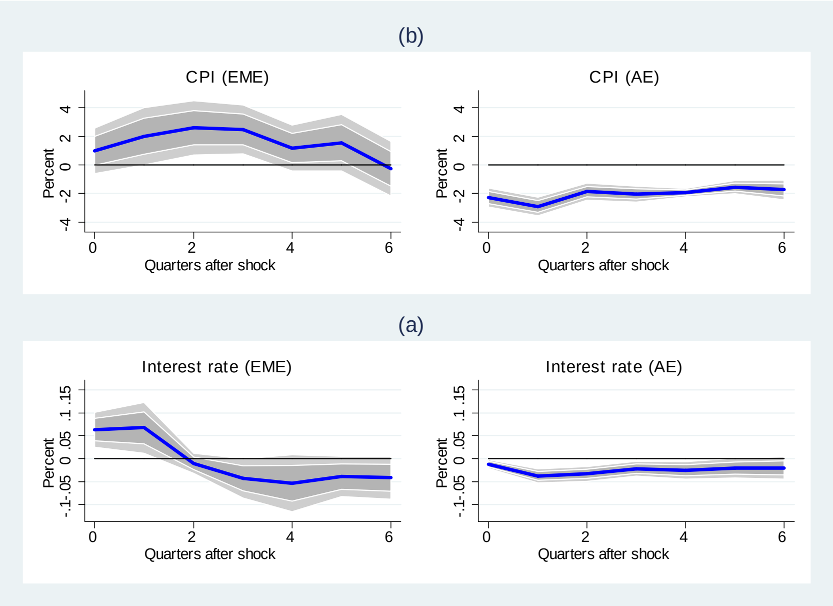
Local projection responses to Net Port.Invst. & Net FA



Local projection responses to NER & REER



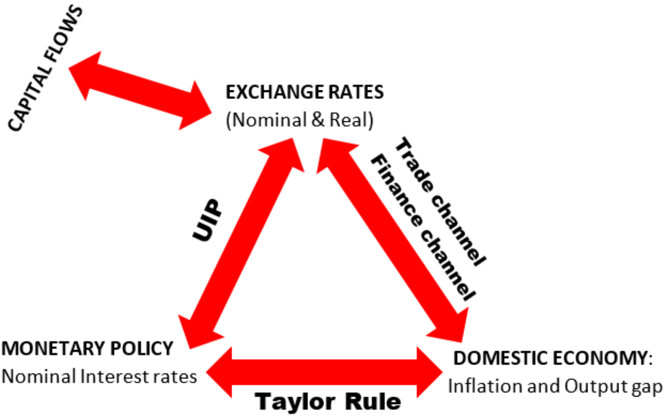
Local projection responses of Monetary policy variables



Introduction II

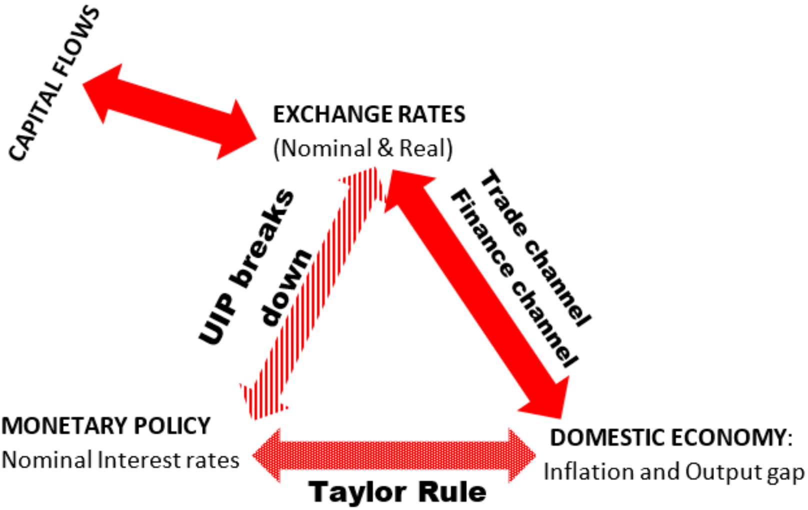
- Real economy adversely affected through two channels here:
  - Precautionary motives (save more, consume less): Lower demand
  - **Finance channel of depreciation:** Cook (2004), Burstein et al. (2004) and Forbes et al. (2012), Cespesdes and Swallow (2013) .
- The CPI increase due to **depreciation**
- Monetary policy *trade-off* in inflation and output stabilization
  - Inflation rising, output falling
- Stabilizing exchange rates is imperative and interest rates fail to do so
- Interest rate rules are ineffective in stabilizing the economy
- To understand the transmission we build a SOE NK-DSGE model

Uncovered Interest rate Parity (UIP): Standard channel



<<UIP>>

Uncertainty shocks: UIP breaks (Benigno et al. (2012))



".......The textbook version of the inflation targeting framework, which prescribes pursuing inflation stability with floating exchange rates through adjustments of a short-term interest rate, is obviously too narrow for EME central banks. In particular, the financial channel of the exchange rate gives rise to difficult trade-offs for monetary policy, while at the same time complicating the conduct of monetary policy by weakening its transmission.................Going forward, EME central banks will need to further develop their toolbox for dealing with the challenges of exchange rate and capital flow gyrations........" (Agustín Carstens, 2nd May 2019, LSE)

Features of the baseline model

- Two country open economy NK-DSGE model. Let us assume a continuum of households in the world with domestic sector D and foreign sector F from $[0, 1]$ The a continuum of domestic sector exist over $[0, n]$ and that of foreign sector from $[n, 1]$ (see Benigno et al (2012))
- We follow Benigno and De Paoli (2010)
  - Size of domestic economy $n \longrightarrow 0:$ SOE (EME)
  - Thus foreign economy is the World economy (AE)
- Flexible exchange rates and perfect capital mobility
- Heterogeneity of preference and demand structure between domestic and foreign households
- Second moment shock/ uncertainty shock to the foreign household demand

Households utility function

Following Fernandez-Villaverde et al. (2011) ,

- Domestic households utility function

$$\max U(C_t, H_{D,t}) = \frac{(C_t)^{1-\nu_D}}{1-\nu_D} - \omega_D \frac{(H_{D,t})^{1+\eta_D}}{1+\eta_D} \quad (1)$$

- Foreign households utility function

$$\max U(C_t^*, H_{F,t}) = \frac{\Upsilon_{F,t}\,(C_t^*)^{1-\nu_F}}{1-\nu_F} - \omega_F \frac{(H_{F,t})^{1+\eta_F}}{1+\eta_F} \quad (2)$$

  - Here, preference shock, $\Upsilon_{F,t}$,

$$\Upsilon_{F,t} = \left(1-\rho_{\Upsilon_F}\right)\overline{\Upsilon} + \rho_{\Upsilon_F}\Upsilon_{t-1} + v_{F,t-1}\varpi_{F,t} \quad (3)$$

$$v_{F,t} = \left(1-\rho_{\sigma_{\Upsilon_F}}\right)\overline{\Upsilon} + \rho_{\sigma_{\Upsilon_F}} v_{F,t-1} + \varkappa_{\Upsilon_F}\xi_{F,t} \quad (4)$$

<<**Model Details**>>

Aggregates I

- Goods market equilibrium,

Domestic country:

$$Y_{D,t} = (T_{D,t})^{-\xi_D} \left[\mu_D C_t + \mu_F \left(\frac{1-n}{n} \right) (T_{D,t})^{\xi_D - \xi_F} (Q_t)^{\xi_F} C_t^* \right] \quad (5)$$

Foreign country

$$Y_{F,t} = (T_{F,t})^{-\xi_D} \left[(1-\mu_D) \left(\frac{n}{1-n} \right) C_t + (1-\mu_F) (T_{F,t})^{\xi_D - \xi_F} (Q_t)^{\xi_F} C_t^* \right] \quad (6)$$

Note here, under autarky, $Y_{D,t} = C_t$ and $Y_{F,t} = C_t^*$
$<<$**Model Details**$>>$

Net Exports

Net export = Export - Import,
$NX_{D,t}$ and $NX_{F,t}$ are the real net export for domestic country and foreign country respectively.

$$NX_{D,t} = T_{D,t} C^*_{D,t} - T_{F,t} C_{F,t}$$

$$NX_{F,t} = \frac{T_{F,t}}{Q_t} C_{F,t} - \frac{T_{D,t}}{Q_t} C^*_{D,t}$$

<<**Model Details**>>

Interest rate rule

- Domestic country

$$R_{D,t} = \left(\frac{1}{\beta}\right)\left(\frac{\pi_t}{\overline{\pi}}\right)^{\phi_\pi}\left(\frac{Y_{D,t}}{Y_{D,t}^{fp}}\right)^{\phi_y} - 1 \qquad (7)$$

- Foreign country

$$R_{F,t} = \left(\frac{1}{\beta}\right)\left(\frac{\pi_t^*}{\overline{\pi}^*}\right)^{\phi_\pi}\left(\frac{Y_{F,t}}{Y_{F,t}^{fp}}\right)^{\phi_y} - 1 \qquad (8)$$

- Welfare

$$Welfare_{D,t} = U(C_t, H_{D,t}) + \beta Welfare_{D,t+1} \qquad (9)$$

$$Welfare_{F,t} = U(C_t^*, H_{F,t}) + \beta Welfare_{F,t+1} \qquad (10)$$

$U_{D,t}$ and $U_{F,t}$ are utility of domestic and foreign households respectively.

<<**Model Details**>>

Third order approximation for volatility shocks

- First order cannot capture the dynamics due to volatility shocks as the policy functions would only depend on level shocks.
- The second order only captures the partial effects of the volatility shocks through interactions with other shocks.
- Third order approximation of the model will fully capture the effect of rise in volatility
- See Basu and Bundick (2017) , Fernández-Villaverde et al. (2011), Benigno et al. (2013), Grohe and Uribe (2004).

Solution

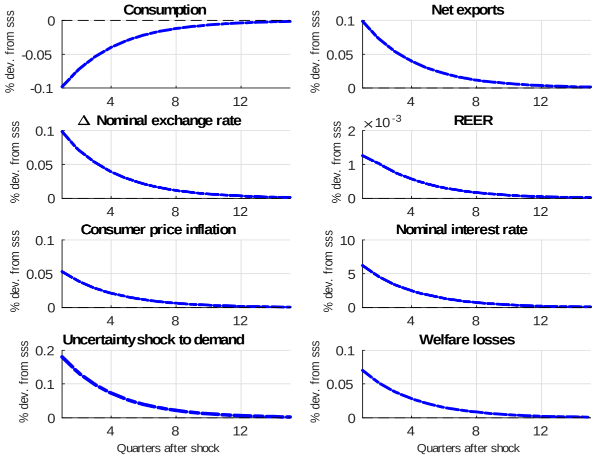
Very difficult to solve it analytically therefore we need to solve it numerically in MATLAB

Baseline calibration

Parameter	Value	Source
Degree of openness(χ)	0.6	Calculated by author
CRRA $(\nu_D; \nu_F)$	5; 5	Fernandez et al.(2011) ; Benigno et al. (2012)
Inverse of FES$(\eta_D; \eta_F)$	25;10	""
Inter-good elasticity of substitution $(\xi_D; \xi_F)$	1.5; 1.5	Benigno et al. (2012)
Discount factor (β)	0.994	Basu & Bundick (2017)
Stickiness parameter $(\alpha_D; \alpha_F)$	0.75; 0.65	" Benigno et al. (2012)
Elasticity of substitution among differentiated goods(θ)	6	Benigno et al. (2010)
Asset market condition(κ)	3.8	Calculated by author
All shock parameters		Basu and Bundick (2017)

Uncertainty/ Volatility shocks I

IRF's for one period uncertainty shock to foreign country demand



<<**Flex vs Sticky**>>

Monetary policy rules: Taylor type interest rate rules

- Taylor rule with CPI/ PPI

$$\text{Benchmark TR-CPI} \quad : \quad R_{D,t} = \overline{R}_D \left(\frac{\pi_t}{\overline{\pi}}\right)^{\phi_\pi} \left(\frac{Y_{D,t}}{Y_{D,t}^{fp}}\right)^{\phi_y} \tag{11}$$

$$\text{TR-PPI} \quad : \quad R_t = \overline{R}_D \left(\frac{\pi_{D,t}}{\overline{\pi}_D}\right)^{\phi_\pi} \left(\frac{Y_{D,t}}{Y_{D,t}^{fp}}\right)^{\phi_y} \tag{12}$$

- Taylor rule with CPI/ PPI and exchange rate

$$\text{TR-CPI-ER} \quad : \quad R_t = \overline{R}_D \left(\frac{\pi_t}{\overline{\pi}}\right)^{\phi_\pi} \left(\frac{Y_{D,t}}{Y_{D,t}^{fp}}\right)^{\phi_y} \left(\frac{X_t}{X_{t-1}}\right)^{\phi_X} \tag{13}$$

$$\text{TR-PPI-ER} \quad : \quad R_t = \overline{R}_D \left(\frac{\pi_{D,t}}{\overline{\pi}_D}\right)^{\phi_\pi} \left(\frac{Y_{D,t}}{Y_{D,t}^{fp}}\right)^{\phi_y} \left(\frac{X_t}{X_{t-1}}\right)^{\phi_X} \tag{14}$$

Monetary policy rules: Exchange rate rules + PEG

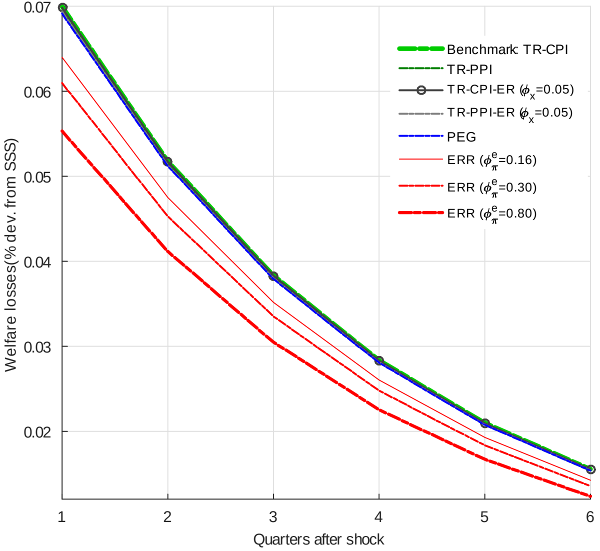
- Exchange rate rule: (see McCallum (2004) , Heipertz et al. (2017))

$$\text{ERR}: \frac{X_t}{X_{t-1}} = \left(\frac{Y_{D,t}}{Y_{D,t}^{fp}}\right)^{-\phi_y^e} \left(\frac{\pi_t}{\overline{\pi}}\right)^{-\phi_\pi^e} \tag{15}$$

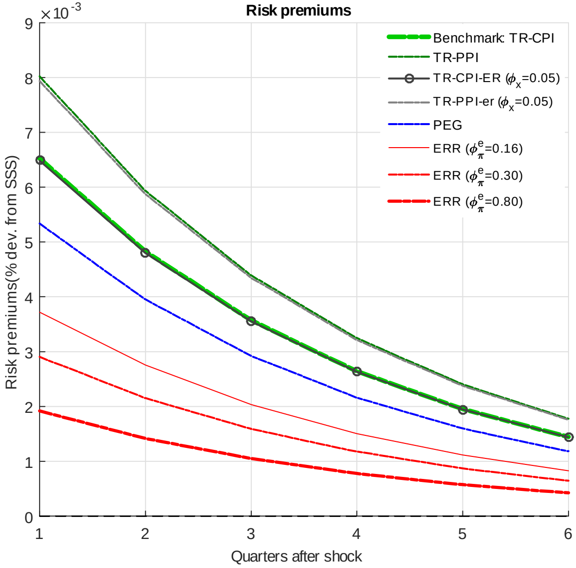
- Fixed exchange rate

$$\text{PEG}: \frac{X_t}{X_{t-1}} = 1 \tag{16}$$

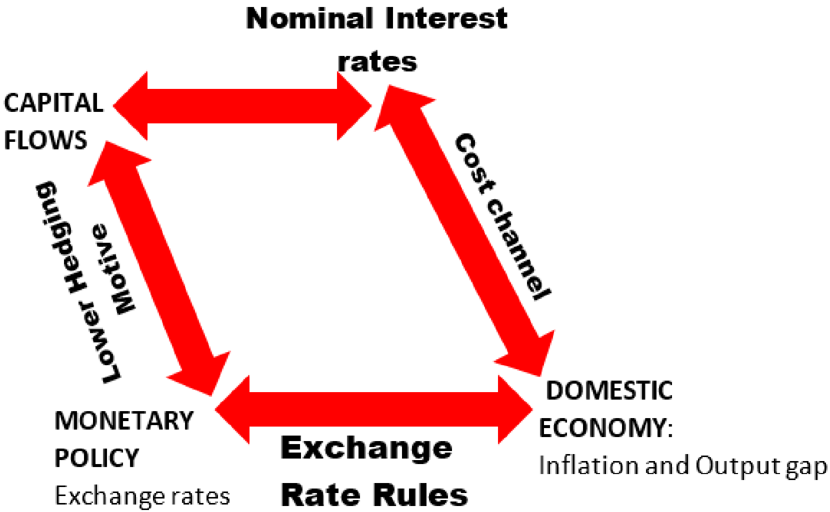
Comparing Welfare losses under different MPRs



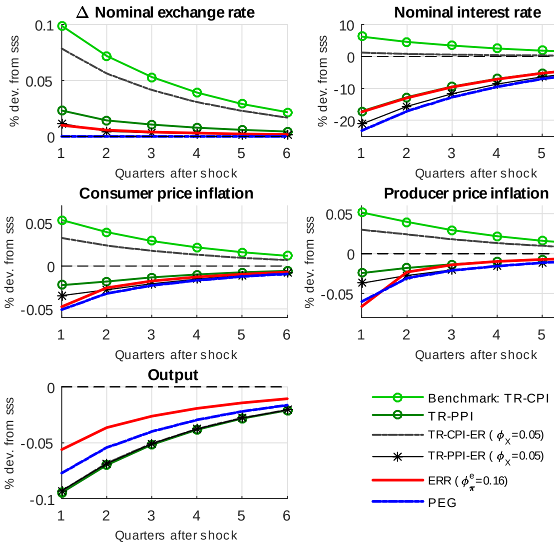
Comparing risk premiums under different MPRs



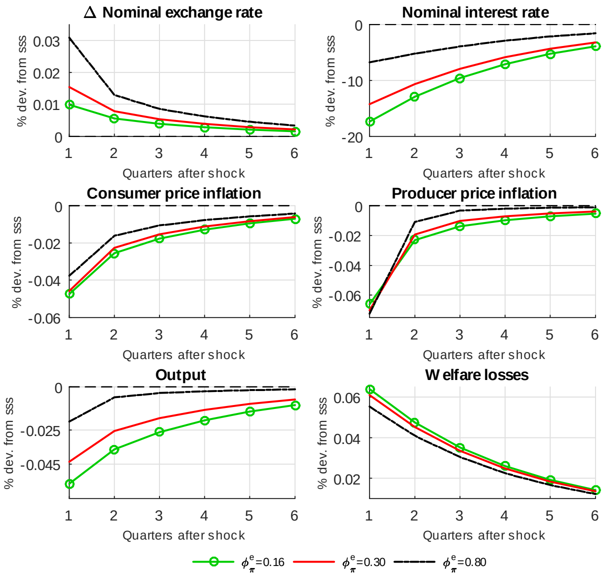
Channel with exchange rate rules



Comparing IRFs under different MPRs



Comparing IRFs under ERRs



Variable	Standard deviation $\times$ 100					
	TR-CPI	TR-PPI	TR-CPI-ER ϕ_X=0.05	TR-PPI-ER ϕ_X=0.05	ERR ϕ^e_π=0.16	PEG
	(1)	(2)	(3)	(4)	(5)	(6)
Consumption	2.484	2.483	2.484	2.483	2.476	2.484
Output	2.502	2.471	2.473	2.446	1.602	2.182
Net exports	1.450	1.451	1.450	1.451	1.455	1.450
Inflation (PPI)	3.081	3.041	2.940	2.914	1.673	1.911
Inflation (CPI)	3.057	3.064	2.923	2.943	1.695	1.933
Nominal ER	1.656	1.607	1.478	1.449	0.246	0.000
REER	0.561	0.550	0.561	0.511	0.507	0.109
Interest rates	3.689	3.711	3.584	3.614	2.645	2.877

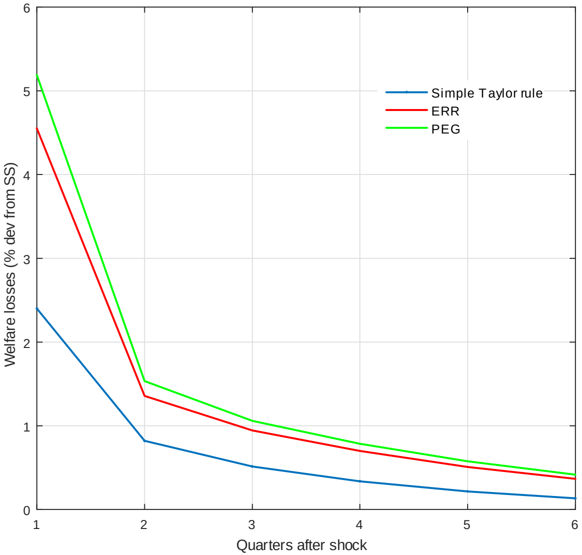
Table: Comparing second empirical moments for different monetary policy rules

Variable	Standard deviation $\times 100$								
	TR-CPI-ER : $\phi_X =$			TR-PPI-ER : $\phi_X =$			ERR : $\phi_\pi^e =$		
	0.05	0.2	0.5	0.05	0.2	0.5	0.16	0.3	0.8
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Consumption	2.484	2.484	2.484	2.483	2.483	2.483	2.476	2.472	2.465
Output	2.473	2.412	2.344	2.446	2.394	2.334	1.602	1.261	0.661
Net exports	1.450	1.450	1.450	1.451	1.451	1.451	1.455	1.458	1.465
Inflation (PPI)	2.940	2.662	2.394	2.914	2.659	2.403	1.673	1.489	1.081
Inflation (CPI)	2.923	2.660	2.406	2.943	2.696	2.446	1.695	1.510	1.089
Nominal ER	1.478	1.116	0.749	1.449	1.124	0.784	0.246	0.430	0.857
REER	0.561	0.552	0.533	0.511	0.542	0.526	0.507	0.282	0.182
Interest rates	3.584	3.385	3.202	3.614	3.425	3.243	2.645	2.476	2.095

Table: Comparing second empirical moments for varying parameters in monetary policy rules

Comparing welfare losses for MPRs

For first moment shock to domestic productivity



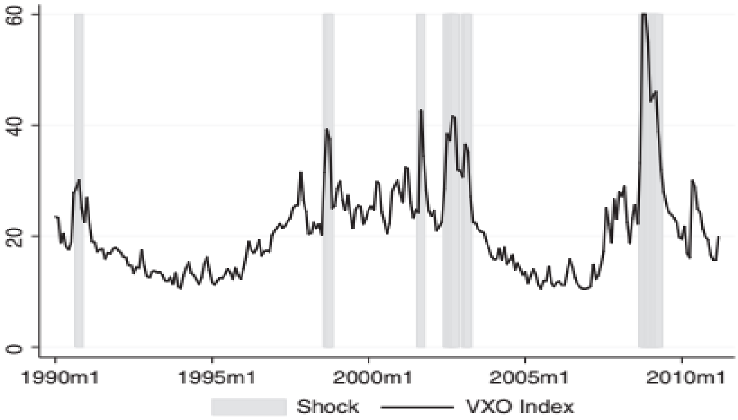
Conclusion

- Monetary policy framework in EMEs following flexible inflation targeting regime (FIT) using **interest rate rules, is not effective** when these economies are hit with global uncertainty shocks
- Global uncertainty shocks affect **real economy in EMEs adversely**
- **Depreciating exchange rates** can amplify the adverse effects
- Monetary policy fails to stabilize exchange rates and the economy
    - Policymakers observe **trade-off** in inflation and output gap stabilization
    - **UIP fails** and risk premiums are positive with interest rate rules
- ERR gives 20% less welfare losses than simple Taylor rules
    - **Lower hedging motive** and thus lower risk premiums with ERRs
- The second order moments from the model show that the **variability** of nominal exchange rates, output and CPI is **reduced** by 85%, 36% and 45% respectively
- Finally, this paper proposes a **dual instrument (interest rate, exchange rate) approach** by the central bank within **flexible inflation targeting (FIT) regime**

Thank you.

Measure of global uncertainty shocks

VXO series, CBOE S&P 100 Volatility Index



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Jorda Local Projections

- Oscar Jorda (2005) , Moller and Wolf (2019)
- Local projection regression

$$\begin{aligned} Y_{i,t+h} - Y_{i,t-1} &= \theta_{i,h} vxo_t + control\ \ variables + \zeta_{t+h} \\ \text{where } h &= 0, 1, ...6 \end{aligned}$$

$\theta_{i,h}$ is the estimate of the IRF at horizon h. We estimate a simple regression for each horizon h.

<<**back**>>

Some price relations

$$\text{Terms of trade, } T_t = \frac{P_{F,t}}{P_{D,t}} \tag{17}$$

$$\pi_t^* = \pi_{F,t}^* \frac{\left[\mu_F (T_t)^{\xi_F - 1} + (1-\mu_F)\right]^{\left(\frac{1}{1-\xi_F}\right)}}{\left[\mu_F (T_{t-1})^{\xi_F - 1} + (1-\mu_F)\right]^{\left(\frac{1}{1-\xi_F}\right)}} \tag{18}$$

$$T_{F,t} = \left[\frac{1-\mu_D (T_{D,t})^{(1-\xi_D)}}{1-\mu_D}\right]^{\left(\frac{1}{1-\xi_D}\right)} \tag{19}$$

$$\text{Real exchange rate,} \tag{20}$$

$$Q_t = \frac{\left[\mu_F + (1-\mu_F)(T_t)^{1-\xi_F}\right]^{\left(\frac{1}{1-\xi_F}\right)}}{\left[\mu_D + (1-\mu_D)(T_t)^{1-\xi_D}\right]^{\left(\frac{1}{1-\xi_D}\right)}}, \tag{21}$$

Special cases

- **Autarky:** $\chi = 0$, when no trade of goods is possible following conditions hold,
  - $\mu_D = 1$, $\mu_F = 0$, $C_{F,t} = 0$ and $C^*_{D,t} = 0$, implying $C_{D,t} = C_t$ and $C^*_{F,t} = C^*_t$
  - $T_{D,t} = 1$, $T_{F,t} = T_t$ (Terms of trade) $= Q_t$ (Real exchange rate)
- **Small open economy,** $n \longrightarrow 0$ following Benigno & De Paoli (2010) ,
  - $\mu_D = 1 - \chi$ and $\mu_F = 0$, also $Q_t = T_{F,t}$

Uncovered Interest rate parity (UIP) condition

- UIP condition,

$$r_{D,t} - r_{F,t} = E_t \left\{ \triangle e_{t+1} \right\} \tag{22}$$

where, $r_{D,t} = \ln R_{D,t}$ and $R_{D,t} = \overline{R}_D \left(\frac{\pi_t}{\overline{\pi}}\right)^{\phi_\pi} \left(\frac{Y_{D,t}}{Y^{fp}_{D,t}}\right)^{\phi_y}$,

- Risk premium

$$rp_t = r_{D,t} - \left(r_{F,t} - E_t \left\{ \triangle e_{t+1} \right\}\right) \tag{23}$$

  - When UIP holds at t, $rp_t = 0$
  - When UIP breaks down, $rp_t \neq 0$

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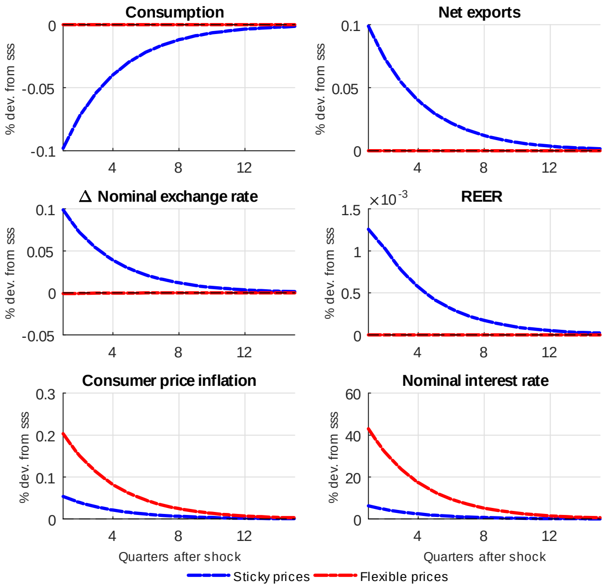

Consumption
Net exports
Δ Nominal exchange rate
REER
Consumer price inflation
Nominal interest rate
% dev. from sss
Quarters after shock
×10^-3
Sticky prices
Flexible prices

<<back>>

Households (Domestic)

The domestic households maximize following utility function (see Fernandez-Villaverde et al. (2011))

$$\max U(C_t, H_{D,t}) = \frac{(C_t)}{1-\nu_D}^{1-\nu_D} - \omega_D \frac{(H_{D,t})}{1+\eta_D}^{1+\eta_D} \tag{24}$$

subject to constraint,

$$W_{D,t} H_{D,t} + profit_t = P_t C_t + B_t - E_t \left\{ B_{t+1} M_{t,t+1} \right\} \tag{25}$$

where $M_{t,t+1}$ is the stochastic discount factor. In above equation, C_t is the aggregate basket of goods a domestic household consumes,

$$C_t = \left[(\mu_D)^{1/\xi_D} (C_{D,t})^{\frac{\xi_D - 1}{\xi_D}} + (1-\mu_D)^{1/\xi_D} (C_{F,t})^{\frac{\xi_D - 1}{\xi_D}} \right]^{\frac{\xi_D}{\xi_D - 1}} \tag{26}$$

<<**back**>>

First order conditions-Domestic households

- First optimization:

$$\lambda_{D,t} = (C_t)^{-(\nu_D)} \tag{27}$$

$$\lambda_{D,t} = \omega^D \frac{(H_{D,t})^{\eta_D}}{w_{D,t} T_{D,t}} \tag{28}$$

$$\lambda_{D,t} E_t \{\pi_{t+1}\} = \beta (1 + R_{D,t}) E_t \{\lambda_{D,t+1}\} \tag{29}$$

where $T_{D,t} = \frac{P_{D,t}}{P_t}$.

- Second optimization (demand function): $C_{D,t}$ and $C_{F,t}$,

$$C_{D,t} = \mu_D \left(\frac{P_{D,t}}{P_t}\right)^{-\xi_D} C_t \tag{30}$$

$$C_{F,t} = (1 - \mu_D) \left(\frac{P_{F,t}}{P_t}\right)^{-\xi_D} C_t \tag{31}$$

where aggregate prices are,

$$P_t = \left[\mu_D (P_{D,t})^{1-\xi_D} + (1 - \mu_F) (P_{F,t})^{1-\xi_D}\right]^{\frac{1}{1-\xi_D}}$$

<<**back**>>

Households (Foreign)

Similarly, the foreign households maximize following utility function

$$\max U(C_t^*, H_{F,t}) = \frac{\Upsilon_{F,t}\left(C_t^*\right)}{1-\nu_F}^{1-\nu_F} - \omega_F \frac{\left(H_{F,t}\right)}{1+\eta_F}^{1+\eta_F} \quad (32)$$

subject to constraint,

$$W_{F,t}H_{F,t} + profit_t^* = P_t^* C_t^* + B_t^* - E_t\left\{B_{t+1}^* M_{t,t+1}^*\right\} \quad (33)$$

where $M_{t,t+1}^*$ is the stochastic discount factor. In above equation, C_t^* is the aggregate basket of goods a foreign household consumes,

$$C_t^* = \left[\left(\mu_F\right)^{1/\xi_F}\left(C_{D,t}^*\right)^{\frac{\xi_F-1}{\xi_F}} + \left(1-\mu_F\right)^{1/\xi_F}\left(C_{F,t}^*\right)^{\frac{\xi_F-1}{\xi_F}}\right]^{\frac{\xi_F}{\xi_F-1}} \quad (34)$$

Following Benigno (2010),

$$1-\mu_D = (1-n)\chi \quad ; \quad \mu_F = n\chi \quad (35)$$

where $\chi \in [0,1]$ is the parameter capturing degree of openness. $\chi = 0$ means autarky and $\chi = 1$ means free trade.

<<**back**>>

First order conditions-Foreign households

- First optimization:

$$\lambda_{F,t} = \Upsilon_{F,t} \left(C_t^*\right)^{-(\nu_F)} \quad (36)$$

$$\lambda_{F,t} = \omega^F \frac{Q_t \left(H_{F,t}\right)^{\eta_F}}{w_{F,t} T_{F,t}} \quad (37)$$

$$\lambda_{F,t} E_t \left\{\pi_{t+1}^*\right\} = \beta \left(1 + R_{F,t}\right) E_t \left\{\lambda_{F,t+1}\right\} \quad (38)$$

where $T_{F,t} = \frac{P_{F,t}}{P_t}$ and $Q_t = \frac{X_t P_t^*}{P_t}$ (Q_t is real exchange rate and X_t is the nominal exchange rate)

- Second optimization (demand functions): $C_{D,t}^*$ and $C_{F,t}^*$ are,

$$C_{D,t}^* = \mu_F \left(\frac{P_{D,t}^*}{P_t^*}\right)^{-\xi_F} C_t^* : \; C_{F,t}^* = (1 - \mu_F) \left(\frac{P_{F,t}^*}{P_t^*}\right)^{-\xi_F} C_t^* \quad (39)$$

where aggregate prices are,

$$P_t^* = \left[\mu_F \left(P_{D,t}^*\right)^{1-\xi_F} + (1 - \mu_F) \left(P_{F,t}^*\right)^{1-\xi_F}\right]^{\frac{1}{1-\xi_F}}$$

<<**back**>>

Complete asset market condition

Complete asset market condition, here it is assumed that there are state-contingent bonds denominated in home currency. Both the domestic as well as the foreign households hold these bonds denominated in home currency (see Chari et. al. (2001)).

$$\frac{U'_{C,t+1}}{U'_{C,t}}\frac{P_t}{P_{t+1}} = \frac{U'_{C^*,t+1}}{U'_{C^*,t}}\frac{X_t}{X_{t+1}}\frac{P^*_t}{P^*_{t+1}}$$

Writing recursively it we get,

$$\begin{aligned} Q_{t+1} &= X_0 \frac{U'_{C,0}}{U'_{C^*,0}}\frac{P^*_0}{P_0}\frac{U'_{C^*,t+1}}{U'_{C,t+1}} \\ Q_{t+1} &= \kappa \frac{\Upsilon_{F,t+1}\left(C^*_{t+1}\right)^{-\nu_F}}{\left(C_{t+1}\right)^{-\nu_D}} \end{aligned}$$

Note, $Q_{t+1} = \frac{P^*_{t+1}X_{t+1}}{P_{t+1}}$ (implying that the real exchange rate depends on the ratios of marginal utilities) and κ is the initial condition, the ratio of marginal utilities at the beginning.

<<**back**>>

Production (Domestic households)

Assume each households produce all the varieties i of good D

$$Y_{D,t}(i) = A_{D,t} H_{D,t}(i) \tag{40}$$

where,

$$A_{D,t} = (1 - \rho_D)\,\overline{A}_D + \rho_D A_{D,t-1} + \epsilon_{D,t} \tag{41}$$

$$\max_{\underline{P}_{D,t}(i)} \sum_{k=0}^{\infty} \alpha_D^k M_{t,t+k} \left(\underline{P}_{D,t}(i) Y_{D,t+k}(i) - MC_{D,t+k} Y_{D,t+k}(i) \right) \tag{42}$$

$$\text{where } Y_{D,t+k}(i) = \left(\frac{\underline{P}_{D,t}(i)}{P_{D,t+k}} \right)^{-\sigma} Y_{D,t+k} \tag{43}$$

<<**back**>>

Production (Foreign households)

$$Y_{F,t}(i) = A_{F,t} H_{F,t}(i) \tag{44}$$

$$\text{where, } A_{F,t} = (1-\rho_F)\,\overline{A}_F + \rho_F A_{F,t-1} + u_{F,t-1}\epsilon_{F,t} \tag{45}$$

$$u_{F,t} = (1-\rho_{\sigma F})\,\overline{u}_F + \rho_{\sigma_F} u_{F,t-1} + \varkappa_F \zeta_{F,t} \tag{46}$$

$$\max_{\underline{P}_{D,t}(i)} \sum_{k=0}^{\infty} \alpha_F M^*_{t,t+k} \left(\underline{P}^*_{F,t}(i) Y_{F,t+k}(i) - MC_{F,t+k} Y_{F,t+k}(i) \right) \tag{47}$$

$$\text{where } Y_{F,t+k}(i) = \left(\frac{\underline{P}^*_{F,t}(i)}{P^*_{F,t+k}} \right)^{-\sigma} Y_{F,t+k} \tag{48}$$

<<**back**>>

Recursive form price setting I

For domestic firms,

$$\underline{\pi}_{D,t} = \frac{\sigma}{(\sigma-1)} \pi_{D,t} \frac{X_{D,t}}{Z_{D,t}} \tag{49}$$

$$X_{D,t} = \lambda_t Y_{D,t} T_{D,t} mc_{D,t} + \alpha_D \beta \left(\pi_{D,t+1}\right)^{\sigma} X_{D,t+1} \tag{50}$$

$$Z_{D,t} = \lambda_t Y_{D,t} T_{D,t} + \alpha_D \beta \left(\pi_{D,t+1}\right)^{\sigma+1} Z_{D,t+1} \tag{51}$$

$$\pi_{D,t} = \left[\alpha_D + (1-\alpha_D)\left(\underline{\pi}_{D,t}\right)^{(1-\sigma)}\right]^{\left(\frac{1}{1-\sigma}\right)} \tag{52}$$

<<**back**>>

Recursive form price setting II

For foreign firms,

$$\underline{\pi}_{F,t} = \frac{\sigma}{(\sigma-1)} \pi^*_{F,t} \frac{X_{F,t}}{Z_{F,t}} \tag{53}$$

$$X_{F,t} = \lambda^*_t Y_{F,t} \frac{T_{F,t}}{Q_t} mc_{F,t} + \alpha_F \beta \left(\pi^*_{F,t+1}\right)^{\sigma} X_{F,t+1} \tag{54}$$

$$Z_{F,t} = \lambda^*_t Y_{F,t} \frac{T_{F,t}}{Q_t} + \alpha_F \beta \left(\pi^*_{F,t+1}\right)^{\sigma+1} Z_{F,t+1} \tag{55}$$

$$\pi^*_{F,t} = \left[\alpha_F + (1-\alpha_F)\left(\underline{\pi}_{F,t}\right)^{(1-\sigma)}\right]^{\left(\frac{1}{1-\sigma}\right)} \tag{56}$$

<<**back**>>