Uncertainty shocks and monetary policy rules in a small open economy*

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This version: 24 November, 2019

Abstract

This paper explores the role of exchange rates (both nominal and real) and monetary policy in amplifying/ stabilizing the real effects of global uncertainty shocks in a small open economy. Post global financial crisis (GFC) of 2008-2009, there has been a surge in the macroeconomics literature on aggregate uncertainty. The recent literature has recognized that global uncertainty shocks reduces private consumption and investment severely in emerging market economies (EMEs). Using data we reproduce stylized facts showing significant movements in exchange rates when EMEs are hit with a global uncertainty shock. We find that interest rate rules are ineffective in stabilizing the exchange rates as well as the domestic economy. With interest rate rules there arises trade-off in inflation and output stabilization. Using a small open economy NK-DSGE model, we show that exchange rate rules (ERRs) reduce welfare losses significantly compared to interest rate rules. ERRs also reduce variability of exchange rates, inflation and output remarkably. This occurs because exchange rate rules generate a lower risk premium than interest rate rules.

Keywords: Uncertainty shocks; monetary policy; interest rate rules; exchange rate rules; uncovered interest rate parity (UIP), risk premiums

JEL Codes: E31, E42, E43, E52, E58, F41

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*I thank Susanto Basu, Chetan Ghate, Morten O. Ravn, seminar participants at the Delhi School of Economics-CDE Seminar (September, 2019), Reserve Bank of India, Mumbai (August, 2019), Conference on Economic Theory and Policy (Ambedkar University, Delhi-March, 2019), 14^{th} Annual Conference on Economic Growth and Development (Indian Statistical Institute, Delhi- December, 2018) and 7^{th} Delhi Macroeconimcs Workshop (ISI-Delhi, October, 2018) for their extremely helpful comments. This version is a part of conference proceedings, 12^{th} International Research Conference (December 2019), Central Bank of Sri Lanka, Colombo.

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1 Introduction

There has been a surge in the macroeconomics literature on aggregate uncertainty post global financial crisis (GFC) of 2008-2009. The role of uncertainty shocks in slowing down the real economy and driving business cycles is getting well recognized in the literature. Using a reduced form VAR, Bloom (2009) estimates that global uncertainty shocks reduce U.S. industrial production by 1 per cent. Gourio et al. (2013) show a similar result for G7 countries. Bloom et al. (2018) show that uncertainty rises sharply during recessions and it reduces GDP by 2.5 per cent. Basu and Bundick (2017), using a new-Keynesian DSGE model, show that demand-determined output is the key mechanism for generating comovements observed in the data as a response to uncertainty fluctuations in US. Ravn and Sterk (2017) exposits the role of job uncertainty in amplifying adverse effect of GFC, using a model featuring labour market with matching frictions and inflexible wages.

While the literature on the impact of uncertainty shocks on emerging market economies macroeconomic outcomes is less developed, Fernández-Villaverde et al. (2011) show adverse real effects of an increase in real interest rate volatility (uncertainty in real interest rates) on output, consumption and investment. Cespedes and Swallow (2013) argue that global uncertainty shocks not only impact consumption and investment demand in advance economies (AEs) but also in emerging market economies (EMEs). Their estimation shows that the impact of such shocks on EMEs is much more severe than AEs. Moreover emerging markets take much longer time to recover due to credit constraints present in these economies. Chatterjee (2018) discusses the role of trade openness in explaining a disproportionately larger real effects of uncertainty shocks on EMEs compared to AEs, especially during a recessionary period.[1] To the best of our knowledge, the role of monetary policy in offsetting the adverse effects of global uncertainty shock in an EME and its link with the exchange rates is not explored in the literature. This paper addresses this gap.

We examine the role of exchange rates and monetary policy rules in transmitting the effect of uncertainty shocks in a small open economy (EME). We observe that exchange rate movements are significant in EMEs vis-a-vis AEs, when global uncertainty rises. To be specific, the data distinctly shows that exchange rates, both nominal as well real, depreciate strongly during periods of high global uncertainty. This happens because capital moves out of EMEs as an immediate response to higher global uncertainty. Typically, when global risks are high investors move their risky asset portfolio into safer assets like US treasury bill and that's why EMEs experience a net portfolio outflow. This is consistent with the

[1] In the trade literature, Magrini et al. (2018) also show that there are *ex-ante* risks due to trade exposure in Vietnam and these risks affect consumption growth. An *ex-ante* shock in the trade literature is closely associated with an uncertainty shock in the macroeconomics literature.

flight-to-safety hypothesis. Fratzscher (2012) finds strong empirical evidence showing that during the time of global financial crises when global risks (same as high global uncertainty) were high, emerging markets economies showed a significant net portfolio outflow. They also argue that global risks have been a key 'push factor' driving capital flows from EMEs.[2] A depreciating currency in an EME does not lead to an expansion of output, due to expenditure switching via trade channel, because increasing global uncertainty contracts world output too. Instead, the depreciating currency is contractionary here. This follows from the existing literature which has emphasized on the contractionary effect of a depreciating currency (see Agenor and Montiel (1999), Cook (2004) and Korinek (2018)).[3]

Further, due to a currency depreciation, domestic consumer prices increase due to an increase in the import prices in EMEs. As a response to increasing inflationary expectations, the central bank in EMEs increases the nominal interest rate.[4] Other possible reasons for increasing interest rates could be to put a check on the outflow of capital. Our stylized facts show that emerging markets grapple with a fall in private consumption and investment during episodes of increasing uncertainty, as shown in the recent literature described above. An increase in the nominal interest rate can further destabilize a contracting small open economy by reinforcing the adverse real effects of uncertainty shocks. A monetary policy (implemented using Taylor type interest rate rules) is thus faced with a strong trade-offs in inflation and output stabilization.

Benigno et al. (2012) explore a link between uncertainty and exchange rates and show that the time variations in uncertainty is an important source of fluctuation in exchange rates. They also argue that when an uncertainty shock hits an economy, fluctuations in exchange rates are guided by a *hedging motive* and uncovered interest rate parity (UIP) does not hold, generating time varying risk premiums.[5] As shown in the left chart of

[2]Fratzscher (2012) also argues that country specific features including structural issues only affect the cross-country heterogeneity effects of common global shocks emanating from advanced economies. In other words, country specific features have been important determinants of 'pull factors' as a driver of capital flows.

[3]This happens because most of the external debt held by firms in emerging market economies is denominated in dominant currencies such as the US dollar. A depreciation (both nominal and real) of the currency would worsens the balance sheets of firms. With worsening balance sheets, foreign investors pull out their funds and firms hit a borrowing/ credit constraint. This can further make things worse if the currency depreciates further with capital moving out of the country. This point has also been emphasized in Cespedes and Swallow (2013) to explain a longer recovery time period for a fall in investment in emerging markets when hit with a global uncertainty shock.

[4]All the countries considered for the empirical analysis are inflation targeters and monetary policy follows an interest rate rule as an instrument to stabilize the economy. The results are based on using short-term interest rates as a proxy to policy rates.

[5]When an uncertainty shock hits the economy, capital looks out for a safer currency which leads to fluctuations in the exchange rates. See Menkhoff et al. (2012) for the link between deviation from the UIP and time varying risk premiums. Backus et al. (2010) have also shown that Taylor rules are associated with

Figure 1 below, when an economy deviates from UIP, the link between nominal interest rates (monetary policy instrument) and the nominal exchange rate breaks down. Thus any attempt to use an interest rate rule to stabilize the economy through the nominal exchange rate is unsuccessful.[6] To summarize, a depreciating domestic currency in EMEs aggravates the contractionary real effects of an increase in global uncertainty and leads to increase in inflation. Thus, in a small open economy (EME), stabilization of exchange rates is imperative to offset the adverse effects of increasing global uncertainty, and interest rate rules fail to do so.

Finally, we build a small open economy new-Keynesian DSGE model with an uncertainty shock to the world demand and examine the response of real macroeconomic variables under a variety of monetary policy rules. The purpose of this exercise is to look for a monetary policy rule which minimizes the welfare losses since interest rate rules are ineffective here. Singh and Subramanian (2008) have shown that an essential feature that determines the optimal choice of monetary policy instrument is the nature of shocks affecting the economy. Following this we consider response of the economy under an alternate monetary policy instrument.

A most obvious alternate policy to be considered here is a fixed exchange rate regime. Cook (2004) has argued that a fixed exchange rate regime (PEG) offers greater stability than an interest rate rule (or flexible exchange rate regime) when currency depreciation destabilizes the business cycle. We show that a fixed exchange rate regime does only slightly better than an interest rate rule, in terms of welfare losses, as it brings high variability to other nominal variables in the economy like consumer price inflation (CPI), which adjusts more. Although fixed exchange rate does bring a greater stability to macroeconomic variables then interest rate rules in the long run. This is different from Corsetti et al. (2017), who argues that flexible exchange rate regimes perform better then a fixed exchange rate regime when the domestic economy faces a negative demand shock (level shock) from abroad. This happens because a flexible exchange rate regime stabilizes the demand via depreciation of the domestic currency which a PEG regime does not allow for. This is in contrast to the results we get in this paper for a second moment shock to the demand abroad. The difference in the results is primarily driven by non-zero risk premiums generated for second moment shocks as UIP does not hold. Since flexible exchange rate regimes are associated with higher risk premiums than PEG, the latter performs better under high global uncertainty.[7]

high risk premiums.

[6] This point is also emphasized in Heiperzt et al. (2017).

[7] In Corsetti et al. (2017) a depreciation of domestic currency stabilizes demand. This paper looks at two other channels of depreciation which can affect an economy adversely in the baseline case of flexible exchange rates. Firstly when the domestic currency depreciates this increases inflation in the domestic country. Assuming the domestic country is an inflation targeter and is not at the zero lower bound (ZLB)

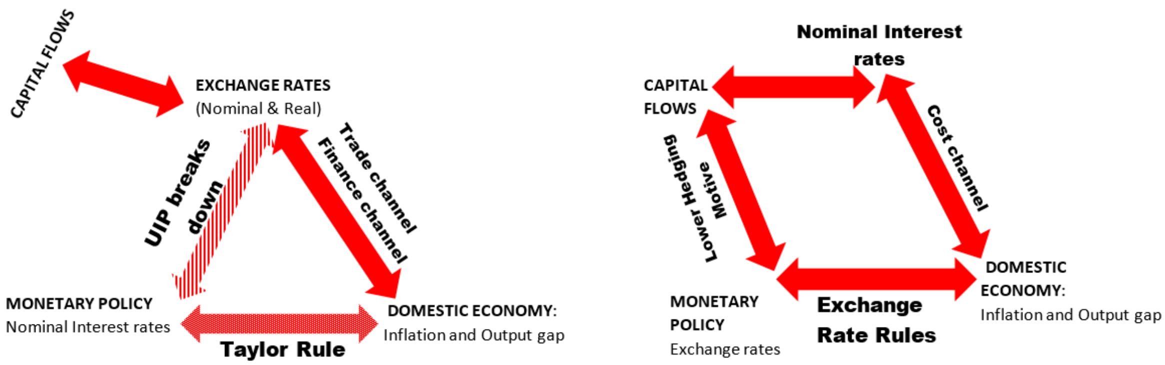

Figure 1: In presense of global uncertainty shock (a) Monetary policy using nominal interest rates as instrument (left); (b) Monetary policy using nominal exchange rates as instrument (right)

We find that a monetary policy rule that gives the lowest welfare losses when a small open economy is hit with a global uncertainty shock is an exchange rate rule. When a monetary policy uses the exchange rate as an instrument, the exchange rate follows a rule and is guided by key fundamentals governing the domestic economy, like inflation and output. Since the exchange rate follows a rule and does not float freely, the hedging motive mentioned above is weakened. Thus, nominal exchange rates are stabilized and welfare losses are reduced significantly. Heiperzt et al. (2017) also show that exchange rate rules outperform interest rate rules in a small open economy for shocks to the first moment. The risk premiums associated with exchange rate rules are also lower, due to a lower hedging motive. The right chart in Figure 1 shows how a link between monetary policy, exchange rates and key real macro variables like inflation and output is restored when exchange rate rules are followed. Exchange rate rules not only reduce welfare losses but also reduce the variability of nominal exchange rates, output and inflation remarkably.

1.1 Empirical evidence

We use a local projection method proposed by Jorda (2005) to look for the effects of global uncertainty shocks on a wide variety of variables for both AEs and EMEs.[8] To capture global uncertainty we use the VXO index series as proxied in Bloom (2009) and

constraint (the EMEs considered here are not at the ZLB constraint), monetary policy increases the policy rate which has a negative affect on domestic demand. The second channel is the fall in the investment demand and drying up of the working capital in domestic firms due to depreciation, as discussed in the Introduction to this paper.

[8]We use STATA 13 to do our empirical analysis.

Cespedes and Swallow (2013). For the VXO series, we use the CBOE S&P 100 Volatility Index's daily series accessed from the Federal Reserve Bank of St. Louis database from 1996 to 2018.[9] For further analysis, we create a quarterly panel dataset for 12 economies from 1996:Q1 to 2018:Q4. We consider six AEs (US, UK, Canada, Japan, Australia and South Korea) and six EMEs (Brazil, Indonesia, India, Mexico, Russia and South Africa).[10] The primary source for most of the macroeconomic series is the quarterly national accounts data compiled by the Organization for Economic Cooperation and Development (OECD). The macroeconomic series we consider are: real GDP, real consumption, real investment, trade balance, nominal exchange rate, real effective exchange rate and short term interest rates.[11] We get the country wise series on net portfolio investment from the International Monetary Fund's International Financial Statistics (IFS).[12] A detailed data description is provided in the Data Appendix 4.1.

We estimate panel local projections for horizon, $h = 0, 1, 2, 3, 4, 5, 6$ as described below,

$$Y_{i,t+h} - Y_{i,t-1} = \alpha_{i,h} + \theta_{i,h} vxo_t + \sum_q \beta^q_{i,h} X_{i,t-q} + \varsigma_{i,t+h}$$

Here, for country i, $\varsigma_{i,t+h}$ is the projection residual, $\alpha_{i,h}$, $\theta_{i,h}$ and $\beta^q_{i,h}$ are the projection coefficients. The vector Y_t is a set of response variables including real GDP, real consumption, real investment, trade balance, nominal exchange rate, real effective exchange rate, net portfolio investment, inflation and short term interest rates. The vector X_t is a set of control variables including lagged dependent variables and policy variables. The local projection impulse response of Y_t with respect to vxo_t at horizon h for country i is given by $\{\theta_{i,h}\}$ for $h \succeq 0$. The lag of control variables, q, is set to upto four periods. We control for the country fixed effects in our panel regression.

Figures 2, 3, 4 and 5 show local projection responses using OLS for six quarters after the shock to global uncertainty.[13] We plot impulse response functions with 90 per cent and 80 per cent confidence bands. Figures 2a and 2b show the response of GDP and private consumption to an increase in global uncertainty. GDP and private consumption decrease

[9] Chicago Board Options Exchange, CBOE S&P 100 Volatility Index: VXO [VXOCLS], is retrieved from FRED, Federal Reserve Bank of St. Louis;
https://fred.stlouisfed.org/series/VXOCLS, January 10, 2019.

[10] The choice of EMEs depends on availability of data. For AEs we choose six large economies. All the series are seasonally adjusted using X-12-ARIMA routine provided by the U.S. Census Bureau, and detrended using the Hodrick–Prescott filter.

[11] Data is accessed in January, 2019 from https://stats.oecd.org/#

[12] Data is accessed in January, 2018 from http://data.imf.org/?sk=388DFA60-1D26-4ADE-B505-A05A558D9A42&sId=1479329334655

[13] The values on the y-axis show a percentage change from the trend. All the graphs are local projection responses with VXO impulse for EMEs (on the left) and AEs (on the right) using OLS.

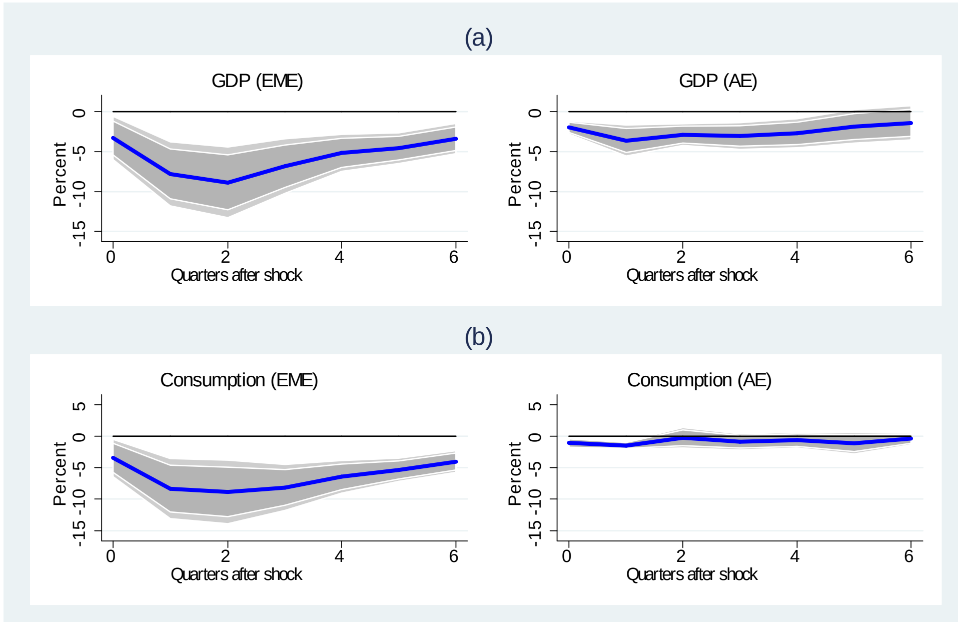

Figure 2: Local projection responses for (a) GDP; (b) Consumption with VXO impulse

in both EMEs and AEs, but the decrease is much higher (upto 10 per cent from the trend) in EMEs compared to AEs. This result is consistent with the empirical facts observed in Cespedes and Swallow (2013). Figure 3a shows capital (net portfolio investment) outflows from EMEs immediately after the shock.[14] About 30 per cent of the capital, as a deviation from the trend, in EMEs flows out when global uncertainty increases. AEs do not experience much change in there capital movement as compared to EMEs. The literature has identified high global risk as one of the most important push factor in determining capital outflows from EMEs (see Fratzscher (2012) , Forbes and Warnock (2012)). As a result of capital outflows, the domestic currency (nominal exchange rate) in EMEs depreciates up to 10 per cent in two quarters after the shock (see Figure 4a). The real effective exchange rate (REER) also depreciates and remains depreciated up to four quarters after the shock in EMEs (see Figure 4b).[15] No significant exchange rate movements are observed in AEs as compared to EMEs. A sustained real or nominal depreciation of the currency amplifies the reduction in real activity and brings instability to the business cycle in EMEs as argued in Korinek (2018) and Cook (2004).

The primary reason emphasized in papers mentioned above is the presence of large external debt denominated in foreign currency in EMEs. When the currency depreciates,

[14]The series used here is net portfolio investment to GDP ratio. This is done to normalize the series before HP filtering.

[15]Since the REER is measure in terms of US dollars, any decrease here indicates real effective depreciation.

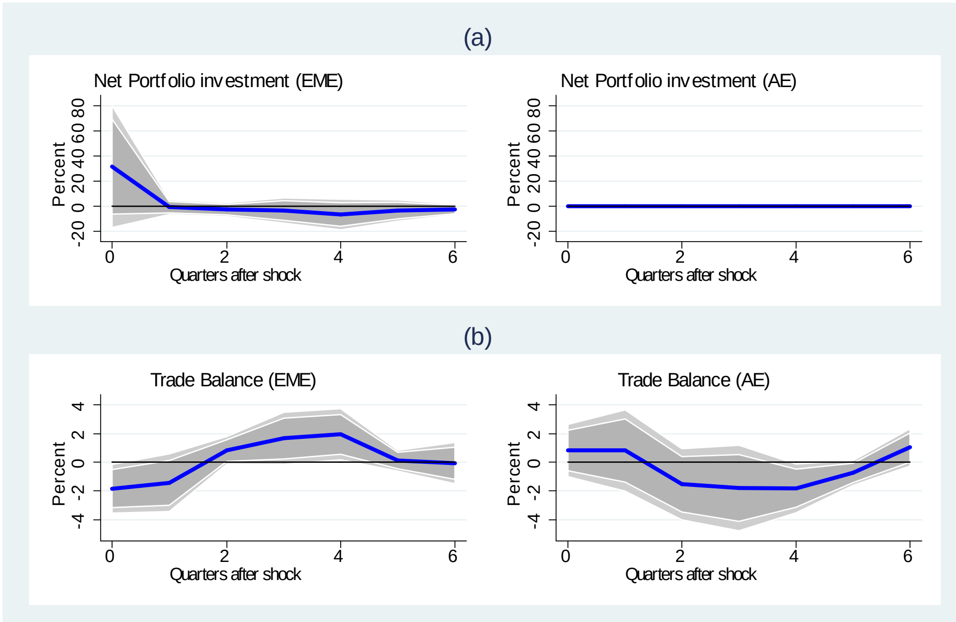

Figure 3: Local projection responses for (a) Net portfolio investment; (b) Trade balance with VXO impulse

balance sheets of firms in EMEs worsens, and this leads to foreign investors pulling out their investments. EMEs also experience a trade deficit in the first two quarters after a shock before the trade balance starts improving due to currency depreciation (see Figure 3a).[16] Initially, the trade balance falls due to a fall in foreign demand for domestic goods (exports) as consumption in the foreign economy is also low due to higher global uncertainty. Currency depreciation in EMEs leads to a rise in inflation due to a rise in the import good price (see Figure 5a). AEs on the other hand, experience a fall in consumer prices as their aggregate demand falls (see Figure 5b). All countries considered for the present analysis have an inflation targeting mandate with interest rates as a monetary policy instrument. Interest rates thus fall in AEs as a policy response to a contracting economy and deflation (Figure 5a).[17] For EMEs, a contracting economy would suggest reduction in the interest rates (expansionary monetary policy), and an increase in consumer prices with exchange rate depreciation would suggest an increase in the interest rates (contractionary monetary policy). Policymakers in EMEs are thus faced with the trade-off between inflation and the

[16]Series used here is the trade balance to GDP ratio. This is done to normalize the series before HP filtering.

[17]Impulse responses for real GDP, real consumption, the trade balance,the real effective exchange rate, inflation and short term interest rates are strongly significant at the 90 per cent confidence level. On the other hand, net portfolio investment and the nominal exchange rate are significant nearly at the 80 per cent confidence level. We suspect this happens due to the averaging out effect in the movement of portfolio investments and exchange rates over a quarter.

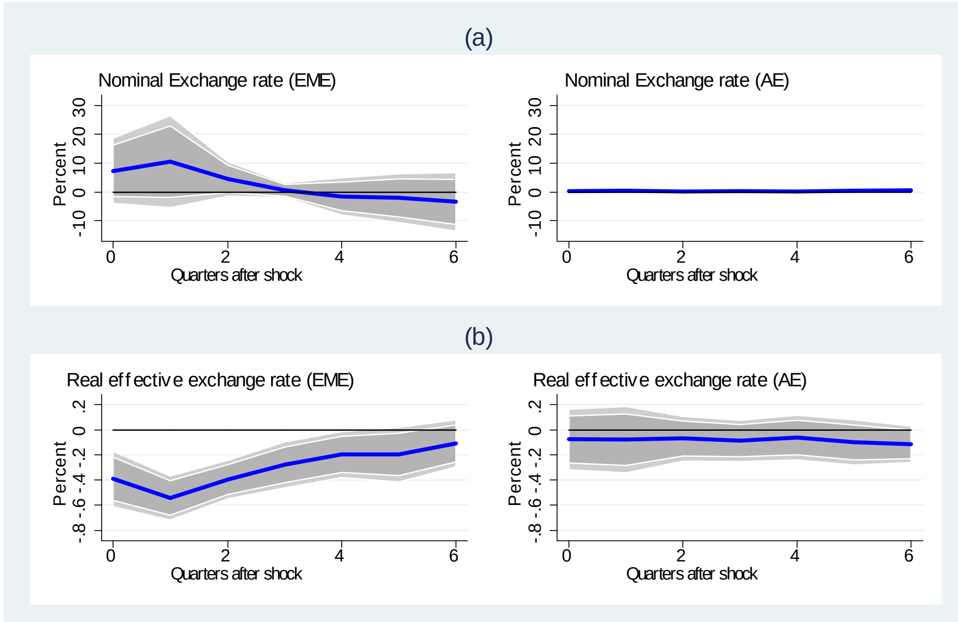

Figure 4: Local projection responses for (a) Nominal exchange rate; (b) Real effective exchange rate with VXO impulse

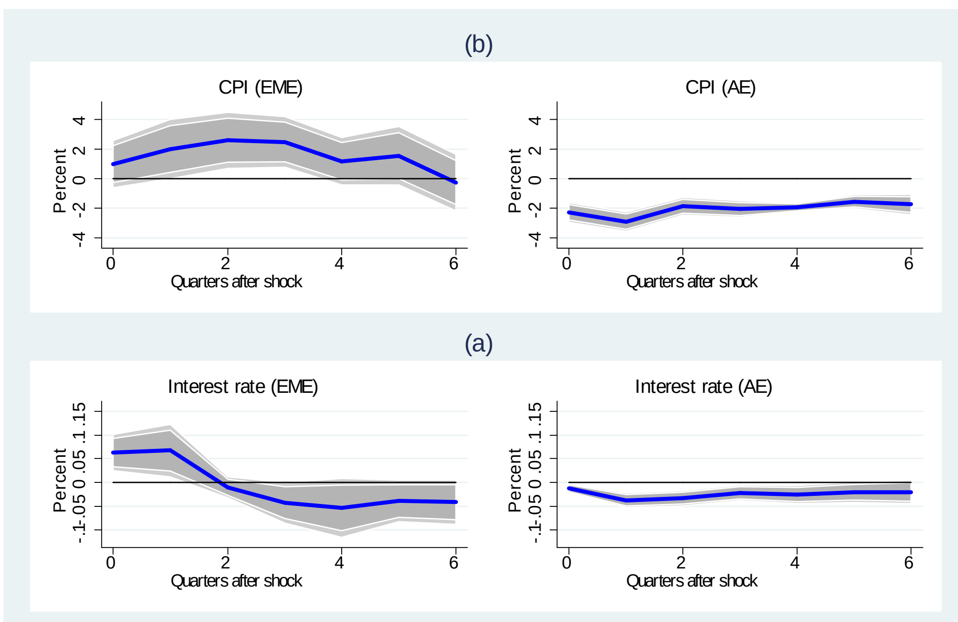

Figure 5: Local projection responses for (a) Consumer price index; (b) Nominal interest rates with VXO impulse

output stabilization. Moreover, as the central bank gives more weight to stabilizing inflation in a Taylor type interest rate rule, we observe an increase in the interest rates in EMEs (see Figure 5a).

1.1.1 Summary of stylized facts

The empirical observations explained above can be summarized as following stylized facts:

Fact 1: An increase in global uncertainty reduces real activity in both AEs as well as EMEs. EMEs experience a greater fall in GDP and private consumption compared to AEs and also take more time to recover from the shock.

Fact 2: An increase in global uncertainty pulls capital (net portfolio investment) out from EMEs. The trade balances deteriorates initially before improving due to an exchange rate depreciation.

Fact 3: The capital outflow from EMEs leads to a currency (both nominal and real exchange rates) depreciation. As has been emphasized in the literature, an exchange rate depreciation worsens the balance sheets of firms, which is followed by foreign investors pulling out capital further and thus amplifying the effect of the shock on the real economy.

Fact 4: Consumer prices in EMEs increase due to a depreciation, and monetary policy responds by increasing interest rates. A rise in interest rates can thus reinforce the adverse effects of global uncertainty shock on the real economy.

To explain these facts and understand the role of monetary policy, we build a small open economy NK-DSGE model with uncertainty shocks. The basic framework of the model is adapted from the two country model (foreign and domestic country) discussed in Benigno et al. (2012). While we characterize the domestic economy as a small open economy, the foreign economy is an approximation to the world economy. The uncertainty is present in the preference/ demand shock of households in the foreign economy. We calibrate a small open economy and the world economy to a prototypical EME and AE, respectively.

1.2 Main results

1.2.1 Response to an uncertainty shock to the demand

We find that the calibration results from the model fit well qualitatively with the empirical stylized facts we observe in the data. When a global uncertainty shock hits a SOE, they experience a sudden capital outflow of capital and their nominal exchange rates depreciate. The real effective exchange rates (REER) also depreciates following a nominal exchange rate depreciation. This result is consistent with stylized Fact 3 we observe in the data. Demand contracts in the economy as agents save more (precautionary savings motive) and consume less today in a demand determined new-Keynesian model. Net exports rise due to a fall in imports as a result of the depreciation. This result is in line with empirical Facts 1 and 2, although in the data we observe the trade balance improving only after two quarters. Due to a depreciation of the domestic currency, the import prices of foreign goods consumed by domestic households increases. This increases consumer price inflation in the domestic economy. Since the central bank follows a simple Taylor type interest rate rule, the nominal interest rate also rises to stabilize consumer price inflation in the domestic country. This result too qualitatively matches Fact 4 that we observe in the data. The welfare losses in the domestic economy are positive because of adverse real effects of uncertainty shock.

We also find that the level of price flexibility matters for the extent to which uncertainty shock affect real variables. Under complete price flexibility, real variables are not affected and only nominal variables adjust. This happens because the economy under flexible price equilibrium is supply determined and not demand determined. When savings increase due to an uncertainty shock, the supply side of the economy is unaffected. Only the price level and the nominal interest rate adjusts here. As a result when savings (in assets) go out of the country, with increasing uncertainty, the price of the asset in domestic country falls. This fall in the asset prices leads to a rise in the nominal rate of interest. Consumer prices also increase to ensure that real savings and real interest rate do not show any change in the new equilibrium.

1.2.2 Role of monetary policy

A positive response of interest rates can reinforce the adverse effects of uncertainty shocks on the real economy. Moreover, the interest rate response is ineffective in stabilizing exchange rates, both nominal and real, as the UIP breaks down. Further, to examine the role of monetary policy in stabilizing the effects of a global uncertainty shock, we compare impulse responses from the model under alternate monetary policy rules. We broadly consider two categories of monetary policy rules. The first category rules are modified Taylor type interest rate rules. The second category rules are exchange rate rules. Under exchange rate rules, monetary policy is conducted with exchange rates as a monetary policy instrument. We also consider an extreme case of complete exchange rate stabilization i.e. a fixed exchange rate / PEG rule.

We find that welfare losses are lowest in exchange rate rules, followed by a PEG rule. The Taylor type interest rate rules give highest welfare losses. The welfare losses are reduced upto 21 per cent when a central bank switches to following an exchange rate rule from an interest rate rule. Comparing second order moments in long run simulations from the model under different rules show a remarkable reduction in the variability of variables when exchange rate rules are followed. To be specific, the standard deviation of the nominal exchange rate, output and consumer price inflation (CPI) is reduced by 85 per cent, 36 per cent and 45 per cent, respectively, when exchange rate rules are followed instead of interest rate rules.

This happens primarily because with a flexible exchange regime and monetary policy following an interest rate rule, uncovered interest parity (UIP) condition does not hold. Under increasing global uncertainty, the movement of nominal exchange rates is guided by a hedging motive, instead of interest rates. In other words, the link between exchange rates and interest rates breaks down and uncovered interest rate parity no longer holds. This gives rise to a non-zero time varying risk premium. When monetary policy follows an exchange rate rule, the hedging motive is weak and the movement of the exchange rates is controlled by a rule. This rule restores the lost connection between monetary policy, exchange rates, inflation and output, thus making monetary policy rules effective in stabilizing the economy. Moreover with exchange rate rules, the precautionary motive to save and thus consume less is weak since exchange rate rules are associated with lower risk premiums. This reduces transmission of uncertainty shocks on the real economy through the aggregate demand channel.

2 The Model

Our model is a two-country (domestic and foreign) open economy NK-DSGE model. The domestic country represents an emerging market economy, which is modelled here as a small open economy, and the foreign country represents an advanced economy. The basic framework of the model is adapted from Benigno et al. (2012) with the following modifications. First, in our model the domestic economy is characterized as a small open economy and the foreign economy is thus an approximation to the world economy.[18] Second, we consider a simple preference structure for households following Fernández-Villaverde et al. (2011), rather than a recursive preference structure.[19] Third, we have a second-moment shock (uncertainty shock) on the productivity and the demand processes of only the foreign/ world economy. We do this because the foreign economy represents the world here due to its size and we are interested in effects of global uncertainty shocks on the small open economy. Fourth, we follow Fernández-Villaverde et al. (2011) and take a third-order approximation of the model to solve it. Benigno et al. (2012) follows an approach discussed in Benigno et al. (2013) and take a second-order approximation to solve the model and capture the effects of second-moment shocks.

[18]Benigno et al. (2012) consider the case of two large economies in their paper.

[19]We also assume that the elasticity of substitution between domestic goods and foreign goods is different for domestic and foreign households in our model. But later we calibrate the model for the same values due to limited empirical evidence on the same.

2.1 Households

The world is assumed to consist of two countries, domestic (D) and foreign (F). We assume that domestic economy is a small open economy with size n relative to the world economy, which is modelled as a foreign economy.[20] A continuum of domestic households exist over $[0, n]$, while foreign households from $(n, 1]$, where $n \in (0, 1)$. An agent in each country is both a consumer and a producer, producing a single differentiated good and consuming all the goods produced in both countries. Also, the population size in each country is set equal to the range of goods produced in that country, such that domestic firms produce goods on $[0, n]$, and foreign firms produce goods on $(n, 1]$. The preferences of a representative household in domestic country is captured by the following utility function,

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{(C_t)}{1-\nu_D}^{1-\nu_D} - \omega_D \frac{(H_{D,t})}{1+\eta_D}^{1+\eta_D} \right). \tag{1}$$

Here C_t denotes the aggregate consumption index, $H_{D,t}$ denotes hours worked by the representative domestic household, ν_D is a measure of the inverse of the intertemporal elasticity of substitution, η_D is the inverse of the Frisch elasticity of substitution, and $\beta \in (0, 1)$ is the discount factor. The aggregate consumption index, C_t, is defined as,

$$C_t = \left[(\mu_D)^{1/\xi_D} (C_{D,t})^{\frac{\xi_D - 1}{\xi_D}} + (1-\mu_D)^{1/\xi_D} (C_{F,t})^{\frac{\xi_D - 1}{\xi_D}} \right]^{\frac{\xi_D}{\xi_D - 1}} \tag{2}$$

where, $C_{D,t}$ and $C_{F,t}$ denotes the consumption index of domestic goods and foreign goods of domestic households, respectively. $\xi_D > 0$ is the elasticity of substitution between domestic goods and foreign goods for domestic households and $\mu_D \in (0, 1)$ is the weight given to domestic goods in the aggregate consumption basket, C_t.[21] Analogous to equation (1), the utility function for a representative household in a foreign country is given by,

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{\Gamma_{F,t} (C_t^*)}{1-\nu_F}^{1-\nu_F} - \omega_F \frac{(H_{F,t})}{1+\eta_F}^{1+\eta_F} \right) \tag{3}$$

[20] We later limit $n \to 0$ to characterize the domestic economy as a small open economy.

[21] When $\nu_D > n$ means a home-bias for domestic goods since the weight given to domestic goods is higher than the size of the country.

where C_t^* denotes the aggregate consumption index, $H_{F,t}$ denotes hours worked and $\Gamma_{F,t}$ is the preference/ demand shock process. The aggregate consumption bundle C_t^* is given by,

$$C_t^* = \left[(\mu_F)^{1/\xi_F} \left(C_{D,t}^* \right)^{\frac{\xi_F - 1}{\xi_F}} + (1-\mu_F)^{1/\xi_F} \left(C_{F,t}^* \right)^{\frac{\xi_F - 1}{\xi_F}} \right]^{\frac{\xi_F}{\xi_F - 1}} \tag{4}$$

where $\mu_F \in (0,1)$ is weight given to domestic goods in the aggregate consumption basket, C_t^*. Following Benigno et al. (2012), the weights mentioned in the aggregate consumption bundles equations (2) and (4) are related to country sizes through:

$$1 - \mu_D = (1-n)\chi \tag{5}$$

$$\mu_F = n\chi. \tag{6}$$

Here, $\chi \in (0,1)$ is the (common) degree of openness between the domestic and foreign country. When $\chi = 0$, there is no trade of either goods or assets happening across the two countries and it represents an autarky case. $\chi = 1$, represents a case of complete free trade of both goods and assets between the two countries. Consumption bundles, $C_{D,t}$, $C_{F,t}, C_{D,t}^*$ and $C_{F,t}^*$ are Dixit-Stiglitz aggregates of differentiated goods produced in two countries and are defined as,

$$C_{D,t} = \left[\left(\frac{1}{n}\right)^{\frac{1}{\sigma}} \int_0^n \left(C_{D,t}(i)\right)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}} ; C_{F,t} = \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\sigma}} \int_n^1 \left(C_{F,t}(i)\right)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}} \tag{7}$$

$$C_{D,t}^* = \left[\left(\frac{1}{n}\right)^{\frac{1}{\sigma}} \int_0^n \left(C_{D,t}^*(i)\right)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}} ; C_{F,t}^* = \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\sigma}} \int_n^1 \left(C_{F,t}^*(i)\right)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}} \tag{8}$$

Here σ is the elasticity of substitution between the varieties, where a variety is indexed by $i \in [0,1]$.[22] The demand for each variety of a differentiated domestic and foreign good by each country's household is given as follows,[23]

$$C_{D,t}(i) = \left(\frac{1}{n}\right) \left(\frac{P_{D,t}(i)}{P_{D,t}}\right)^{-\sigma} C_{D,t} ; C_{F,t}(i) = \left(\frac{1}{1-n}\right) \left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\sigma} C_{F,t} \tag{9}$$

$$C_{D,t}^*(i) = \left(\frac{1}{n}\right) \left(\frac{P_{D,t}^*(i)}{P_{D,t}^*}\right)^{-\sigma} C_{D,t}^* ; C_{F,t}^*(i) = \left(\frac{1}{1-n}\right) \left(\frac{P_{F,t}^*(i)}{P_{F,t}^*}\right)^{-\sigma} C_{F,t}^* \tag{10}$$

[22]Note that the elasticity of substitution between the varieties, σ, is assumed to be same in both the countries.

[23]Refer to the Technical Appendix 4.2 for derivations.

where, $P_{D,t}(i)$ and $P^*_{D,t}(i)$ are prices of a variety i of a good produced in the domestic country in domestic and foreign currency, respectively. Similarly, $P_{F,t}(i)$, and $P^*_{F,t}(i)$ are prices of a variety i of a good produced in the foreign country in domestic and foreign currency, respectively. $P_{D,t}$, $P_{F,t}$, $P^*_{D,t}$ and $P^*_{F,t}$ are the price aggregates of the aggregate consumption baskets, $C_{D,t}$, $C_{F,t}$, $C^*_{D,t}$ and $C^*_{F,t}$, respectively and are defined as follows,

$$P_{D,t} = \left[\left(\frac{1}{n}\right)\int_0^n P_{D,t}(i)^{1-\sigma}\,di\right]^{\frac{1}{1-\sigma}} \;;\; P_{F,t} = \left[\left(\frac{1}{1-n}\right)\int_n^1 P_{F,t}(i)^{1-\sigma}\,di\right]^{\frac{1}{1-\sigma}} \quad (11)$$

$$P^*_{D,t} = \left[\left(\frac{1}{n}\right)\int_0^n P^*_{D,t}(i)^{1-\sigma}\,di\right]^{\frac{1}{1-\sigma}} \;;\; P^*_{F,t} = \left[\left(\frac{1}{1-n}\right)\int_n^1 P^*_{F,t}(i)^{1-\sigma}\,di\right]^{\frac{1}{1-\sigma}} \quad (12)$$

The law of one price is assumed to hold across all individual goods, such that, $P_{D,t}(i) = X_t P^*_{D,t}(i)$, and $P_{F,t}(i) = X_t P^*_{F,t}(i)$, where X_t is the nominal exchange rate (price of foreign currency in terms of domestic currency). Using this relation with the price aggregates in equations (11) and (12) we also get, $P_{D,t} = X_t P^*_{D,t}$ and $P_{F,t} = X_t P^*_{F,t}$. Demand functions for the consumption aggregates, $C_{D,t}$, $C_{F,t}$, $C^*_{D,t}$ and $C^*_{F,t}$ are as follows,

$$C_{D,t} = \mu_D \left(\frac{P_{D,t}}{P_t}\right)^{-\xi_D} C_t \;;\; C_{F,t} = (1-\mu_D)\left(\frac{P^*_{F,t}}{P^*_t}\right)^{-\xi_D} C_t, \quad (13)$$

$$C^*_{D,t} = \mu_F \left(\frac{T_{D,t}}{Q_t}\right)^{-\xi_F} C^*_t \;;\; C^*_{F,t} = (1-\mu_F)\left(\frac{P^*_{F,t}}{P^*_t}\right)^{-\xi_F} C^*_t \quad (14)$$

where, P_t and P^*_t are the aggregate consumer price indices (CPI) in the domestic and foreign country, in domestic and foreign currency, respectively, and are defined as,

$$P_t = \left[\mu_D \left(P_{D,t}\right)^{1-\xi_D} + (1-\mu_D)\left(P_{F,t}\right)^{1-\xi_D}\right]^{\frac{1}{1-\xi_D}} \quad (15)$$

$$P^*_t = \left[\mu_F \left(P^*_{D,t}\right)^{1-\xi_F} + (1-\mu_F)\left(P^*_{F,t}\right)^{1-\xi_F}\right]^{\frac{1}{1-\xi_F}} \quad (16)$$

It can be seen that due to a heterogenous preference structure across the two countries, purchasing power parity (PPP) does not hold at the aggregate price levels, such that $P_t \neq X_t P^*_t$. PPP holds only when $\mu_D = \mu_F$ and $\xi_D = \xi_F$. Benigno et al. (2012) assume $\mu_D \neq \mu_F$, such that PPP does not hold in their model too. Any deviations from PPP are measured through the real exchange rate, which is defined as the ratio of consumer price indices in the two countries in terms of domestic prices, and is given by,

$$Q_t = \frac{X_t P_t^*}{P_t}. \tag{17}$$

Re-writing equation (17) gives us the following relationship between consumer price inflation in the domestic and foreign country,

$$\pi_t^* = \pi_t \frac{Q_t}{Q_{t-1}\pi_{X,t}}. \tag{18}$$

Here, consumer price inflation in the foreign country and domestic country are defined as $\pi_t^* = \frac{P_t^*}{P_{t-1}^*}$ and $\pi_t = \frac{P_t}{P_{t-1}}$, respectively. Also, the change in the nominal exchange rate is defined as, $\pi_{X,t} = \frac{X_t}{X_{t-1}}$. The terms of trade is defined as a ratio of foreign prices to domestic prices, where both price indices are denominated in domestic currency and is given by,

$$\begin{aligned} T_t &= \frac{P_{F,t}}{P_{D,t}} \\ &= \frac{T_{F,t}}{T_{D,t}} \end{aligned} \tag{19}$$

where we define relative price ratios, $T_{D,t} = \frac{P_{D,t}}{P_t}$ and $T_{F,t} = \frac{P_{F,t}}{P_t}$. Using these definitions of relative price ratios with equation (15), we get the following relation,

$$T_{F,t} = \left[\frac{1-\mu_D (T_{D,t})^{1-\xi_D}}{1-\mu_D}\right]^{\frac{1}{1-\xi_D}}. \tag{20}$$

Similarly, equation (16) can be re-written in terms of gross foreign inflation $\left(\pi_{F,t}^*\right)$, foreign consumer price inflation (π_t^*), and the terms of trade as,

$$\pi_t^* = \pi_{F,t}^* \left[\frac{\mu_F (T_t)^{\xi_F - 1} + (1-\mu_F)}{\mu_F (T_{t-1})^{\xi_F - 1} + (1-\mu_F)}\right]^{\frac{1}{1-\xi_F}} \tag{21}$$

where, $\pi_{F,t}^* = \frac{P_{F,t}^*}{P_{F,t-1}^*}$. For the above described preferences, the total demand for each variety i of the domestic produce is given by,

$$Y_{D,t}(i) = nC_{D,t}(i) + (1-n) C_{D,t}^*(i)$$

where $nC_{D,t}(i)$ and $(1-n) C_{D,t}^*(i)$ is the aggregate demand of all households in the domestic and foreign country, respectively, for variety i of the domestic produce. Using the demand functions described in (9) and (10), we get

$$Y_{D,t}(i) = \left(\frac{P_{D,t}(i)}{P_{D,t}}\right)^{-\sigma} Y_{D,t} \tag{22}$$

where, aggregate demand for domestic good (all varieties) is given by, $Y_{D,t} = C_{D,t} + \left(\frac{1-n}{n}\right) C^*_{D,t}$. Further, using (13) and (14) in equation (22), we can re-write $Y_{D,t}$ in terms of aggregate consumption bundles in the two countries, as given by

$$Y_{D,t} = (T_{D,t})^{-\xi_D} \left[\mu_D C_t + \left(\frac{1-n}{n}\right) \mu_F Q_t^{\xi_F} (T_{D,t})^{\xi_D - \xi_F} C^*_t\right] \tag{23}$$

Similar to the domestic country, aggregate demand for a variety i of the foreign good is given by,

$$Y_{F,t}(i) = \left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\sigma} Y_{F,t} \tag{24}$$

where, aggregate demand for the foreign good (all varieties), $Y_{F,t} = \frac{n}{(1-n)} C_{F,t} + C^*_{F,t}$. Aggregate demand, $Y_{F,t}$, can be re-written in terms of aggregate consumption bundles in the two countries as,

$$Y_{F,t} = (T_{F,t})^{-\xi_D} \left[\frac{n}{(1-n)} (1-\mu_D) C_t + (1-\mu_F) Q_t^{\xi_F} (T_{F,t})^{\xi_D - \xi_F} C^*_t\right] \tag{25}$$

Households in the domestic and foreign country maximize (1) and (3) subject to the following flow budget constraints,

$$W_{D,t} H_{D,t} + \varpi_{D,t} \geq P_t C_t - B_{D,t} + E_t \{B_{D,t+1} M_{t,t+1}\}, \tag{26}$$

$$W_{F,t} H_{F,t} + \varpi_{F,t} \geq P^*_t C^*_t - B_{F,t} + E_t \{B_{F,t+1} M^*_{t,t+1}\} \tag{27}$$

respectively. Here $W_{D,t}$ and $W_{F,t}$ are nominal wages in the domestic and foreign country, respectively. The nominal wages are decided in a common labour market in each country. Also, $\varpi_{D,t}$ and $\varpi_{F,t}$ are the nominal profits which households receive from owning monopolistically competitive firms in the domestic and foreign country, respectively. Each household in each country holds equal shares in all firms and there is no trade in firm shares. The asset markets are assumed to be complete both at domestic and at international levels. Households trade in state-contingent nominal securities denominated in the domestic currency. $B_{D,t+1}$ is the state-contingent payoff at time $t+1$ of a portfolio of state-contingent nominal securities held by a household in the domestic country at the end of period t. The value of this portfolio can be written as $E_t \{B_{D,t+1} M_{t,t+1}\}$, where $M_{t,t+1}$ is the nominal stochastic discount factor for discounting wealth denominated in the domestic currency.

Households in the foreign country also trade in state-contingent securities denominated in the domestic currency. Let B_{t+1} be the state-contingent payoff (denominated in domestic currency) in period $t+1$ of the state-contingent portfolio held by foreign households at the end of period t. The payoff in the foreign currency in period $t+1$ is given by, $B_{F,t+1} = \frac{B_{t+1}}{X_{t+1}}$. Also the value of the portfolio today in foreign currency in period t is given by $\frac{E_t\{B_{t+1}M_{t,t+1}\}}{X_t} = \frac{E_t\left\{B_{F,t+1}X_{t+1}M_{t,t+1}\right\}}{X_t}$. The nominal stochastic discount factor for discounting wealth denominated in the foreign currency can thus be defined as,

$$M^*_{t,t+1} = \frac{X_{t+1}}{X_t}M_{t,t+1}. \tag{28}$$

The first order conditions for maximizing utility functions (1) and (3) for consumption (C_t, C^*_t), labour $(H_{D,t}, H_{F,t})$ and asset holdings $(B_{D,t+1}, B_{F,t+1})$ subject to the flow budget constraints (26) and (27) respectively are given by:

$$\text{Euler's equation } (D) \quad : \quad \beta\frac{E_t\left\{C_{t+1}^{-\nu_D}\right\}}{C_t^{-\nu_D}} = E_t\left\{M_{t,t+1}\pi_{t+1}\right\}$$

$$\Rightarrow \quad \beta\frac{E_t\left\{C_{t+1}^{-\nu_D}\right\}}{C_t^{-\nu_D}} = \frac{E_t\left\{\pi_{t+1}\right\}}{(1+R_t)} \tag{29}$$

$$(F) \quad : \quad \beta\frac{E_t\left\{\Gamma_{F,t+1}C_{t+1}^{*-\nu_F}\right\}}{\Gamma_{F,t}C_t^{*-\nu_F}} = E_t\left\{M^*_{t,t+1}\pi^*_{t+1}\right\}$$

$$\Rightarrow \quad \beta\frac{E_t\left\{\Gamma_{F,t+1}C_{t+1}^{*-\nu_F}\right\}}{\Gamma_{F,t}C_t^{*-\nu_F}} = \frac{E_t\left\{\pi^*_{t+1}\right\}}{(1+R^*_t)} \tag{30}$$

$$\text{Labour supply equation } (D) \quad : \quad w_{D,t} = \frac{\omega_D\left(H_{D,t}\right)^{\eta_D}}{\left(C_t\right)^{-\nu_D}T_{D,t}} \tag{31}$$

$$(F) \quad : \quad w_{F,t} = \frac{\omega_F\left(H_{F,t}\right)^{\eta_F}Q_t}{\Gamma_{F,t}\left(C^*_t\right)^{-\nu_F}T_{F,t}} \tag{32}$$

Here, the gross nominal interest rate in domestic country is given by, $(1+R_t) = \frac{1}{E_t\{M_{t,t+1}\}}$ and the gross nominal interest rate in foreign country is given by, $(1+R^*_t) = \frac{1}{E_t\left\{M^*_{t,t+1}\right\}}$. Real wages in the domestic and foreign country are defined respectively as, $w_{D,t} = \frac{W_{D,t}}{P_{D,t}}$ and $w_{F,t} = \frac{W_{F,t}}{P_{F,t}}$. We also define the Lagrangian multiplier denoting the marginal utility of income for the above maximization exercise as,

$$\lambda_{D,t} = \left(C_t\right)^{-\nu_D} \quad ; \lambda_{F,t} = \Gamma_{F,t}\left(C^*_t\right)^{-\nu_F} \tag{33}$$

Here $\lambda_{D,t}$ and $\lambda_{F,t}$ are Lagrangian multipliers for domestic and foreign country households, respectively. Combining the Euler equation from equation (29) and (30) with equation (28), we get the following complete asset market condition,

$$Q_{t+1} = \kappa \frac{E_t \left\{ \Gamma_{F,t+1} C_{t+1}^{*-\nu_F} \right\}}{E_t \left\{ C_{t+1}^{-\nu_D} \right\}}. \tag{34}$$

where, $\kappa = Q_0 \frac{C_0^{-\nu_D}}{\Gamma_{F,0} C_0^{*-\nu_F}}$ is the ratio of marginal utilities of nominal income across countries in the initial period. Equation (28) when combined with definitions of nominal stochastic discount factors i.e. $E_t \{M_{t,t+1}\} = \frac{1}{(1+R_t)}$ and $E_t \left\{M_{t,t+1}^*\right\} = \frac{1}{(1+R_t^*)}$, gives the following uncovered interest rate parity (UIP) condition (log-linearized),

$$r_t - r_t^* = E_t \{\Delta e_{t+1}\} \tag{35}$$

where, r_t, r_t^* and $E_t \{\Delta e_{t+1}\}$ are logs of $(1 + R_t)$, $(1 + R_t^*)$ and $E_t \left\{ \frac{X_{t+1}}{X_t} \right\}$, respectively. Following Menkhoff et al. (2012), Backus et al. (2010) and Benigno et al. (2012), we define time-varying risk premiums as deviations from the UIP condition, mentioned in equation (35). The log-linearized time-varying risk premiums, rp_t, are excess returns on holding domestic currency and written as follows,

$$rp_t = r_t - r_t^* - E_t \{\Delta x_{t+1}\}. \tag{36}$$

2.2 Firms

The domestic country produces goods on the interval $[0, n]$ and the foreign country on $(n, 1]$. A firm producing variety i of a good in the domestic and foreign country follows a production function linear in labour, given by,

$$Y_{D,t}(i) = A_{D,t} H_{D,t}(i) \tag{37}$$

$$Y_{F,t}(i) = A_{F,t} H_{F,t}(i), \tag{38}$$

respectively. Here, $A_{D,t}$ and $A_{F,t}$ are the productivity levels (common) following exogenous processes. $H_{D,t}(i)$ and $H_{F,t}(i)$ are composites of all the differentiated labour supplied by household h in each country, as given by,

$$H_{D,t}(i) = \frac{1}{n} \int_0^n H_{D,t}^h(i) \; dh \; ; H_{F,t}(i) = \frac{1}{1-n} \int_n^1 H_{F,t}^h(i) \; dh \tag{39}$$

where $H^h_{D,t}(i)$ and $H^h_{F,t}(i)$ are the labour supplied by household h to firm i in the domestic and foreign country, respectively.

2.2.1 Price setting

In the benchmark model we assume that firms in both the countries have nominal price rigidities in the form of price stickiness. We follow Calvo (1983) to capture price stickiness here. In each period only $(1-\alpha_D)$ fraction of firms in the domestic country can reset their prices independent of whether they had a chance to reset them in the last period. A firm i which gets a chance to reset its prices, $\overline{P}_{D,t}(i)$, maximizes a discounted sum of current and future expected values of profit, given by

$$\max_{\overline{P}_{D,t}(i)} \sum_{k=0}^{\infty} \alpha_D^k M_{t,t+k} \left(\overline{P}_{D,t}(i) Y_{D,t_{+k}}(i) - MC_{D,t+k} Y_{D,t_{+k}}(i) \right) \tag{40}$$

where $MC_{D,t+k}$ is the nominal marginal cost of domestic firms in period $t+k$ and is the same for all firms as the nominal wage is decided in a common labour market and all firms face a common productivity level realization. The demand function $Y_{D,t+k}(i)$, for each firm i in period $t+k$ is given by,

$$Y_{D,t+k}(i) = \left(\frac{\overline{P}_{D,t}(i)}{P_{D,t+k}} \right)^{-\sigma} Y_{D,t+k}$$

The optimal price chosen by firms re-setting prices is given by,

$$\overline{P}_{D,t}(i) = \frac{\sigma}{\sigma-1} \frac{\sum\limits_{k=0}^{\infty} \alpha_D^k M_{t,t+k} MC_{D,t+k} Y_{D,t+k}(i)}{\sum\limits_{k=0}^{\infty} \alpha_D^k M_{t,t+k} Y_{D,t+k}(i)} \tag{41}$$

where $\frac{\sigma}{\sigma-1}$ is the constant markup charged by firms. As can be seen from equation (41), the optimal price today depends on not just current but future marginal costs, and also demand conditions in the economy. A firm i, which does not reset its price is assumed to keep the prices same as last year's prices, $P_{D,t-1}(i)$. Thus, the law of motion for the aggregate producers price index (PPI) in the domestic country for Calvo's model can be written as,

$$P_{D,t} = \left[\alpha_D \left(P_{D,t-1} \right)^{1-\sigma} + (1-\alpha_D) \left(\overline{P}_{D,t} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \tag{42}$$

Using the domestic household's optimization problem we can write the stochastic discount factor $M_{t,t+k}$ as,

$$M_{t,t+k} = \beta^k \frac{\lambda_{D,t+k}}{\lambda_{D,t}} \frac{P_t}{P_{t+k}} \tag{43}$$

where $\lambda_{D,t}$ is the Lagrangian multiplier denoting the marginal utility of income. Combined with equation (43), the price setting equation (41) can be written recursively as,

$$\overline{\pi}_{D,t} = \frac{\sigma}{\sigma - 1} \pi_{D,t} \frac{X_{D,t}}{Z_{D,t}} \tag{44}$$

where $X_{D,t}$ and $Z_{D,t}$ are defined as follows,

$$X_{D,t} = \lambda_{D,t} Y_{D,t} mc_{D,t} T_{D,t} + \alpha_D \beta \left(\pi_{D,t+1}\right)^{\sigma} E_t \left\{X_{D,t+1}\right\} \tag{45}$$

$$Z_{D,t} = \lambda_{D,t} Y_{D,t} T_{D,t+k} + \alpha_D \beta \left(\pi_{D,t+1}\right)^{\sigma-1} E_t \left\{Z_{D,t+1}\right\} \tag{46}$$

Here, the reset domestic price inflation is defined as, $\overline{\pi}_{D,t} = \frac{\overline{P}_{D,t}}{P_{D,t-1}}$, and domestic price inflation is defined as, $\pi_{D,t} = \frac{P_{D,t}}{P_{D,t-1}}$. The real marginal cost for domestic firms in terms of domestic prices is given by, $mc_{D,t} = \frac{MC_{D,t}}{P_{D,t}}$. The law of motion for the domestic producer's prices in equation (42) can be written in terms of inflation as follows,

$$\pi_{D,t} = \left[\alpha_D + (1 - \alpha_D) \left(\overline{\pi}_{D,t}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}. \tag{47}$$

Since labour is the only input into production, the nominal marginal cost for domestic firms, $MC_{D,t}$, can also be written as,

$$MC_{D,t} = \frac{W_{D,t}}{A_{D,t}}.$$

The real marginal cost for domestic firms, $mc_{D,t}$, in terms of domestic prices would then be,

$$mc_{D,t} = \frac{w_{D,t}}{A_{D,t}} \tag{48}$$

where $w_{D,t} = \frac{w_{D,t}}{P_{D,t}}$ are real wages in the domestic country.

The price-setting behavior of firms in the foreign country is similar to the price-setting behavior of firms in the domestic country, as described from equation (40) – (63). In the foreign country, $(1 - \alpha_F)$ proportion of the firms reset their prices to $\overline{P}_{F,t}$ and the rest α_F proportion keep it the same as last year prices, $P^*_{F,t-1}$. Maximizing the current and future stream of profits by firms in the foreign country yields the following equation on reset foreign inflation, similar to equation (44)

$$\overline{\pi}_{F,t} = \frac{\sigma}{\sigma - 1} \pi^*_{F,t} \frac{X_{F,t}}{Z_{F,t}} \tag{49}$$

where $X_{F,t}$ and $Z_{F,t}$ are defined as follows,

$$X_{F,t} = \lambda_{F,t} Y_{F,t} mc_{F,t} T_{F,t} + \alpha_F \beta \left(\pi^*_{F,t+1}\right)^{\sigma} E_t \left\{X_{F,t+1}\right\} \tag{50}$$

$$Z_{F,t} = \lambda_{F,t} Y_{F,t} T_{F,t+k} + \alpha_F \beta \left(\pi^*_{F,t+1}\right)^{\sigma-1} E_t \left\{Z_{F,t+1}\right\} \tag{51}$$

Here the reset foreign price inflation is defined as, $\overline{\pi}_{F,t} = \frac{\overline{P}_{F,t}}{P^*_{F,t}}$, and the foreign price inflation is defined as, $\pi^*_{F,t} = \frac{P^*_{F,t}}{P^*_{F,t-1}}$. The real marginal cost for the foreign firms in terms of foreign prices is given by, $mc_{F,t} = \frac{MC_{F,t}}{P_{F,t}}$. The law of motion for the foreign producer's inflation is given by,

$$\pi^*_{F,t} = \left[\alpha_F + (1 - \alpha_F) \left(\overline{\pi}_{F,t}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}. \tag{52}$$

The real marginal cost for the foreign firms, $mc_{F,t}$, in terms of foreign prices would be,

$$mc_{F,t} = \frac{w_{F,t}}{A_{F,t}} \tag{53}$$

where $w_{F,t} = \frac{w_{F,t}}{P_{F,t}}$ denotes real wages in the foreign country.

The terms of trade equation (19) can be written as $T_t = \frac{X_t P^*_{F,t}}{P_{D,t}}$. Re-writing this gives us the following relation between the terms of trade, the nominal exchange rate change and producer price inflation between the two countries,

$$T_t = T_{t-1} \pi_{X,t} \frac{\pi^*_{F,t}}{\pi_{D,t}}. \tag{54}$$

Under a flexible price equilibrium, $\alpha_D = \alpha_F = 0$, such that all firms reset their prices in each period. This would imply, $P_{D,t} = \overline{P}_{D,t}$, $P^*_{F,t} = \overline{P}_{F,t}$ and $Disp_{D,t} = Disp_{F,t} = 1$. The reset price in each period would simply be a markup over marginal cost in both the countries i.e., $P_{D,t} = \frac{\sigma}{\sigma-1} MC_{D,t}$ and $P_{F,t} = \frac{\sigma}{\sigma-1} MC_{F,t}$.

2.3 Equilibrium

2.3.1 Aggregate goods market equilibrium in a small open economy

In this section we will describe the equilibrium for the benchmark case of the small open economy. To characterize the small open economy we follow Benigno and Paoli (2010) and limit $n \to 0$, such that $1 - \mu_D \to \chi$ and $\mu_F \to 0$ from equations (5) and (6). It can be seen that the share of domestic goods in the consumption basket of domestic households, μ_D, now depends only upon the degree of openness (inversely), while the share of domestic goods in the consumption basket of foreign households, μ_F, is negligible.[24] The real exchange rate in equation (17) is now given by,

$$Q_t = \frac{X_t P^*_{F,t}}{P_t} = \frac{P_{F,t}}{P_t} = T_{F,t} \tag{55}$$

(since $P^*_t = P^*_{F,t}$ under the limit $n \to 0$ in consumer price index equation (16)). The demand function equations (13) and (14) , aggregate demand equations (23) and (25) , relative price and inflation relations in equations (20) and (21) reduce to the following,

$$C_{D,t} = (1-\chi)(T_{D,t})^{-\xi_D} C_t \;\; ; \;\; C_{F,t} = \chi (T_{F,t})^{-\xi_D} C_t \tag{56}$$

$$C^*_{D,t} = 0 \;\; ; \;\; C^*_{F,t} = \left(\frac{T_{F,t}}{Q_t}\right)^{-\xi_F} C^*_t \tag{57}$$

$$Y_{D,t} = (T_{D,t})^{-\xi_D} \left[(1-\chi) C_t + \chi Q_t^{\xi_F} (T_{D,t})^{\xi_D - \xi_F} C^*_t\right] \tag{58}$$

$$Y_{F,t} = C^*_t \tag{59}$$

$$T_{F,t} = \left[\frac{1-(1-\chi)(T_{D,t})^{1-\xi_D}}{\chi}\right]^{\frac{1}{1-\xi_D}} \tag{60}$$

$$\pi^*_t = \pi^*_{F,t}, \tag{61}$$

respectively.

2.3.2 Aggregate labour market equilibrium

Equilibrium in the labour market would require aggregate labour supply to be equal to aggregate labour demand. For the domestic country, labour is aggregated as follows,

$$H_{D,t} = \frac{1}{n} \int_0^n H_{D,t}(i)\, di$$

[24] Note that the negligible share of domestic goods in the foreign household's consumption basket does not mean that foreign households do not consume domestic goods. It just means that the size of the domestic country is small compared to the foreign country such that the share of the domestic good in it's basket appears to be negligible.

Using labour demand of a firm i, $H_{D,t}(i)$, from equation (37), and demand for the firms's output, $Y_{D,t}(i)$, from equation (22), we re-write equilibrium in labour market as,

$$H_{D,t} = \frac{Y_{D,t}}{A_{D,t}} Disp_{D,t} \tag{62}$$

where the price dispersion term, $Disp_{D,t} = \frac{1}{n}\int_0^n \left(\frac{P_{D,t}(i)}{P_{D,t}}\right)^{-\sigma} di$ and can be written recursively as,

$$Disp_{D,t} = (\pi_{D,t})^{\sigma} \left[\alpha_D Disp_{D,t-1} + (1-\alpha_D)(\overline{\pi}_{D,t})^{-\sigma}\right] \tag{63}$$

where $Disp_{D,t-1} = \frac{1}{n}\int_0^n \left(\frac{P_{D,t-1}(i)}{P_{D,t-1}}\right)^{-\sigma} di$. Analogously, equilibrium in the foreign labour market implies,

$$H_{F,t} = \frac{Y_{F,t}}{A_{F,t}} Disp_{F,t} \tag{64}$$

where the price dispersion term, $Disp_{F,t} = \frac{1}{1-n}\int_n^1 \left(\frac{P^*_{F,t}(i)}{P^*_{F,t}}\right)^{-\sigma} di$, can be written recursively as,

$$Disp_{F,t} = \left(\pi^*_{F,t}\right)^{\sigma} \left[\alpha_F Disp_{F,t-1} + (1-\alpha_F)(\overline{\pi}_{F,t})^{-\sigma}\right] \tag{65}$$

where $Disp_{F,t-1} = \frac{1}{1-n}\int_n^1 \left(\frac{P^*_{F,t-1}(i)}{P^*_{F,t-1}}\right)^{-\sigma} di$. For a given wages and prices, labour supply equations (31) and (32) along with labour demand equations (62) and (64) determines the labour market equilibrium.

2.3.3 Trade balance

The trade balance is captured through net exports (net trade of goods) in domestic and foreign country. The value of net exports for the domestic country in terms of domestic consumer prices, $NX_{D,t}$, is defined as the value of total imports (in domestic consumer prices) subtracted from the value of total exports (in domestic consumer prices), and is given by,

$$\begin{aligned} NX_{D,t} &= \frac{P_{D,t}C^*_{D,t}}{P_t} - \frac{P_{F,t}C_{F,t}}{P_t} \\ &= T_{D,t}C^*_{D,t} - T_{F,t}C_{F,t} \end{aligned} \tag{66}$$

Similarly, the value of net exports for the foreign country in terms of foreign consumer prices (foreign currency), $NX_{F,t}$, is defined as the value of total imports (in foreign consumer prices) subtracted from the value of total exports (in foreign consumer prices), and is given by

$$NX_{F,t} = \frac{P^*_{F,t} C_{F,t}}{P^*_t} - \frac{P^*_{D,t} C^*_{D,t}}{P^*_t}$$
$$= \frac{T_{F,t}}{Q_t} C_{F,t} - \frac{T_{D,t}}{Q_t} C^*_{D,t} \qquad (67)$$

A positive and a negative net exports are referred to as trade surplus and trade deficit, respectively.

2.4 Welfare losses

The utility based welfare criterion defines welfare as an expected lifetime utility of a representative household (see Chapter-6, Woodford (2003)).[25] The welfare function in the domestic country would thus be a following lifetime utility of a representative domestic household, described in equation (1):

$$Welfare_{D,t} = E_t \sum_{t=0}^{\infty} \beta^t U_{D,t}$$

where, $U_{D,t} = U\left(C_t, H_{D,t}\right) = \frac{(C_t)^{1-\nu_D}}{1-\nu_D} - \omega_D \frac{\left(H_{D,t}\right)^{1+\eta_D}}{1+\eta_D}$. We can write the above welfare function recursively as:

$$Welfare_{D,t} = U_{D,t} + \beta E_t \left\{Welfare_{D,t+1}\right\} \qquad (68)$$

Similarly the welfare function in the foreign country would be a lifetime utility of a representative foreign household, described in equation (3). Writing welfare function recursively we get,

$$Welfare_{F,t+1} = U_{F,t} + \beta E_t \left\{Welfare_{F,t+1}\right\} \qquad (69)$$

where, $U_{F,t} = U\left(C_t, H_{F,t}\right) = \frac{(C_t)^{1-\nu_F}}{1-\nu_F} - \omega_F \frac{\left(H_{F,t}\right)^{1+\eta_F}}{1+\eta_F}$. We define welfare losses in the domestic country and foreign country as $-Welfare_{D,t}$ and $-Welfare_{F,t}$, respectively.

2.5 Monetary Policy Rules

2.5.1 Simple Taylor rule: benchmark policy

In the benchmark case we assume that the central banks in both the domestic and the foreign country set a monetary policy rule on the nominal interest rates using a simple Taylor rule (see Taylor (1993)). Here the central bank attempts to stabilize both inflation and output.

[25]We do not take an approximation of the welfare function in this chapter as we are solving a non-linear model. The welfare described in this section would be used later to compare alternate monetary policy rules.

In this case, we assume that the measure of inflation a central bank targets is the consumer price inflation in their respective countries. The rules are given by,

$$\text{TR-CPI} \quad : \quad (1+R_t) = \overline{R}\left(\frac{\pi_t}{\overline{\pi}}\right)^{\phi_\pi}\left(\frac{Y_{D,t}}{Y_{D,t}^{fp}}\right)^{\phi_y} \tag{70}$$

$$\text{TR-CPI} \quad : \quad (1+R_t^*) = \overline{R}^*\left(\frac{\pi_t^*}{\overline{\pi}^*}\right)^{\phi_\pi^*}\left(\frac{Y_{F,t}}{Y_{F,t}^{fp}}\right)^{\phi_y^*} \tag{71}$$

for the domestic and foreign country, respectively. Here, $\overline{R} = \frac{1}{\beta}$ and $\overline{R}^* = \frac{1}{\beta}$ are the steady state values of nominal interest rate, R_t, and R_t^*, respectively. We get these steady state values from Euler equations (29) and (30). Here, $\overline{\pi}$ and $\overline{\pi}^*$ are the steady state values of consumer price inflation, and $Y_{D,t}^{fp}$ and $Y_{F,t}^{fp}$ are the flexible price equilibrium levels of output, in the domestic and foreign country, respectively. The parameters $\left(\phi_\pi,\ \phi_y\right)$ and $\left(\phi_\pi^*,\ \phi_y^*\right)$ capture the responsiveness of the interest rates to the deviation of inflation from its steady state level and deviation of output from its flexible price level counterpart in the respective countries.

2.5.2 Alternate monetary policy rules

For comparative analysis, we only vary the monetary policy rule in the domestic economy/ SOE. The monetary policy rule for the foreign economy is assumed to be a simple Taylor rule as described in equation (71) for all the alternative monetary policy cases we consider for the domestic economy.

The Taylor rule we consider in the benchmark model, as described in equation (70) is a consumer price inflation (CPI) based rule. The first alternate rule we consider is a Taylor rule with producer's price index (PPI), given by,

$$\text{TR-PPI:} \ (1+R_t) = \overline{R}\left(\frac{\pi_{D,t}}{\overline{\pi}_D}\right)^{\phi_\pi}\left(\frac{Y_{D,t}}{Y_{D,t}^{fp}}\right)^{\phi_y} \tag{72}$$

Here, $\pi_{D,t}$ is producer price inflation in the domestic country and $\overline{\pi}_D$ is it's steady state value. This is an interesting case because it has been shown in Gali and Monacelli (2005) that under a flexible exchange regime it is optimal for the central bank of a small open economy to target producer price inflation. Later, Engel (2011) shows that under local currency pricing, exchange rate flexibility does not matter and the optimal policy for a central bank is to completely stabilize consumer price inflation.[26]

[26]These papers analyze shocks to first moment, while we consider shocks to second moments of the un-

It has been argued in Calvo and Reinhart (2002) and Reinhart (2000) that emerging market economies use their foreign exchange reserves and monetary policy with interest rates as an instrument to stabilize exchange rate movements in a flexible exchange rate regime. There also exists empirical evidence showing that central banks in emerging markets consider exchange rate movements while setting their monetary policy (see Cuevas and Topak (2008), Aizenman et al. (2011)). Given this, the next set of rules we consider are Taylor rules (both CPI and PPI) with nominal exchange rates, as given by

$$\text{TR-CPI-ER:} \qquad (1+R_t) = \overline{R}\left(\frac{\pi_t}{\overline{\pi}}\right)^{\phi_\pi}\left(\frac{Y_{D,t}}{Y_{D,t}^{fp}}\right)^{\phi_y}\left(\frac{X_t}{X_{t-1}}\right)^{\phi_X} \tag{73}$$

$$\text{TR-PPI-ER} \quad : \quad (1+R_t) = \overline{R}\left(\frac{\pi_{D,t}}{\overline{\pi}_D}\right)^{\phi_\pi}\left(\frac{Y_{D,t}}{Y_{D,t}^{fp}}\right)^{\phi_y}\left(\frac{X_t}{X_{t-1}}\right)^{\phi_X} \tag{74}$$

Here, $\frac{X_t}{X_{t-1}}$ denotes a change in the nominal exchange rate and the policy rate responds positively to a positive change in the nominal exchange rate. This is because a depreciation of currency would imply an increase in expected future inflation (due to a rise in import prices) and an increase in output (because of a higher demand for exports and import substitution). A rise in the interest rate is thus required to stabilize the economy from the effects of the depreciation.[27]

From the empirical evidence shown in Section 1.1, it is evident that the movement of the exchange rates (both nominal as well as real) is high and significant in emerging markets with uncertainty shocks. We also observed that the nominal interest rates increase as a response to an increase in global uncertainty and thus can reinforce the adverse effects of uncertainty shock. At the same time the interest rates do not seem to stabilize exchange rates. Aizenman et al. (2011) also show that when monetary policy is geared to stabilize inflation, output and exchange rates, exchange rates are not much stabilized as a part of mixed strategy in an IT (Inflation Targeting) regime. Given the inability of interest rate rules to absorb the effect of the shock under consideration, we examine, an alternative instrument for conducting monetary policy, namely, exchange rates. This puts a rule on exchange rates directly and does not let them float freely. These set of rules are called exchange rate rules (ERR) where a central bank manages exchange rates to target inflation and output. The Monetary Authority of Singapore (MAS) has been following this rule since 1981 (McCallum (2006)). We consider a simple exchange rate rule as described in

derlying process.

[27]The rule would suggest a fall in the nominal interest rates in case of an appreciation.

Heiperzt et al. (2017),

$$\text{ERR: } \frac{X_t}{X_{t-1}} = \left(\frac{Y_{D,t}}{Y_{D,t}^{fp}}\right)^{-\phi_y^e} \left(\frac{\pi_t}{\overline{\pi}}\right)^{-\phi_\pi^e} \tag{75}$$

Here, ϕ_y^e and ϕ_π^e are the response parameters of nominal exchange to the change in output and inflation. Note that the exchange rate responds negatively to an increase in inflation and output to stabilize the economy. This is because increase in inflation and output can be stabilized when nominal exchange rates fall (an appreciation). An appreciation reduces inflation (by reducing the price of imports) and also reduces output (by reducing the foreign demand for domestic goods and reducing the domestic good's demand by domestic households). We also consider an extreme case of a fixed exchange rule (PEG) where the central bank completely stabilizes the nominal exchange rate, as given by

$$\text{PEG: } \frac{X_t}{X_{t-1}} = 1 \tag{76}$$

When, $\phi_y^e \to 0$ and $\phi_\pi^e \to 0$, the exchange rate rule (75) approaches a PEG rule in (76). As values of ϕ_y^e and ϕ_π^e increase, the exchange rate adjusts more to stabilize the economy. Note that interest rates are endogenously determined in the economy under ERR and PEG rule.

2.6 Exogenous shock processes

The technology process for domestic country firms, $A_{D,t}$, in equation (37), follows a standard AR(1), as given by,

$$A_{D,t} = (1 - \rho_D)\overline{A}_D + \rho_D A_{D,t-1} + \epsilon_{D,t} \tag{77}$$

where $\epsilon_{D,t}$ is a shock to the first moment of the technology process. For the present analysis we assume that there are no shocks to technology in the domestic economy, such that the technology $A_{D,t}$ is at its steady state level $\overline{A}_D$. Since we are interested in global uncertainty shocks we assume a shock to the second moment of a foreign country's preference/ demand and technology/ productivity process. We follow Basu and Bundick (2017) and Fernández-Villaverde et al. (2011) to describe the shock processes with uncertainty shocks. The demand shock process in equation (3) and productivity shock processes in equation (37) and (38) take the following form,

$$\Gamma_{F,t} = (1 - \delta_F)\overline{\Gamma}_F + \delta_F \Gamma_{F,t-1} + v_{F,t-1}\varepsilon_{F,t} \tag{78}$$

$$A_{F,t} = (1 - \rho_F)\overline{A}_F + \rho_F A_{F,t-1} + u_{F,t-1}\epsilon_{F,t} \tag{79}$$

where $\varepsilon_{F,t}$ and $\epsilon_{F,t}$ are shocks to the first moment of demand and productivity levels. The standard deviations $v_{F,t-1}$ and $u_{F,t-1}$ in the foreign demand and productivity shocks are not constant and are described by the following AR(1) processes,

$$
\begin{aligned}
v_{F,t} &= (1-\delta_{\sigma_F})\,\overline{v}_F + \delta_{\sigma_F} v_{F,t-1} + \varpi_F \vartheta_{F,t} && (80) \\
u_{F,t} &= \left(1-\rho_{\sigma_F}\right)\overline{u}_F + \rho_{\sigma_F} u_{F,t-1} + \varkappa_F \zeta_{F,t}. && (81)
\end{aligned}
$$

Here, $\vartheta_{F,t}$ and $\zeta_{F,t}$ are shocks to the second moment or an uncertainty shock to the underlying demand and the productivity levels, respectively. In other words, uncertainty shocks here refer to the shocks to standard deviation of the underlying process. It is assumed that the stochastic shocks, $\varepsilon_{F,t}$, $\epsilon_{D,t}$, $\epsilon_{F,t}$, $\vartheta_{F,t}$ and $\zeta_{F,t}$, are independent and normally distributed random variables. In the baseline calibration we show results for uncertainty shocks to the demand process. The results for the uncertain productivity shocks are very similar. Also, $\overline{A}_D = \overline{A}_F = 1$, at the steady state.

2.7 Solution method

We are interested in looking at the effects of shocks to the second moments (or uncertainty shocks) of the demand/ preference levels of the foreign country on a small open economy (domestic country). To capture the complete effect of the second moment shocks on the endogenous variables of the model we need to take the third order approximation of the model equations as explained in Fernández-Villaverde et al. (2011) and later also applied in Basu and Bundick (2017). Following this, we do a third order Taylor series approximation of the model using the Dynare software package in MATLAB to find a solution to our benchmark model.[28] All the approximations are done around the stochastic steady state.

2.8 Calibration

We calibrate the small open economy to a prototypical emerging market economy and the foreign country, which comprises the world, to an advanced economy. We estimate the degree of openness parameter, χ, to be 0.6, as the average trade share to GDP of emerging market economies. To get this we use World Bank's country level trade data for year 2015.[29] The value of κ, which is the initial parameter in the asset market condition is estimated to be 3.8. We calculate this using the OECD database on national accounts.[30]

[28]We use MATLAB 2015 and Dynare 4.4.3 for calibrating the model.
[29]The data was accessed in November, 2018 from:
https://datacatalog.worldbank.org/search?sort_by=title&sort_order=ASC
[30]Data is accessed in January, 2019 from OECD: https://stats.oecd.org/#

Details on the calculation of χ and κ is provided in the Data Appendix 4.1. The inverse of the intertemporal elasticity of substitution parameter, ν_D and ν_F for the domestic and the foreign country, respectively, are set to 5 following Fernández-Villaverde et al. (2011) and Benigno et al. (2012). We make the domestic goods and foreign goods relatively substitutable in the benchmark calibration for both the countries, thus setting the value of the elasticity of substitution between domestic and foreign goods, ξ_D and ξ_F, to be 1.5 as calculated in Benigno et al. (2012). The discount factor, β is assumed to be the same in both the countries and is set to 0.994 following Basu and Bundick (2017). The utility parameter, ω_D and ω_F capturing the weight given to the household's disutility from the labour supply is set to 1 using Fernández-Villaverde et al. (2011). The parameter for the elasticity of substitution between varieties, θ, is set to 6 following Benigno et al. (2012) such that the steady state markup for a firm is 20 per cent. In the baseline calibration we fix the value of stickiness parameter for the foreign country, α_F, to be 0.66 following Sbordone (2002) and Gali et al. (2001). These papers provide empirical evidence for stickiness parameter for the US and Europe, respectively. For the domestic country, the parameter for stickiness, α_D is set slightly higher to 0.75 such that domestic firms revise prices in 4 quarters.[31] We also compare our baseline sticky price calibration results to a completely flexible price calibration, where $\alpha_D = 0$ and $\alpha_F = 0$. The value of the inverse of the Frisch elasticity of substitution (IFES) varies from 0.5 to 1000 in the literature (see Basu and Bundick (2017), Fernández-Villaverde et al. (2011)). Here we set IFES, η_D, for domestic households to 25 and IFES, η_F, for foreign households to 50.[32] The preference shock parameters for the preference shock (both first moment and second moment), for the foreign country are calibrated from Basu and Bundick (2017), and are set as follows: $\delta_F = 0.94$, $\delta_{\sigma_F} = 0.74$. The steady state values for the demand shock, $\overline{\Gamma}_F$, and its standard deviation, $\overline{\upsilon}_F$, are set to 1 and 0.085 respectively. The scaling parameter for the uncertainty shock ϖ, is set to 0.18 following Benigno et al. (2012).

For the baseline calibration of the Taylor rule as described in equations (70) and (71), for both the countries, we set the weight on inflation to be, $\phi_\pi = \phi_\pi^* = 1.5$ and the weight on output to be, $\phi_y = \phi_y^* = 0.5$. These are the standard values used in the literature (see Taylor (1993)). We also consider models with alternate monetary policies. The parameter for Taylor rules with an exchange rate where weight on the exchange rate change, ϕ_X, is set to 0.05 uses estimates from Cuevas and Topak (2008). The exchange rate rule parameters, ϕ_π^e, i.e., weight on the inflation gap, and ϕ_y^e, i.e., weight on the output gap are set to 0.16 and 0.04 following estimates from Parrado (2004) and Heiperzt et al. (2017). We also calculate

[31] See Devereux and Engel (2003).

[32] We choose the minimum values for the IFES such that the impulse responses are matched qualitatively.

the second moments of the simulated data from the model by varying the value of ϕ_X to 0.2 and 0.5, and of ϕ_π^e to 0.3 and 0.8. The parameters are summarized in Table 1 below.

Table 1: Summary of parameter values

Parameter	Notation	Value	Source
Households & Firms			
Discount factor	β	0.994	Basu and Bundick (2017)
Inverse of intertemporal elasticity of substitution	ν_D ; ν_F	5 ; 5	Fernández-Villaverde et al. (2011)
Inverse of Frisch elasticity of substitution	η_D ; η_F	25 ; 50	Author
Stickiness parameter	α_D ; α_F	0.75 ; 0.66	Author; Sbordone (2002)
General			
Degree of openness	χ	0.6	Author
Elas. of substitution between domestic and foreign goods	ξ_D ; ξ_F	1.5 ; 1.5	Benigno et al. (2012)
Elas. of substitution between varieties	θ	6	Benigno et al. (2012)
Shocks: preference shock			
Level parameters	δ_F ; $\overline{\Gamma}$	0.94 ; 1	Basu and Bundick (2017)
Uncertain shock parameters	δ_{σ_F},$\overline{\upsilon}$	0.74 ; 0.085	Basu and Bundick (2017)
	ϖ	0.18	Benigno et al. (2012)
Policy : Taylor rule coefficients			
Inflation	ϕ_π ; ϕ_π^*	1.5 ; 1.5	Taylor (1993)
Output gap	ϕ_y ; ϕ_y^*	0.5 ; 0.5	Taylor (1993)
Exchange rate change	ϕ_X	0.05	Cuevas and Topak (2008)
Policy: Exchange rate rule coefficients			
Inflation	ϕ_π^e	0.16	Parrado (2004)
Output gap	ϕ_y^e	0.04	Parrado (2004)

3 Impulse Response Functions

3.1 Effects of an uncertainty shock to the foreign demand

In this section we discuss the macroeconomic effects of a one standard deviation shock to uncertainty in demand of the foreign households as described in equation (80).

Figure 6 shows the impulse responses of the macroeconomic variables for the domestic economy/ SOE when the foreign/ world economy experiences an uncertainty shock to its demand. As described in Basu and Bundick (2017) the uncertainty shock to demand

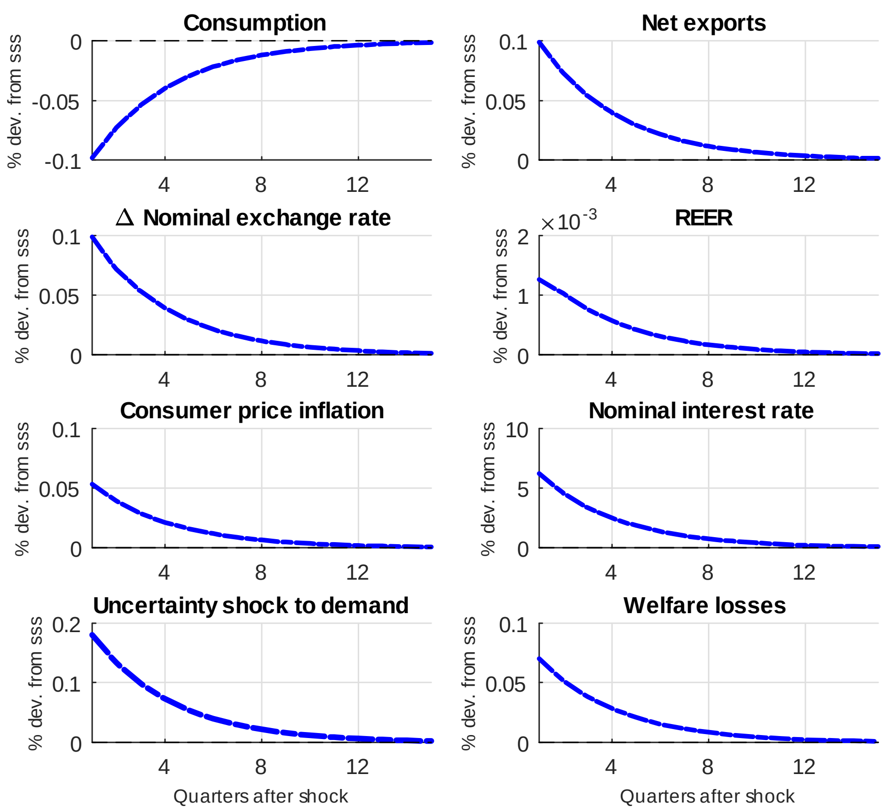

Figure 6: IRFs for a SOE to a one standard deviation shock to uncertainty in the foreign demand

contracts the economy as agents save more (precautionary savings) and consume less today. Ravn and Sterk (2017) argues that a higher risk of job loss and worsening job finding prospects during unemployment depress consumption goods demand today because of a precautionary savings motive. Note that both the domestic as well as foreign economy have a new-Keynesian feature of nominal rigidities in the form of price stickiness and thus output is demand determined. When an uncertainty shock hits the foreign economy the households save more and consume less today which leads to a fall in aggregate demand and hence prices in the foreign economy. When a SOE (domestic) is connected to the world through trade of goods and assets, the exogenous uncertainty shock to foreign demand also affects them.

The domestic country experiences a sudden outflow of capital and its nominal currency depreciates. Subplot (2,1) of Figure 6 shows the depreciation of the nominal exchange rate. Since prices are sticky in both the countries, the REER also depreciates following a nominal exchange rate depreciation (Subplot (2,2)). This result is consistent with Fact 3 we observe in the data. Due to an uncertain future demand, households in the domestic economy too save more (precautionary savings) and consume less today because of which consumption demand falls (Subplot (1,1)). Net exports rise due a fall in imports as a result of a depreciation (Subplot (1,2)).[33] This result is in line with empirical Facts 1 and 2, although in the data we observe the trade balance improves only after two quarters. The consumption basket in the SOE has a share of imported goods proportional to the degree of openness as shown in equation (56.2). Due to a depreciation of the currency, the import prices of the foreign goods consumed by domestic households increases. This increases the consumer price inflation in the domestic economy (Subplot (3,2)). Since the central bank follows a simple interest rate rule described in equation (70) , the nominal interest rate also rises to stabilize consumer price inflation in the domestic country.[34] This result too qualitatively matches empirical Fact 4 we observe in the data. The welfare losses in the domestic economy are positive because of the real effects of the shock. To summarize, the calibration results from the model fit well qualitatively with the empirical stylized facts.

Figure 7 compares the impulse responses for uncertainty shocks to foreign demand under a flexible price allocation (red line) with the sticky price allocation (blue line). The calibration under flexible price allocation is interesting because it can affect the way real variables respond to the uncertainty shock. It has been shown in Basu and Bundick (2017), that a standard model with flexible prices does not generate a negative comovement in uncertainty

[33] The initial value for the net exports is negative here such that the country starts with a trade deficit.

[34] Note that the output gap would be negative here which would require the central bank to reduce the nominal interest rates but the net change depends on the Taylor parameters and the size of the change in inflation and output.

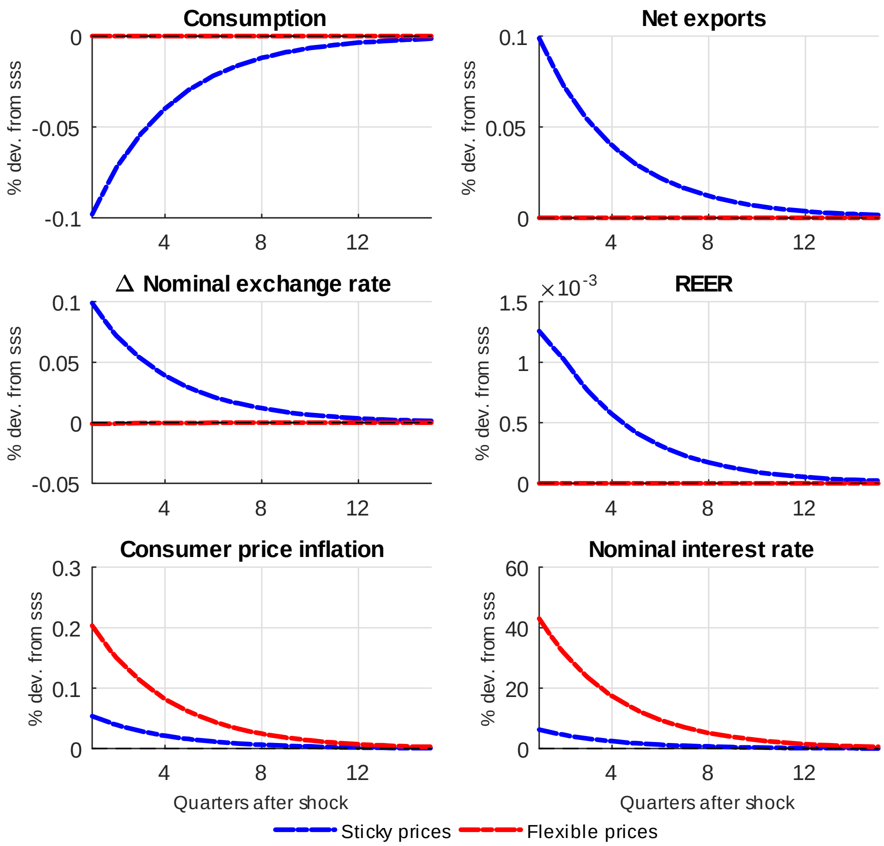

Figure 7: IRFs comparing the sticky price and flexible price allocation for a one standard deviation shock to uncertainty in foreign demand

and real demand in the economy as observed in the data, which nominal rigidities in the form of sticky prices are able to generate. Figure 7 shows that only nominal variables change as a response to an increase in the uncertainty and none of the real variables are affected under a flexible price allocation. This happens because the economy under flexible price equilibrium is supply determined and not demand determined. When savings increase due to an uncertainty shock the supply side of the economy is unaffected as the savings in the present model are not investible (no capital in the model). This is in contrast to Basu and Bundick (2017) where a flexible price allocation results in the expansion of economy with an uncertainty shock. This happens because they assume a model with capital such that when savings increase, investment increases in the economy, leading to a capital driven expansion of output. Since we consider a model without capital, this channel does not exist. The nominal variables, price level and the nominal interest rate, adjusts here as can be seen in Subplot (3,1) and (3,2) respectively. This happens because savings (in assets) have a tendency to go out of the country which reduces the price of an asset in the domestic country and thus increases the nominal rate of interest. To satisfy the Taylor rule, we would observe that consumer prices also rise with increasing nominal interest rates. Moreover, increasing consumer prices also ensures that the real savings and the real interest rate do not show much change in the new equilibrium.

3.2 Role of monetary policy

In the model calibration so far we have assumed that the central bank of a small open economy (domestic country) follows a simple Taylor rule (TR-CPI) described in equation (70). As discussed earlier a positive response of the interest rate rule in the EMEs amplifies the contractionary effect of an uncertainty shock on the real economy. In this section we consider alternate monetary policy rules to ascertain the role of monetary policy in determining the post shock (uncertainty shock) equilibrium. For comparative analysis we set TR-CPI as the benchmark case. The other monetary policy rules we consider for comparison can broadly be grouped into two categories. The first category correspond to modified Taylor rules. Here we consider a simple Taylor rule with PPI (TR-PPI), a CPI Taylor rule with an exchange rate mandate (TR-CPI-ER), a PPI Taylor rule with an exchange rate mandate (TR-CPI-ER), as specified in equations (72), (73) and (74), respectively. In all the above mentioned cases, we have free movement of assets across countries and an independent monetary policy. Following the impossible trinity, the exchange rate is completely flexible.

The second category is a different class of monetary policy rules, where the exchange rate is the monetary policy instrument. Here we consider a very simple exchange rate rule

(ERR) and an extreme case of fixed exchange rates (PEG), as specified in equations (75) and (76) , respectively. A detailed description of the alternate monetary policy rules is given in Section 2.5.

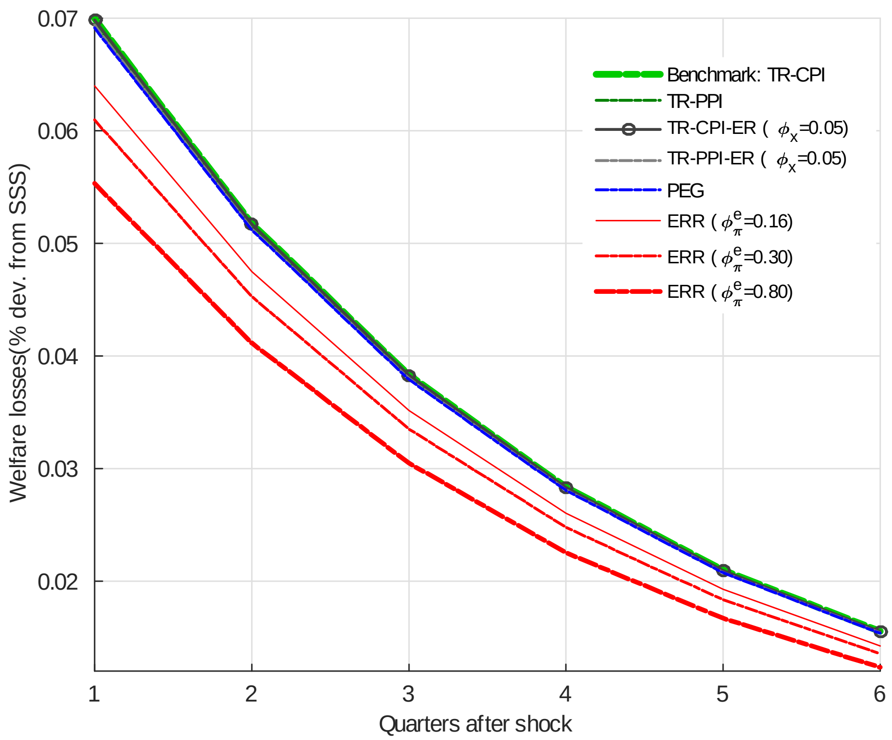

Figure 8: Welfare loss responses in a SOE under different monetary policy rules to one standard deviation shock to uncertainty in foreign demand

Figure 8 compares the impulse response functions for welfare losses for the above described monetary policy rules. As can be seen, welfare losses do not vary significantly among modified Taylor rules (TR-CPI, TR-PPI, TR-CPI-ER and TR-PPI-ER) and the PEG rule, for the given calibration. Flexible exchange rate regimes and fixed exchange rate regimes give very similar welfare losses with the present calibration. We do find however that the PEG rule does slightly better (not significantly) than interest rate rules. On impact, exchange rate rules reduce welfare losses by 21 per cent, when the inflation parameter in exchange rate rule, ϕ_π^e, is 0.8.[35] The reduction in welfare losses is 9 per cent and 13 per cent when ϕ_π^e equals 0.16 and 0.30, respectively. This happens because in the presence of uncertainty, with the central bank following an interest rate rule (flexible exchange rates), the movement in exchange rates are primarily driven by a *hedging motive* (see Benigno et al. (2012)). Thus the

[35]The comparisons are made from the benchmark policy.

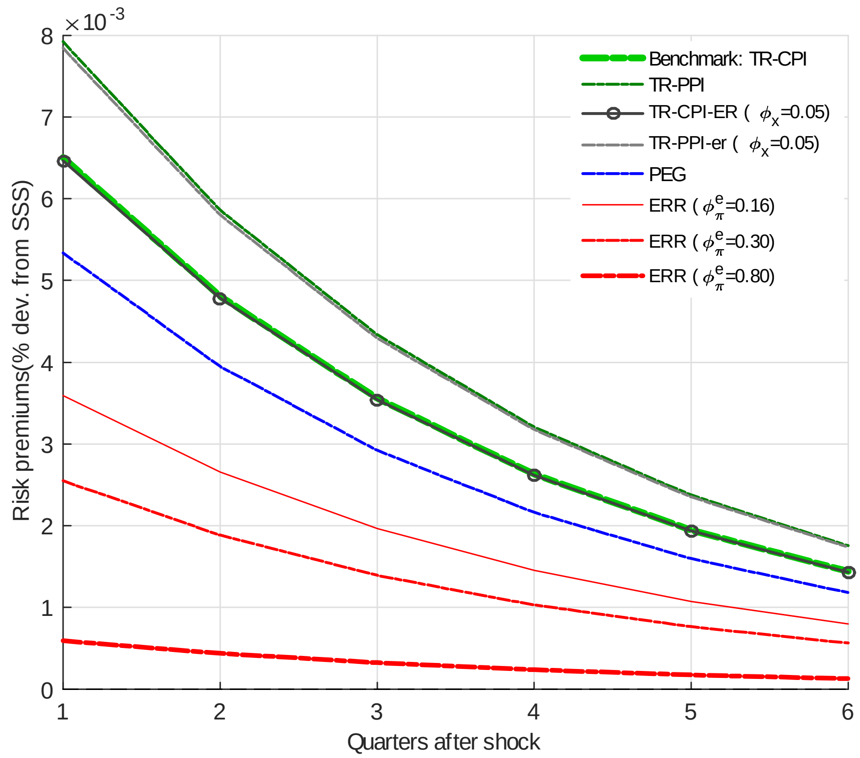

Figure 9: Risk premium responses in a SOE under different monetary policy rules to one standard deviation shock to uncertainty in foreign demand

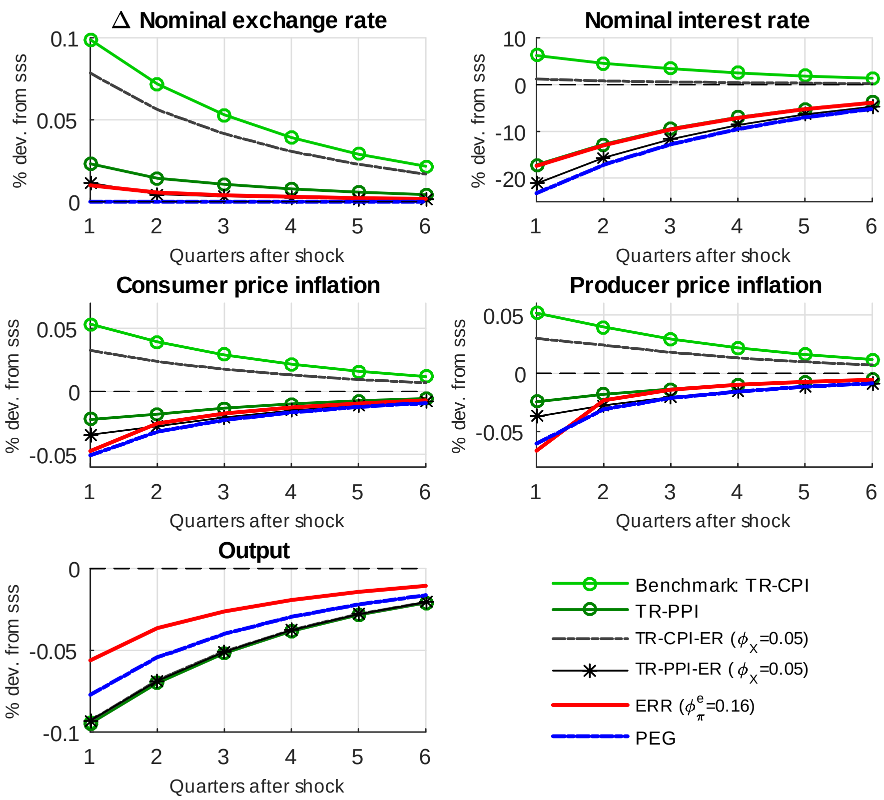

Figure 10: IRFs for a SOE under different monetary policy rules to a one standard deviation shock to uncertainty in foreign demand

link between exchange rate and the monetary policy through interest rates (UIP condition) breaks down with higher-order moment shocks. When the link between monetary policy (through interest rate rules) and exchange rate breaks down, the monetary policy becomes ineffective in stabilizing the economy via stabilizing exchange rates. Due to this, we observe higher welfare losses when monetary policy is implemented with interest rate rules as an instrument.[36] When we compared the welfare losses for the economy under same monetary policy rules with a standard domestic first moment shock to demand or production, we find that the Taylor rules are performing better then the ERRs. The PEG rule performs the worst in this case. This implies that the choice of instrument for monetary policy clearly depends on the source and type of the shock affecting an economy.

Exchange rate rules give the least welfare losses as they are associated with lower risk premiums. Figure 9 above compares the risk premiums under different monetary policy rules.[37] The risk premiums with exchange rate rules are strictly lower than the considered Taylor rules and the PEG rule. In particular, the risk premiums reduce by 45 per cent, 61 per cent and 91 per cent from the benchmark rule when ϕ_π^e equals 0.16, 0.30 and 0.80, respectively, in an ERR. This result is consistent with Heiperzt et al. (2017), who show that ERRs are associated with lower risk premiums than interest rate rules. The risk premiums are lower with ERRs because movements in exchange rate are no longer guided by a *hedging motive*, but rather by a rule as shown in the equation (75). This restores the broken link between monetary policy, exchange rates and other real variables in the domestic economy like inflation and output. Subplot (3,1) in Figure 10 shows that the output fall is the least in ERR vis-a-vis other rules considered. This happens because ERRs are associated with lower risk premiums and thus lead to a lower precautionary motive to save. Thus the adverse impact on aggregate demand triggered by an uncertainty shock in a demand determined economy is weakened when monetary policy follows an exchange rate rule. Consumer price inflation is negative as the currency does not depreciate much and PPI falls in the domestic country (due to fall in the demand). Under ERR, an initial depreciation of the currency with an expectation of future currency appreciation leads to an increased demand for bonds denominated in the home currency. This increases the price of bonds, which leads to a fall in nominal interest rates.

On the other extreme, the fixed exchange rate regime will not generate any movement in the exchange rates or the risk premiums when the economy is hit with uncertainty shocks.

[36]When we compared the welfare losses for the economy under same monetary policy rules with a standard domestic first moment shock to demand or producntion, we find that the Taylor rules are performing better then the ERRs. The PEG rule performs the worst in this case.

[37]The risk premiums plotted here are levels and not logs. Values less than 1 here signify negative log risk premiums, rp_t.

This implies that other nominal variable like consumer price inflation and nominal interest rate adjusts more to stabilize the economy, as shown in Figure 10. Subplot (2,1) in Figure 10 shows that the movement of consumer price inflation under PEG rule is the highest compared to other rules. Moreover, output fluctuates less under PEG than in interest rate rules considered (see Subplot (3,1)). Due to this balanced trade-off between inflation and output stabilization, we get similar welfare losses with a PEG rule and interest rate rules. Among the interest rate rules, Taylor rules with CPI as inflation measure performs the worse. Although the welfare losses are similar among Taylor rules, the nominal exchange rate movements with TR-CPI and TR-CPI-ER are very high. In fact it is highest in the benchmark case of TR-CPI (see Subplot (1,1)). This is consistent with the literature which shows that with producer currency pricing, a Taylor rule with CPI brings more inefficiency (see Gali and Monacelli (2005), Engel (2011), Devereux and Engel (2003)). Taylor rules with an exchange rate mandate perform slightly better than those without it but they do not significantly reduce welfare losses.

Table 2: Comparing second empirical moments for different monetary policy rules

Variable	Standard deviation×100					
	TR-CPI	TR-PPI	TR-CPI-ER	TR-PPI-ER	ERR	PEG
			ϕ_X=0.05	ϕ_X=0.05	ϕ_π^e=0.16	
	(1)	(2)	(3)	(4)	(5)	(6)
Consumption	2.484	2.483	2.484	2.483	2.476	2.484
Output	2.502	2.471	2.473	2.446	1.602	2.182
Net exports	1.450	1.451	1.450	1.451	1.455	1.450
Inflation (PPI)	3.081	3.041	2.940	2.914	1.673	1.911
Inflation (CPI)	3.057	3.064	2.923	2.943	1.695	1.933
Nominal ER	1.656	1.607	1.478	1.449	0.246	000
REER	0.561	0.550	0.561	0.511	0.507	0.109
Interest rates	3.689	3.711	3.584	3.614	2.645	2.877

We investigate the response of the economy under different monetary policy rules to further examine how the economy responds in the long run. We simulate data from the model for 100 periods (25 years) under the considered monetary policy rules.[38] Table 2 compares the standard deviation of some important variables under different monetary policy rules. The ERR (column 5) outperforms all monetary policy rules and gives strictly lower standard deviation of all variables. The standard deviation of the nominal exchange rate, output and

[38]The economy is assumed to be at the steady state in the initial period.

CPI is reduced by 85 per cent, 36 per cent and 45 per cent respectively from the benchmark case (column 1). The PEG rule (column 6) stands out as the second best monetary policy rule with the fall in the standard deviation of output, inflation and nominal exchange rates upto 13 per cent, 37 per cent and 100 per cent, respectively. Among Taylor type interest rate rules, TR-PPI-ER does the best. This is consistent with the results shown in Cook (2004) where, he argues that the fixed exchange rate regimes offer greater stability than interest rate rules. The ranking of monetary policy rules based on second moments is consistent with the impulse response for welfare losses discussed above.

Table 3: Comparing second empirical moments for varying parameters in monetary policy rules

Variable	Standard deviation×100								
	TR-CPI-ER : $\phi_X =$			TR-PPI-ER : $\phi_X =$			ERR : $\phi^e_\pi =$		
	0.05	0.2	0.5	0.05	0.2	0.5	0.16	0.3	0.8
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Consumption	2.484	2.484	2.484	2.483	2.483	2.483	2.476	2.472	2.465
Output	2.473	2.412	2.344	2.446	2.394	2.334	1.602	1.261	0.661
Net exports	1.450	1.450	1.450	1.451	1.451	1.451	1.455	1.458	1.465
Inflation (PPI)	2.940	2.662	2.394	2.914	2.659	2.403	1.673	1.489	1.081
Inflation (CPI)	2.923	2.660	2.406	2.943	2.696	2.446	1.695	1.510	1.089
Nominal ER	1.478	1.116	0.749	1.449	1.124	0.784	0.246	0.430	0.857
REER	0.561	0.552	0.533	0.511	0.542	0.526	0.507	0.282	0.182
Interest rates	3.584	3.385	3.202	3.614	3.425	3.243	2.645	2.476	2.095

Table 3 compares the Taylor rules with an exchange rate mandate with varying degrees of the exchange rate parameter (ϕ_X), and exchange rate rules with varying degree of the inflation parameter (ϕ^e_π). Among the Taylor rules (column 1-6), TR-PPI-ER with $\phi_X = 0.5$, gives the least standard deviations of the variables. However, ERR with the lowest value of $\phi^e_\pi = 0.16$, performs better than TR-PPI-ER with ϕ_X as high as 0.5. When the response parameter of the exchange rates to inflation, ϕ^e_π, increases, both output and inflation are stabilized more at the cost of increasing variability in nominal exchange rates. When the inflation parameter, ϕ^e_π, increases from 0.16 to 0.30, the standard deviation of output and inflation reduces by 21 per cent and 11 per cent respectively, but the exchange rate variability is increased by 75 per cent. In an extreme case, the nominal exchange rate variability increases by 248 per cent when the inflation parameter is increased from 0.16 to 0.80. Note

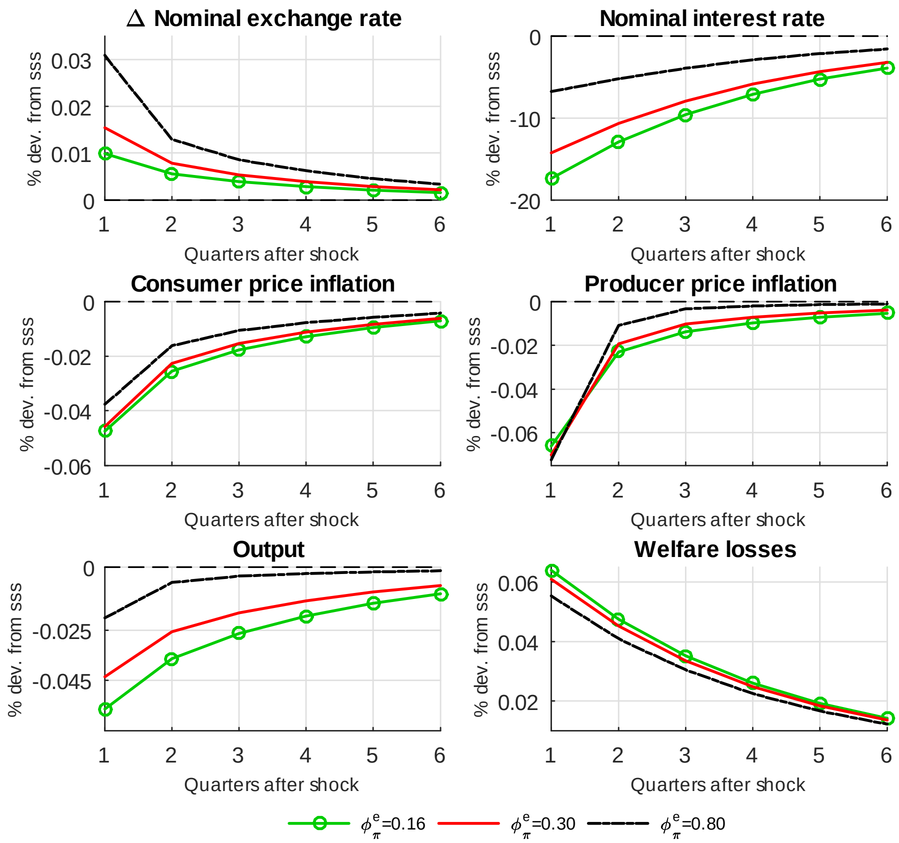

Figure 11: IRFs for a SOE under exchange rate rules with varying sensitivity to inflation (ϕ_π^e) for a one standard deviation shock to uncertainty in foreign demand

that even with ϕ_π^e as high as 0.8, the variability of the nominal exchange rate is much lower compared to Taylor interest rate rules.

To summarize, there exists a trade-off between stabilizing the nominal exchange rate and inflation-output with exchange rate rules. The choice of ϕ_π^e by a central bank should thus depend on the weight it puts on variability of the nominal exchange rates and the inflation in its objective function. Furthermore, the trade off can be noticed in Figure 11, in Subplots (1,1), (2,1), (3,1) corresponding to the nominal exchange rate, consumer price inflation and output, respectively. Welfare losses reduce by 14 per cent when ϕ_π^e increases from 0.16 to 0.80 due to more stabilized consumer price inflation and output. The higher value of ϕ_π^e ensures that exchange rates respond more to the change in key fundamental variables governing the domestic economy.

4 Conclusion

Current monetary policy framework in most central banks of EMEs follows a flexible inflation targeting regime with interest rate rule as an instrument. This paper attempts to show that the present approach of monetary policy is not effective in the presence of global uncertainty shocks for EMEs. This happens because the movement of capital and nominal exchange is primarily driven by a hedging motive in the presence of global uncertainty shocks and the uncovered interest rate parity condition does not hold in this scenario. We build a small open economy NK-DSGE model to qualitatively fit the stylized facts from the data and compare responses of an economy with alternate monetary policy rules. To the best of our knowledge this is the first paper analyzing the effects of an uncertainty shock in a small open economy NK-DSGE model. The small open economy is calibrated to a prototypical EME.

It is observed that a monetary policy using an exchange rate as an instrument to stabilize price and output in an economy, under flexible inflation targeting regime, reduces the welfare losses by upto 21 per cent. The exchange rate rules also reduce the variability of nominal and real variables in the long-run when an economy faces global uncertainty shocks. To be specific, the second order moments from the model show that the variability of nominal exchange rates, output and CPI is reduced by 85 per cent, 36 per cent and 45 per cent, respectively, when exchange rate rules are followed instead of interest rate rules. This happens because, exchange rate rules are associated with a lower risk premium which reduces the real effect of uncertainty shocks on the domestic economy. This paper thus proposes a dual instrument approach under current flexible inflation targeting regime in EMEs where the two instruments are namely, nominal interest rate and nominal exchange rate. The monetary policy can switch between the two instruments depending on the expected future domestic and global economic scenarios.

Since the instruments available with the central banks are limited, the future research agenda includes looking at the role of macroprudential regulation within this framework in stabilizing an EME during periods of high global uncertainty. Moreover, the current model framework does not feature some of the frictions standard in the literature (like imperfections in domestic financial markets or transactions costs) typical of an EME. For future research, we believe that adding the following features to the model can make the framework richer: (1) Adding trend inflation rate to a small open economy (EME). This would allow us to analyze the case of a zero lower bound (ZLB) in the foreign economy (AE) leaving the domestic economy (EME) unconstrained (i.e. no ZLB). (2) Introducing foreign borrowing by domestic firms as working capital loans. This way external debt in major currencies can be introduced.

References

Agenor, P.R. & Montiel, P.J. (1999). *Development Macroeconomics*, 2^{nd} Edition. Princeton University Press, Princeton.

Aizenman, J. Hutchison, M. & Noy, I. (2011). *World Development.* Vol. 39, No. 5, pp. 712–724.

Backus, D.K., Gavazzoni, F., Telmer, C. & Zin, S.E. (2010). Monetary Policy and the Uncovered Interest Parity Puzzle. NBER Working Paper 16218.

Basu, S. & Bundick, B.(2017). Uncertainty Shocks in a Model of Effective Demand. *Econometrica*, 85(3), 937–958.

Benigno, G. & De Paoli, B. (2010). On the International Dimension of Fiscal Policy. *Journal of Money, Credit and Banking*, Vol. 42, No. 8, 1523-1542.

Benigno, G., Benigno, P. & Nisticò, S. (2012). Risk, Monetary Policy and The Exchange Rate. *NBER Macroeconomics Annual* 2011, 247-309.

Benigno, G., Benigno P., & Nisticò, S. (2013). Second- Order Approximation of Dynamic Models with Time-Varying Risk. *Journal of Economic Dynamics and Control,* 37 (7), 1231-1247.

Bloom, N. (2009). The Impact of Uncertainty Shocks. *Econometrica*, 77(3), 623–685.

Bloom, N., Floetotto, M., Jaimovich N., Saporta-Eksten, I. & Terry, S.J. (2018). Really uncertain business cycles. *Econometrica*. 86(3). 1031–1065.

Calvo, G. (1983). Staggered prices in a utility maximizing framework. *Journal of Monetary Economics, 12* (3), 383-398.

Calvo, G.A. & Reinhart, C.M. (2002). Fear of Floating. *The Quarterly Journal of Economics.* 117(2). 379-408.

Cespedes, L.F. & Swallow, Y.C. (2013). The impact of uncertainty shocks in emerging economies. *Journal of International Economics*, 90, 316–325.

Chatterjee, P. (2018). Asymmetric impact of uncertainty in recessions: are emerging countries more vulnerable? *Studies in Nonlinear Dynamics & Econometrics.* 20160148

Corsetti, G., Kuester, K. & Muller G.J. (2017). Fixed on Flexible: Rethinking Exchange Rate Regimes after the Great Recession. *IMF Economic Review,* 65(3). 586-632.

Cook, D. (2004). Monetary policy in emerging markets: Can liability dollarization explain contractionary devaluations? *Journal of Monetary Economics*. 51. 1155–1181.

Cuevas, A. & Topak S. (2008). Monetary Policy and Relative Price Shocks in South Africa and Other Inflation Targeters. IMF Working Paper, 289/2008. Washington, DC: International Monetary Fund.

Dedola, L., Rivolta, G. & Stracca, L. (2017) . If the Fed sneezes, who catches a cold? *Journal of International Economics*, 108, S23–S41.

Devereux, M.B. & Engel, C. (2003). Monetary Policy in the Open Economy Revisited: Price Setting and Exchange-Rate Flexibility. *Review of Economic Studies*. 70. 765–783.

Engel, C. (2011). Currency Misalignments and Optimal Monetary Policy: A Reexamination. *American Economic Review*. 101. 2796–2822.

Fernández-Villaverde, J., Guerrón-Quintana, P., Rubio-Ramírez, J. & Uribe, M., (2011). Risk matters: the real effects of volatility shocks. *American Economic Review,* 101, 2530–2561.

Fratzscher, M. (2012). Capital flows, push versus pull factors and the global financial crisis. *Journal of International Economics*. 88. 341–356.

Forbes, K.J. & Warnock, F.E. (2012). Capital flow waves: Surges, stops, flight, and retrenchment *Journal of International Economics*. 88. 235–251.

Gali, J. Gertler, M. & Lopez-Salido, J.D. (2001). European inflation dynamics. *European Economic Review*. 45. 1237-1270.

Galí, J. & Monacelli T. (2005). Monetary Policy and Exchange Rate Volatility in a Small Open Economy. *Review of Economic Studies*. 72(3). 707–34

Gali, J., Gertler, M., & Lopez-Salido, J. D. (2007). Markups, Gaps, and the Welfare Costs of Business Fluctuations. *The Review of Economics and Statistics*, 89(1), 4459.

Gourio, F., Siemer, M. & Verdelhan, A., (2013). International risk cycles. *Journal of International Economics* 89 (2), 471-484.

Heiperzt, J., Mihov, I. & Santacreu, A. M. (2017). The exchange rate as an instrument of monetary policy. Federal Reserve Bank of St. Loius. Working Paper 2017-028A.

Jorda, O. (2005). Estimation and Inference of Impulse Responses by Local Projections. *The American Economic Review*. 95(1). 161-182.

Korinek, A. (2018) Regulating capital flows to emerging markets: An externality view. *Journal of International Economics*, 111. 61–80.

Magrini, E., Montalbano, P., Winters, A. (2018). Household's vulnerability from trade in Vietnam. *World Development,* 112. 46–58.

McCallum, B. T. (2006). Singapore's exchange rate-centered monetary policy regime and its relevance for China. Staff paper 43. Monetary Authority of Singapore.

Menkhoff L., Sarno L., Schmeling M. & Schrimpf A. (2012). Carry trades and global foreign exchange rate volatility. *Journal of Finance*, 67 (2), pp. 681-718.

OECD National Accounts Database. DOI: https://stats.oecd.org/# Data accessed in January, 2019.

Parrado, E. (2004). Singapore's Unique Monetary Policy: How Does It Work? IMF Working Paper. WP/04/10.

Ravn, M.O. & Sterk, V. (2017). Job uncertainty and deep recessions. *Journal of Monetary Economics*, 90, 125–141.

Reinhart, C. (2000). The Mirage of Floating Exchange Rates. *American Economic Review*, 90 (2000), pp. 65-70.

Sbordone, A.M. (2002). Prices and unit labor costs: a new test of price stickiness. *Journal of Monetary Economics*. 49. 265–292.

Singh, R. & Subramanian, C. (2008). The optimal choice of monetary policy instruments in a small open economy. *Canadian Journal of Economics*. 41(1). 105-137.

Taylor, J. (1993). Discretion versus policy rules in practice. *Carnegie-Rochester conference series on public policy, 39,* 195–214.

World Bank (2018). *World Bank National Accounts Data*. DOI: https://datacatalog.worldbank.org/search?sort by=title&sort order=ASC. Data accessed in November, 2018.

Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy.* Princeton University Press.

Appendix

4.1 Data Appendix

4.1.1 Data description for empirical evidence

For the VXO series, we use the CBOE S&P 100 Volatility Index's daily series accessed from the Federal Reserve Bank of St. Louis database from 1996 to 2018. The series is available with daily frequency which we convert to quarterly series by taking simple quarterly averages. We create a quarterly panel data for 12 economies from 1996:Q1 to 2018:Q4. We consider six AEs (US, UK, Canada, Japan, Australia and South Korea) and six EMEs (Brazil, Indonesia, India, Mexico, Russia and South Africa).

The primary source for most of the macroeconomic series is the quarterly national accounts data compiled by the Organization for Economic Cooperation and Development (OECD). We consider a seasonally adjusted volume index for the following series: GDP, private consumption, government consumption and private investment (GFCF). The reference year for the all the data series in the dataset is 2010. For India we consider the nominal series data (for GDP, private consumption, government consumption and private investment (GFCF)) at current prices instead of the volume index data because the volume index data for India is available from 2011:Q1. We later adjust the nominal data series with the CPI (consumer price index) for India to get real indices for all the variables mentioned above.

We create trade balance (total exports-total imports) series from the quarterly nominal data series on total imports and total exports. To normalize the trade balance series we take the ratio of the trade balance to GDP. We get monthly series on nominal exchange rates (currency per US dollar) from the OECD. We create quarterly nominal exchange rate series by taking quarterly averages of the monthly series. The relative consumer price indices (in terms of US dollars) data is used to capture the real effective exchange rate. Any increase (decrease) in the index would thus mean currency appreciation (depreciation).

We use short term interest rate (per annum) series to approximate the nominal interest rate series (policy rate). We also consider money supply measures including broad money and narrow money as control variables for local projections. We consider seasonally adjusted narrow and broad money quarterly indices and adjust them with CPI series to get real narrow and broad money series.

We get the country wise quarterly series on net portfolio investment (US dollars) from the International Monetary Fund's International Financial Statistics (IFS). We also consider the net financial account (except exceptional financing) series as a control in local projections. Finally we create a ratio of net portfolio investment to GDP and net financial account to

GDP to normalize the series.

We HP filter following series for the analysis: VXO, real GDP, real private consumption, real government consumption, real private investment, trade balance ratio to GDP, net portfolio investment ratio to GDP, net financial account ratio to GDP, real narrow money and real broad money. The non-filtered series used during the analysis are CPI, nominal exchange rates, relative CPI and short term interest rates.

We run panel data local projections on the above described dataset. To get the impulse response on a single variable, with VXO being an impulse variable, we control for all the variables with lag upto 4 periods over a horizon of 6 periods.

4.1.2 Data description for calibration

We estimate the degree of openness parameter, χ, to be 0.6, as the average trade share to GDP of emerging market economies. To get this we use the World Bank's country level trade data for year 2015. We take the average for 13 emerging market economies, namely: Argentina, Brazil, Chile, Colombia, Costa Rica, Hungary, India, Indonesia, Mexico, Poland, Russia, South Africa and Turkey to get average value as 0.6. We get the trade share of each country as a ratio of the total value of trade of a country with the world to the value of country's GDP, for year 2015.

The value of the initial parameter in the asset market condition, κ, is estimated to be 3.8. From the asset market condition, $\kappa = Q_0 \frac{C_0^{-\nu_D}}{\Gamma_{F,0} C_0^{*-\nu_F}}$ is a function of the initial (beginning of the time period) ratio of marginal utility of the domestic country to the foreign country and real exchange rates. We calculate this using the OECD database on annual national accounts. First, using the exchange rate and the consumption series at constant prices of 2015, we get real consumption series in US dollars. We then calculate the average for EMEs and AEs from 2005-2015. We consider 13 EMEs namely: Argentina, Brazil, Chile, Colombia, Costa Rica, Hungary, India, Indonesia, Mexico, Poland, Russia, South Africa, Turkey, and 31 AEs namely: Australia, Austria, Belgium, Canada, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Iceland, Ireland, Israel, Italy, Japan, Korea, Latvia, Lithuania, Luxembourg, Netherlands, New Zealand, Norway, Portugal, Slovakia, Slovenia, Spain, Sweden, Switzerland, UK and the US. We then calculate the marginal utilities ratio using the utility parameter (inverse of IES) as 1.5. [Calculation: $\kappa = (109293.4/266609)^{-1.5} = 3.8$].

4.2 Technical Appendix

4.2.1 Derivation of the demand functions

Demand for a variety i of domestic good by domestic households

$$\max_{C_{D,t}(i)} C_{D,t} = \left[\left(\frac{1}{n}\right)^{\frac{1}{\sigma}} \int_0^n \left(C_{D,t}\left(i\right)\right)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}}$$

subject to constraint,

$$\int_0^n P_{D,t}\left(i\right) C_{D,t}\left(i\right) di = Z_{D,t}$$

$$\mathcal{L}_t = \max_{C_{D,t}(i)} \left[\left[\left(\frac{1}{n}\right)^{\frac{1}{\sigma}} \int_0^n \left(C_{D,t}\left(i\right)\right)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}} - \lambda_{D,t} \left(\int_0^n P_{D,t}\left(i\right) C_{D,t}\left(i\right) di - Z_t \right) \right]$$

First order condition,

$$\frac{\partial \mathcal{L}}{\partial C_{D,t}\left(i\right)} = \frac{\sigma}{\sigma-1} \left(C_{D,t}\right)^{\frac{1}{\sigma-1}} \left(\frac{1}{n}\right)^{\frac{1}{\sigma}} \left(C_{D,t}\left(i\right)\right)^{\frac{\sigma-1}{\sigma}-1} - \lambda_{D,t} P_{D,t}\left(i\right) = 0$$

For any two variety i_1, i_2, we get,

$$\begin{aligned} \frac{\left(C_{D,t}\left(i_1\right)\right)^{-\frac{1}{\sigma}}}{\left(C_{D,t}\left(i_2\right)\right)^{-\frac{1}{\sigma}}} &= \frac{P_{D,t}\left(i_1\right)}{P_{D,t}\left(i_2\right)} \\ C_{D,t}\left(i_1\right) &= \left(\frac{P_{D,t}\left(i_1\right)}{P_{D,t}\left(i_2\right)}\right)^{-\sigma} C_{D,t}\left(i_2\right) \end{aligned}$$

Substituting the value in $C_{D,t}$,

$$\begin{aligned} C_{D,t} &= \left[\left(\frac{1}{n}\right)^{\frac{1}{\sigma}} \int_0^n \left(C_{D,t}\left(i_1\right)\right)^{\frac{\sigma-1}{\sigma}} di_1 \right]^{\frac{\sigma}{\sigma-1}} \\ &= \left[\left(\frac{1}{n}\right)^{\frac{1}{\sigma}} \int_0^n \left(\left(\frac{P_{D,t}\left(i_1\right)}{P_{D,t}\left(i_2\right)}\right)^{-\sigma} C_{D,t}\left(i_2\right) \right)^{\frac{\sigma-1}{\sigma}} di_1 \right]^{\frac{\sigma}{\sigma-1}} \\ &= \left[\left(\frac{1}{n}\right)^{\frac{1}{\sigma}} \int_0^n P_{D,t}\left(i_1\right)^{1-\sigma} di_1 \right]^{\frac{\sigma}{\sigma-1}} \frac{C_{D,t}\left(i_2\right)}{\left(P_{D,t}\left(i_2\right)\right)^{-\sigma}} \end{aligned}$$

let $\left[\left(\frac{1}{n}\right)\int_0^n P_{D,t}\left(i_2\right)^{1-\sigma} di_2\right]^{\frac{1}{1-\sigma}} = P_{D,t}$.

$$C_{D,t} = \left(\frac{1}{n}\right)^{-1} \left(P_{D,t}\right)^{-\sigma} \frac{C_{D,t}\left(i_2\right)}{\left(P_{D,t}\left(i_2\right)\right)^{-\sigma}}$$

Above equation can be re-arranged for a variety i as,

$$C_{D,t}\left(i\right) = \left(\frac{1}{n}\right)\left(\frac{P_{D,t}\left(i\right)}{P_{D,t}}\right)^{-\sigma} C_{D,t}$$

where,

$$P_{D,t} = \left[\left(\frac{1}{n}\right)\int_0^n P_{D,t}\left(i\right)^{1-\sigma} di\right]^{\frac{1}{1-\sigma}}$$

Substituting the value of $C_{D,t}\left(i\right) = \left(\frac{1}{n}\right)\left(\frac{P_{D,t}(i)}{P_{D,t}}\right)^{-\sigma} C_{D,t}$ in the constraint,

$$\begin{aligned}
\int_0^n P_{D,t}\left(i\right)\left(\frac{1}{n}\right)\left(\frac{P_{D,t}\left(i\right)}{P_{D,t}}\right)^{-\sigma} C_{D,t} di &= Z_{D,t} \\
\left(P_{D,t}\right)^{1-\sigma}\left(P_{D,t}\right)^{\sigma} C_{D,t} &= Z_{D,t} \\
P_{D,t}C_{D,t} &= Z_{D,t}
\end{aligned}$$

Similarly it can be shown,

$$\begin{aligned}
C_{F,t}\left(i\right) &= \left(\frac{1}{1-n}\right)\left(\frac{P_{F,t}\left(i\right)}{P_{F,t}}\right)^{-\sigma} C_{F,t} \text{ ; where } P_{F,t} = \left[\left(\frac{1}{1-n}\right)\int_n^1 P_{F,t}\left(i\right)^{1-\sigma} di\right]^{\frac{1}{1-\sigma}} \\
C^*_{D,t}\left(i\right) &= \left(\frac{1}{n}\right)\left(\frac{P_{D,t}\left(i\right)}{P_{D,t}}\right)^{-\sigma} C^*_{D,t}\text{; where } P^*_{D,t} = \left[\left(\frac{1}{n}\right)\int_0^n P^*_{D,t}\left(i\right)^{1-\sigma} di\right]^{\frac{1}{1-\sigma}} \\
C^*_{F,t}\left(i\right) &= \left(\frac{1}{1-n}\right)\left(\frac{P_{F,t}\left(i\right)}{P_{F,t}}\right)^{-\sigma} C^*_{F,t}\text{; where } P^*_{F,t} = \left[\left(\frac{1}{1-n}\right)\int_n^1 P^*_{F,t}\left(i\right)^{1-\sigma} di\right]^{\frac{1}{1-\sigma}}
\end{aligned}$$

by maximizing $C_{F,t} = \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\sigma}}\int_n^1 \left(C_{F,t}\left(i\right)\right)^{\frac{\sigma-1}{\sigma}} di\right]^{\frac{\sigma}{\sigma-1}}$ subject to $\int_n^1 P_{F,t}\left(i\right)C_{F,t}\left(i\right)di = Z_{F,t}$, $C^*_{D,t} = \left[\left(\frac{1}{n}\right)^{\frac{1}{\sigma}}\int_0^n \left(C^*_{D,t}\left(i\right)\right)^{\frac{\sigma-1}{\sigma}} di\right]^{\frac{\sigma}{\sigma-1}}$ subject to $\int_0^n P^*_{D,t}\left(i\right)C^*_{D,t}\left(i\right)di = Z^*_{D,t}$ and $C^*_{F,t} = \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\sigma}}\int_n^1 \left(C^*_{F,t}\left(i\right)\right)^{\frac{\sigma-1}{\sigma}} di\right]^{\frac{\sigma}{\sigma-1}}$ subject to $\int_n^1 P^*_{F,t}\left(i\right)C^*_{F,t}\left(i\right)di = Z^*_{D,t}$, respectively. It can also be shown that expenditure $Z_{F,t} = P_{F,t}C_{F,t}$, $Z^*_{D,t} = P^*_{D,t}C^*_{D,t}$, $Z^*_{F,t} = P^*_{F,t}C^*_{F,t}$.

For the domestic and foreign goods in the total consumption basket

$$\max_{C_{D,t},C_{F,t}} C_t = \left[(\mu_D)^{1/\xi_D}(C_{D,t})^{\frac{\xi_D-1}{\xi_D}} + (1-\mu_D)^{1/\xi_D}(C_{F,t})^{\frac{\xi_D-1}{\xi_D}}\right]^{\frac{\xi_D}{\xi_D-1}}$$

subject to,

$$P_{D,t}C_{D,t} + P_{F,t}C_{F,t} = Z_t$$

$$\mathcal{L}_t = \left[(\mu_D)^{1/\xi_D}(C_{D,t})^{\frac{\xi_D-1}{\xi_D}} + (1-\mu_D)^{1/\xi_D}(C_{F,t})^{\frac{\xi_D-1}{\xi_D}}\right]^{\frac{\xi_D}{\xi_D-1}} - \lambda_{D,t}\left[P_{D,t}C_{D,t} + P_{F,t}C_{F,t} - Z_t\right]$$

The first order conditions are,

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial C_{D,t}} &= (C_t)^{\frac{1}{\xi_D-1}}(\mu_D)^{1/\xi_D}(C_{D,t})^{\frac{\xi_D-1}{\xi_D}-1} - \lambda_{D,t}P_{D,t} = 0 \\
\frac{\partial \mathcal{L}}{\partial C_{F,t}} &= (C_t)^{\frac{1}{\xi_D-1}}(1-\mu_D)^{1/\xi_D}(C_{F,t})^{\frac{\xi_D-1}{\xi_D}-1} - \lambda_{D,t}P_{F,t} = 0
\end{aligned}$$

Combining the above two conditions we get,

$$C_{F,t} = \frac{(1-\mu_D)}{\mu_D}\left(\frac{P_{F,t}}{P_{D,t}}\right)^{-\xi_D} C_{D,t}$$

Substituting this value in the consumption bundle, we get

$$\begin{aligned}
C_t &= \left[(\mu_D)^{1/\xi_D}(C_{D,t})^{\frac{\xi_D-1}{\xi_D}} + (1-\mu_D)^{1/\xi_D}\left(\frac{(1-\mu_D)}{\mu_D}\left(\frac{P_{F,t}}{P_{D,t}}\right)^{-\xi_D} C_{D,t}\right)^{\frac{\xi_D-1}{\xi_D}}\right]^{\frac{\xi_D}{\xi_D-1}} \\
&= \left[\frac{(\mu_D)^{\frac{\xi_D-1}{\xi_D}}(\mu_D)^{1/\xi_D}(P_{D,t})^{1-\xi_D} + (1-\mu_D)^{1/\xi_D}(1-\mu_D)^{\frac{\xi_D-1}{\xi_D}}(P_{F,t})^{1-\xi_D}}{(\mu_D)^{\frac{\xi_D-1}{\xi_D}}(P_{D,t})^{1-\xi_D}}\right]^{\frac{\xi_D}{\xi_D-1}} C_{D,t} \\
&= (\mu_D)^{-1}(P_{D,t})^{\xi_D} C_{D,t}\left[\mu_D (P_{D,t})^{1-\xi_D} + (1-\mu_D)(P_{F,t})^{1-\xi_D}\right]^{\frac{\xi_D}{\xi_D-1}}
\end{aligned}$$

Assuming,

$$P_t = \left[\mu_D (P_{D,t})^{1-\xi_D} + (1-\mu_D)(P_{F,t})^{1-\xi_D}\right]^{\frac{1}{1-\xi_D}}$$

$$C_t \;=\; (\mu_D)^{-1}(P_{D,t})^{\xi_D} C_{D,t} (P_t)^{-\xi_D}$$

$$\therefore \;\; C_{D,t} = \mu_D (T_{D,t})^{-\xi_D} C_t$$

Similarly substituting,

$$C_{D,t} = \frac{\mu_D}{(1-\mu_D)} \left(\frac{P_{D,t}}{P_{F,t}}\right)^{-\xi_D} C_{F,t}$$

in C_t we get,

$$\begin{aligned} C_t &= \left[(\mu_D)^{1/\xi_D}\left(\frac{\mu_D}{(1-\mu_D)}\left(\frac{P_{D,t}}{P_{F,t}}\right)^{-\xi_D} C_{F,t}\right)^{\frac{\xi_D-1}{\xi_D}} + (1-\mu_D)^{1/\xi_D}(C_{F,t})^{\frac{\xi_D-1}{\xi_D}}\right]^{\frac{\xi_D}{\xi_D-1}} \\ &= (P_t)^{-\xi_D}\frac{C_{F,t}(P_{F,t})^{\xi_D}}{(1-\mu_D)} \end{aligned}$$

Re-arranging the above equation,

$$\begin{aligned} C_{F,t} &= (1-\mu_D)\left(\frac{P_{F,t}}{P_t}\right)^{-\xi_D} C_t \\ C_{F,t} &= (1-\mu_D)(T_{F,t})^{-\xi_D} C_t \end{aligned}$$

Substituting the demand functions in the constraint,

$$\begin{aligned} P_{D,t}\mu_D\left(\frac{P_{D,t}}{P_t}\right)^{-\xi_D} C_t + P_{F,t}(1-\mu_D)\left(\frac{P_{F,t}}{P_t}\right)^{-\xi_D} C_t &= Z_t \\ \left[\frac{\mu_D(P_{D,t})^{1-\xi_D} + (1-\mu_D)(P_{F,t})^{1-\xi_D}}{(P_t)^{-\xi_D}}\right] C_t &= Z_t \\ P_t C_t &= Z_t \end{aligned}$$

Similarly, maximizing the aggregate consumption bundle C_t^* subject to the expenditure on the bundle:

$$\max_{C_{D,t}^8, C_{F,t}^*} C_t^* = \left[(\mu_F)^{1/\xi_F}\left(C_{D,t}^*\right)^{\frac{\xi_F-1}{\xi_F}} + (1-\mu_F)^{1/\xi_F}\left(C_{F,t}^*\right)^{\frac{\xi_F-1}{\xi_F}}\right]^{\frac{\xi_F}{\xi_F-1}}$$

subject to,

$$P^*_{D,t}C^*_{D,t} + P^*_{F,t}C^*_{F,t} = Z^*_t.$$

We get the following,

$$\begin{aligned}
C^*_{D,t} &= \mu_F \left(\frac{P^*_{D,t}}{P^*_t}\right)^{-\xi_F} C^*_t \\
&\text{and} \\
C^*_{F,t} &= (1-\mu_F)\left(\frac{P^*_{F,t}}{P^*_t}\right)^{-\xi_F} C^*_t \\
\text{where } P^*_t &= \left[\mu_F \left(P^*_{D,t}\right)^{1-\xi_F} + (1-\mu_F)\left(P^*_{F,t}\right)^{1-\xi_F}\right]^{\frac{1}{1-\xi_F}}
\end{aligned}$$

It can also be shown that total expenditure $Z^*_t = P^*_t C^*_t$.

4.2.2 Derivation of Euler's equation and labour supply equation

For domestic households,

$$\max U(C_t, H_{D,t}) = \frac{(C_t)}{1-\nu_D}^{1-\nu_D} - \omega_D \frac{(H_{D,t})}{1+\eta_D}^{1+\eta_D}$$

subject to the constraint,

$$W_{D,t}H_{D,t} + profit_{D,t} = P_tC_t - B_{D,t} + E_t\left\{B_{D,t+1}M_{t,t+1}\right\}$$

Writing the above constraints in real terms implies,

$$\begin{aligned}
\frac{W_{D,t}H_{D,t} + profit_t}{P_t} &= \frac{P_tC_t}{P_t} - \frac{B_{D,t}}{P_t} + \frac{E_t\left\{B_{D,t+1}M_{t,t+1}\right\}}{P_t} \\
w_{D,t}T_{D,t}H_{D,t} + \ \Omega_{D,t} &= C_t - \frac{B_{D,t}}{P_t} + \frac{E_t\left\{B_{D,t+1}M_{t,t+1}\right\}}{P_t}
\end{aligned}$$

where $w_{D,t} = \frac{W_{D,t}}{P_{D,t}}$, $T_{D,t} = \frac{P_{D,t}}{P_t}$ and $\Omega_{D,t}$ are real profits.

Maximizing the utility subject to constraint,

$$\begin{aligned}
\mathcal{L}_t &= \max_{C_t, H_{D,t}, B_{D,t+1}} \sum_{t=0}^{\infty} \beta^t \Big[U(C_t, H_{D,t}) + \lambda_{D,t}\Big(w_{D,t}T_{D,t}H_{D,t} + \ \Omega_{D,t} - \\
&\quad C_t + \frac{B_{D,t}}{P_t} - \frac{E_t\left\{B_{D,t+1}M_{t,t+1}\right\}}{P_t}\Big)\Big]
\end{aligned}$$

The first order conditions are as follows,

$$
\begin{aligned}
\frac{\partial \mathcal{L}_t}{\partial C_t} &= U'_{C_t} - P_t \lambda_{D,t} = 0 \\
\frac{\partial \mathcal{L}_t}{\partial H_{D,t}} &= U'_{H_{D,t}} + \lambda_{D,t} w_{D,t} T_{D,t} = 0 \\
\frac{\partial \mathcal{L}}{\partial B_{D,t+1}} &= -\frac{\lambda_{D,t} E_t \{M_{t,t+1}\}}{P_t} + \beta E_t \left\{ \frac{\lambda_{D,t+1}}{P_{t+1}} \right\} = 0
\end{aligned}
$$

where for the considered utility function, $U'_{C_t} = (C_t)^{-\nu_D}$, $U'_{H_{D,t}} = -\omega_D (H_{D,t})^{\eta_D}$, thus

$$
\begin{aligned}
\lambda_{D,t} &= (C_t)^{-\nu_D} \\
\lambda_{D,t} &= \frac{\omega_D (H_{D,t})^{\eta_D}}{w_{D,t} T_{D,t}} \\
E_t \{\pi_{t+1} M_{t,t+1}\} &= \beta \frac{E_t \{\lambda_{D,t+1}\}}{\lambda_{D,t}} \\
\text{where } E_t \{M_{t,t+1}\} &= \frac{1}{(1+R_t)}
\end{aligned}
$$

Similarly for foreign households,

$$
\max U(C_t^*, H_{F,t}) = \frac{\Gamma_{F,t} (C_t^*)}{1-\nu_F}^{1-\nu_F} - \omega_F \frac{(H_{F,t})}{1+\eta_F}^{1+\eta_F}
$$

subject to the constraint,

$$
W_{F,t} H_{F,t} + profit_{F,t} = P_t^* C_t^* - B_{F,t} + B_{F,t+1} E_t \left\{ M_{t,t+1}^* \right\}
$$

Writing the above constraints in real terms,

$$
\begin{aligned}
\frac{W_{F,t} H_{F,t} + profit_t^*}{P_t^*} &= \frac{P_t^* C_t^*}{P_t^*} - \frac{B_{F,t}}{P_t^*} + \frac{E_t \left\{ M_{t,t+1}^* B_{F,t+1} \right\}}{P_t^*} \\
w_{F,t} \frac{T_{F,t}}{Q_t} H_{F,t} + \ \Omega_t^* &= C_t^* - \frac{B_{F,t}}{P_t^*} + \frac{E_t \left\{ M_{t,t+1}^* B_{F,t+1} \right\}}{P_t^*}
\end{aligned}
$$

where $w_{F,t} = \frac{W_{F,t}}{P_{F,t}^*}$, $T_{F,t} = \frac{P_{F,t}}{P_t}$, $Q_t = \frac{X_t P_t^*}{P_t}$, $\pi_t^* = \frac{P_{t+1}^*}{P_t^*}$ and Ω_t^* are real profits.

Maximizing the utility subject to constraint,

$$
\mathcal{L}_t = \max_{C_t^*, H_{F,t}, B_{F,t+1}} \sum_{t=0}^{\infty} \beta^t \left[U(C_t^*, H_{F,t}) + \lambda_{F,t} \left(w_{F,t} \frac{T_{F,t}}{Q_t} H_{F,t} + \ \Omega_t^* - C_t^* + \frac{B_{F,t}}{P_t^*} - \frac{E_t \left\{ M_{t,t+1}^* B_{F,t+1} \right\}}{P_t^*} \right) \right]
$$

The first order conditions are as follows,

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial C_t^*} &= U'_{C_t^*} - \lambda_{F,t} = 0 \\
\frac{\partial \mathcal{L}}{\partial H_{F,t}} &= U'_{H_{F,t}} + \lambda_{F,t} w_{F,t} \frac{T_{F,t}}{Q_t} = 0 \\
\frac{\partial \mathcal{L}}{\partial B_{F,t+1}} &= -\frac{\lambda_{F,t} E_t \left\{ M_{t,t+1}^* \right\}}{P_t^*} + \beta \frac{\lambda_{F,t+1}}{P_{t+1}^*} = 0
\end{aligned}$$

where for the considered utility function, $U'_{C_t^*} = \Gamma_{F,t} \left(C_t^* \right)^{-\nu_F}, U'_{H_{F,t}} = -\omega_F \left(H_{F,t} \right)^{\eta_F}$

$$\begin{aligned}
\lambda_{F,t} &= \Gamma_{F,t} \left(C_t^* \right)^{-\nu_F} \\
\lambda_{F,t} &= \frac{\omega_F \left(H_{F,t} \right)^{\eta_F} Q_t}{w_{F,t} T_{F,t}} \\
E_t \left\{ \pi_{t+1}^* M_{t,t+1}^* \right\} &= \beta \frac{E_t \left\{ \lambda_{F,t+1} \right\}}{\lambda_{F,t}} \\
\text{where } E_t \left\{ M_{t,t+1}^* \right\} &= \frac{1}{(1 + R_t^*)}
\end{aligned}$$

4.2.3 Derivation of price-setting equations

For domestic firms: since the domestic sector is a sticky price sector, $(1 - \alpha_D)$ firms which can optimize, maximize the following profit function,

$$\max_{\overline{P}_{D,t}(i)} \sum_{k=0}^{\infty} \alpha_D^k M_{t,t+k} \left(\overline{P}_{D,t}(i) Y_{D,t_{+k}}(i) - MC_{D,t+k} Y_{D,t_{+k}}(i) \right)$$

$$\text{where } Y_{D,t+k}(i) = \left(\frac{\overline{P}_{D,t}(i)}{P_{D,t+k}} \right)^{-\sigma} Y_{D,t+k}$$

$$\begin{aligned}
\frac{\partial \mathcal{L}_t}{\partial \overline{P}_{D,t}(i)} &= \sum_{k=0}^{\infty} \alpha_D^k M_{t,t+k} \left(Y_{D,t+k}(i) + \overline{P}_{D,t}(i) \frac{\partial Y_{D,t+k}(i)}{\partial \overline{P}_{D,t}(i)} - MC_{D,t+k} \frac{\partial Y_{D,t+k}(i)}{\partial \overline{P}_{D,t}(i)} \right) = 0 \\
\textit{where } \frac{\partial Y_{D,t+k}(i)}{\partial \overline{P}_{D,t}(i)} &= -\sigma \left(\frac{\overline{P}_{D,t}(i)}{P_{D,t+k}} \right)^{-\sigma} \frac{1}{\overline{P}_{D,t}(i)} Y_{D,t+k} \\
&= -\sigma \frac{Y_{D,t+k}(i)}{\overline{P}_{D,t}(i)}
\end{aligned}$$

Therefore,

$$\sum_{k=0}^{\infty} \alpha_D^k M_{t,t+k} \left(Y_{D,t+k}(i) + \overline{P}_{D,t}(i) \left(-\sigma \frac{Y_{D,t+k}(i)}{\overline{P}_{D,t}(i)} \right) - MC_{D,t+k} \left(-\sigma \frac{Y_{D,t+k}(i)}{\overline{P}_{D,t}(i)} \right) \right) = 0$$

$$\overline{P}_{D,t}(i) = \frac{\sigma}{\sigma - 1} \frac{\sum\limits_{k=0}^{\infty} \alpha_D^k M_{t,t+k} MC_{D,t+k} Y_{D,t+k}(i)}{\sum\limits_{k=0}^{\infty} \alpha_D^k M_{t,t+k} Y_{D,t+k}(i)}$$

The remaining α_D share of the firms keep their price the same as the aggregate of last year prices, such that the aggregate price in the manufacturing sector is

$$\left(P_{D,t}(i)\right)^{-\sigma} = \alpha_D \left(P_{D,t-1}(i)\right)^{-\sigma} + (1 - \alpha_D) \left(\overline{P}_{D,t}(i)\right)^{-\sigma}$$

Writing the price equation recursively, note that the stochastic discount factor, $M_{t,t+k}$, is given by

$$M_{t,t+k} = \frac{1}{(1 + R_t)}$$

Now from the household's optimization,

$$M_{t,t+k} = \beta^k \frac{\lambda_{D,t+k}}{\lambda_{D,t}} \frac{P_t}{P_{t+k}}$$

$$\begin{aligned}
\overline{P}_{D,t}(i) &= \frac{\sigma}{\sigma - 1} \frac{\sum\limits_{k=0}^{\infty} \alpha_D^k M_{t,t+k} \frac{MC_{D,t+k}}{P_{D,t+k}} P_{D,t+k} \left(\frac{\overline{P}_{D,t}(i)}{P_{D,t+k}} \right)^{-\sigma} Y_{D,t+k}}{\sum\limits_{k=0}^{\infty} \alpha_D^k M_{t,t+k} \left(\frac{\overline{P}_{D,t}(i)}{P_{D,t+k}} \right)^{-\sigma} Y_{D,t+k}} \\
&= \frac{\sigma}{\sigma - 1} \frac{\sum\limits_{k=0}^{\infty} \alpha_D^k \beta^k \frac{\lambda_{D,t+k}}{\lambda_{D,t}} \frac{P_t}{P_{t+k}} mc_{D,t+k} \left(P_{D,t+k}\right)^{\sigma+1} Y_{D,t+k}}{\sum\limits_{k=0}^{\infty} \alpha_D^k \beta^k \frac{\lambda_{D,t+k}}{\lambda_{D,t}} \frac{P_t}{P_{t+k}} \left(P_{D,t+k}\right)^{\sigma} Y_{D,t+k}} \\
&= \frac{\sigma}{\sigma - 1} \frac{\sum\limits_{k=0}^{\infty} \left(\alpha_D \beta\right)^k \lambda_{D,t+k} mc_{D,t+k} T_{D,t+k} \left(P_{D,t+k}\right)^{\sigma} Y_{D,t+k}}{\sum\limits_{k=0}^{\infty} \left(\alpha_D \beta\right)^k \lambda_{D,t+k} T_{D,t+k} \left(P_{D,t+k}\right)^{\sigma-1} Y_{D,t+k}}
\end{aligned}$$

$$
\begin{aligned}
\frac{\overline{P}_{D,t}}{P_{D,t-1}} &= \frac{\sigma}{\sigma-1}\frac{\sum\limits_{k=0}^{\infty}\left(\alpha_D\beta\right)^k\lambda_{D,t+k}mc_{D,t+k}T_{D,t+k}\left(\frac{P_{D,t+k}}{P_{D,t-1}}\right)^{\sigma}Y_{D,t+k}}{\sum\limits_{k=0}^{\infty}\left(\alpha_D\beta\right)^k\lambda_{D,t+k}T_{D,t+k}\left(\frac{P_{D,t+k}}{P_{D,t-1}}\right)^{\sigma-1}Y_{D,t+k}} \\
\overline{\pi}_{D,t} &= \frac{\sigma}{\sigma-1}\frac{\sum\limits_{k=0}^{\infty}\left(\alpha_D\beta\right)^k\lambda_{D,t+k}mc_{D,t+k}T_{D,t+k}\left(\pi_{D,t}\times\pi_{D,t+1}\times\pi_{D,t+2}\times\ldots..\pi_{D,t+k}\right)^{\sigma}Y_{D,t+k}}{\sum\limits_{k=0}^{\infty}\left(\alpha_D\beta\right)^k\lambda_{D,t+k}T_{D,t+k}\left(\pi_{D,t}\times\pi_{D,t+1}\times\pi_{D,t+2}\times\ldots..\pi_{D,t+k}\right)^{\sigma-1}Y_{D,t+k}}
\end{aligned}
$$

We can write $\pi^*_{D,t}$ in recursive form,

$$\overline{\pi}_{D,t} = \frac{\sigma}{\sigma-1}\pi_{D,t}\frac{X_{D,t}}{Z_{D,t}}$$

where,

$$
\begin{aligned}
X_{D,t} &= \lambda_{D,t}Y_{D,t}mc_{D,t}T_{D,t} + \alpha_D\beta\left(\pi_{D,t+1}\right)^{\sigma}E_t\left\{X_{D,t+1}\right\} \\
Z_{D,t} &= \lambda_{D,t}Y_{D,t}T_{D,t+k} + \alpha_D\beta\left(\pi_{D,t+1}\right)^{\sigma-1}E_t\left\{Z_{D,t+1}\right\}
\end{aligned}
$$

Aggregate prices for domestically produced goods is given by,

$$
\begin{aligned}
P_{D,t} &= \left[\left(\frac{1}{n}\right)\int_0^n P_{D,t}\left(i\right)^{1-\sigma}di\right]^{\frac{1}{1-\sigma}} \\
&\quad i \text{ is a variety here.} \\
\left(P_{D,t}\right)^{(1-\sigma)} &= \left(\frac{1}{n}\right)\left[\int_0^{n\alpha_D}P_{D,t-1}(i)^{1-\sigma}di + \int_{n\alpha_D}^{n}\overline{P}_{D,t}(i)^{1-\sigma}di\right] \\
&= \left(\frac{1}{n}\right)\left[n\alpha_D\left(P_{D,t-1}\left(i\right)\right)^{1-\sigma} + n\left(1-\alpha_D\right)\left(\overline{P}_{D,t}\left(i\right)\right)^{1-\sigma}\right] \\
&\quad \text{dropping } i \text{ due to symmetry,} \\
\left(P_{D,t}\right)^{(1-\sigma)} &= \alpha_D\left(P_{D,t-1}\right)^{1-\sigma} + \left(1-\alpha_D\right)\left(\overline{P}_{D,t}\right)^{1-\sigma}
\end{aligned}
$$

$$\therefore\ P_{D,t} = \left[\alpha_D\left(P_{D,t-1}\right)^{1-\sigma} + \left(1-\alpha_D\right)\left(\overline{P}_{D,t}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$

Re-writing this in recursive form yields,

$$
\begin{aligned}
\frac{P_{D,t}}{P_{D,t-1}} &= \left[\alpha_D\left(\frac{P_{D,t-1}}{P_{D,t-1}}\right)^{1-\sigma} + \left(1-\alpha_D\right)\left(\frac{\overline{P}_{D,t}}{P_{D,t-1}}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \\
\pi_{D,t} &= \left[\alpha_D + \left(1-\alpha_D\right)\left(\overline{\pi}_{D,t}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}
\end{aligned}
$$

Similarly for foreign firms, where $(1-\alpha_F)$ firms can optimize, they maximize the following profit function,

$$\max_{\overline{P}_{F,t}(i)} \sum_{k=0}^{\infty} \alpha_F^k M_{t,t+k}^* \left(\overline{P}_{F,t}(i) Y_{F,t_{+k}}(i) - MC_{F,t+k} Y_{F,t_{+k}}(i)\right)$$

$$\text{where } Y_{F,t+k}(i) = \left(\frac{\overline{P}_{F,t}(i)}{P_{F,t+k}^*}\right)^{-\sigma} Y_{F,t+k}$$

To get the price setting equation,

$$\overline{P}_{F,t}(i) = \frac{\sigma}{\sigma-1} \frac{\sum_{k=0}^{\infty} \alpha_F^k M_{t,t+k}^* MC_{F,t+k} Y_{F,t+k}(i)}{\sum_{k=0}^{\infty} \alpha_F^k M_{t,t+k}^* Y_{F,t+k}(i)}$$

which can written recursively as,

$$\overline{\pi}_{F,t} = \frac{\sigma}{\sigma-1} \pi_{F,t}^* \frac{X_{F,t}}{Z_{F,t}}$$

where,

$$\begin{aligned} X_{F,t} &= \lambda_{F,t} Y_{F,t} mc_{F,t} \frac{T_{F,t}}{Q_t} + \alpha_F \beta \left(\pi_{F,t+1}^*\right)^{\sigma} E_t \left\{X_{F,t+1}\right\} \\ Z_{F,t} &= \lambda_{F,t} Y_{F,t} \frac{T_{F,t}}{Q_t} + \alpha_F \beta \left(\pi_{F,t+1}^*\right)^{\sigma-1} E_t \left\{Z_{F,t+1}\right\} \end{aligned}$$

The aggregate foreign producer's price inflation is given by,

$$\pi_{F,t}^* = \left[\alpha_F + (1-\alpha_F)\left(\overline{\pi}_{F,t}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$

4.2.4 Equilibrium

Aggregate demand functions for the domestic and foreign produce Total demand for each variety i of the output produced by domestic firms,

$$\begin{aligned} Y_{D,t}(i) &= C_{D,t}(i) = nC_{D,t}(i) + (1-n) C_{D,t}^*(i) \\ &= n\left(\frac{1}{n}\right)\left(\frac{P_{D,t}(i)}{P_{D,t}}\right)^{-\sigma} C_{D,t} + (1-n)\left(\frac{1}{n}\right)\left(\frac{P_{D,t}^*(i)}{P_{D,t}^*}\right)^{-\sigma} C_{D,t}^* \end{aligned}$$

Note, $P_{D,t}(i) = X_t P^*_{D,t}(i)$, $P_{D,t} = X_t P^*_{D,t}$, where X_t is the nominal exchange rate. Real exchange rate $Q_t = \frac{X_t P^*_t}{P_t}$, $T_t = \frac{P_{F,t}}{P_{D,t}}$ Thus,

$$\begin{aligned}
Y_{D,t}(i) &= n\left(\frac{1}{n}\right)\left(\frac{P_{D,t}(i)}{P_{D,t}}\right)^{-\sigma} C_{D,t} + (1-n)\left(\frac{1}{n}\right)\left(\frac{P_{D,t}(i)}{P_{D,t}}\right)^{-\sigma} C^*_{D,t} \\
&= \left(\frac{P_{D,t}(i)}{P_{D,t}}\right)^{-\sigma}\left[C_{D,t} + \left(\frac{1-n}{n}\right) C^*_{D,t}\right] \\
&= \left(\frac{P_{D,t}(i)}{P_{D,t}}\right)^{-\sigma} Y_{D,t}
\end{aligned}$$

Total demand for agricultural produce, $Y_{D,t} = C_{D,t} + \left(\frac{1-n}{n}\right) C^*_{D,t}$. Aggregate demand, $Y_{D,t}$, can be re-written as,

$$\begin{aligned}
Y_{D,t} &= C_{D,t} + \left(\frac{1-n}{n}\right) C^*_{D,t} \\
&= \mu_D \left(\frac{P_{D,t}}{P_t}\right)^{-\xi_D} C_t + \left(\frac{1-n}{n}\right)\mu_F\left(\frac{P^*_{D,t}}{P^*_t}\right)^{-\xi_F} C^*_t \\
&= \left(\frac{P_{D,t}}{P_t}\right)^{-\xi_D}\left[\mu_D C_t + \left(\frac{1-n}{n}\right)\mu_F\left(\frac{P^*_{D,t}}{P^*_t}\right)^{-\xi_F}\left(\frac{P_t}{P_{D,t}}\right)^{-\xi_D} C^*_t\right] \\
&= \left(\frac{P_{D,t}}{P_t}\right)^{-\xi_D}\left[\mu_D C_t + \left(\frac{1-n}{n}\right)\mu_F\left(\frac{X_t P_{D,t}}{X_t P_t Q_t}\right)^{-\xi_F}\left(\frac{P_{D,t}}{P_t}\right)^{\xi_D} C^*_t\right] \\
&= \left(\frac{P_{D,t}}{P_t}\right)^{-\xi_D}\left[\mu_D C_t + \left(\frac{1-n}{n}\right)\mu_F Q_t^{\xi_F}\left(\frac{P_{D,t}}{P_t}\right)^{\xi_D-\xi_F} C^*_t\right] \\
&= (T_{D,t})^{-\xi_D}\left[\mu_D C_t + \left(\frac{1-n}{n}\right)\mu_F Q_t^{\xi_F}(T_{D,t})^{\xi_D-\xi_F} C^*_t\right]
\end{aligned}$$

Similarly, total demand for each variety i of the output produced by foreign firms can be written as,

$$\begin{aligned}
Y_{F,t}(i) &= C_{F,t}(i) = nC_{F,t}(i) + (1-n)\, C^*_{F,t}(i) \\
&= n\left(\frac{1}{1-n}\right)\left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\sigma} C_{F,t} + \left(\frac{1-n}{1-n}\right)\left(\frac{P^*_{F,t}(i)}{P^*_{F,t}}\right)^{-\sigma} C^*_{F,t} \\
&= \left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\sigma} Y_{F,t}
\end{aligned}$$

where total demand for agricultural produce, $Y_{F,t} = \frac{n}{(1-n)} C_{F,t} + C^*_{F,t}$. Aggregate demand, $Y_{F,t}$, can be re-written as,

$$
\begin{aligned}
Y_{F,t} &= \frac{n}{(1-n)} C_{F,t} + C^*_{F,t} \\
&= \frac{n}{(1-n)} (1-\mu_D) \left(\frac{P_{F,t}}{P_t} \right)^{-\xi_D} C_t + (1-\mu_F) \left(\frac{P^*_{F,t}}{P^*_t} \right)^{-\xi_F} C^*_t \\
&= \frac{n}{(1-n)} (1-\mu_D) \left(\frac{P_{F,t}}{P_t} \right)^{-\xi_D} C_t + (1-\mu_F) \left(\frac{X_t P_{F,t}}{X_t Q_t P_t} \right)^{-\xi_F} C^*_t \\
&= \left(\frac{P_{F,t}}{P_t} \right)^{-\xi_D} \left[\frac{n}{(1-n)} (1-\mu_D) C_t + (1-\mu_F) Q_t^{\xi_F} \left(\frac{P_{F,t}}{P_t} \right)^{-\xi_F} \left(\frac{P_{F,t}}{P_t} \right)^{\xi_D} C^*_t \right] \\
&= (T_{F,t})^{-\xi_D} \left[\frac{n}{(1-n)} (1-\mu_D) C_t + (1-\mu_F) Q_t^{\xi_F} (T_{F,t})^{\xi_D - \xi_F} C^*_t \right]
\end{aligned}
$$

Labour market equilibrium For the domestic country, aggregate labour supply would equalize aggregate labour demand in equilibrium,

$$
\begin{aligned}
H_{D,t} &= \frac{1}{n} \int_0^n H_{D,t}(i)\, di \\
&= \frac{1}{n} \int_0^n \frac{Y_{D,t}(i)}{A_{D,t}} di \\
&= \frac{Y_{D,t}}{A_{D,t}} Disp_{D,t} \\
\text{where } Disp_{D,t} &= \frac{1}{n} \int_0^n \left(\frac{P_{D,t}(i)}{P_{D,t}} \right)^{-\sigma} di
\end{aligned}
$$

Re-writing $Disp_{D,t}$ in recursive form,

$$\begin{aligned}
Disp_{D,t} &= \frac{1}{n}\int\limits_0^n \frac{\alpha_D\left(P_{D,t-1}(i)\right)^{-\sigma}+(1-\alpha_D)\left(\overline{P}_{D,t}(i)\right)^{-\sigma}}{\left(P_{D,t}\right)^{-\sigma}}di \\
&= \alpha_D\frac{1}{n}\int\limits_0^n\left(\frac{P_{D,t-1}(i)}{P_{D,t}}\right)^{-\sigma}di+(1-\alpha_D)\frac{1}{n}\int\limits_0^n\left(\frac{\overline{P}_{D,t}(i)}{P_{D,t}}\right)^{-\sigma}di \\
&= \alpha_D\frac{1}{n}\int\limits_0^n\left(\frac{P_{D,t-1}(i)}{P_{D,t}}\frac{P_{D,t-1}}{P_{D,t-1}}\right)^{-\sigma}di+(1-\alpha_D)\left(\frac{\overline{P}_{D,t}}{P_{D,t}}\right)^{-\sigma} \\
&= \alpha_D\left(\frac{P_{D,t-1}}{P_{D,t}}\right)^{-\sigma}Disp_{D,t-1}+(1-\alpha_D)\left(\frac{\overline{P}_{D,t}}{P_{D,t}}\frac{P_{D,t-1}}{P_{D,t-1}}\right)^{-\sigma}
\end{aligned}$$

$$\begin{aligned}
Disp_{D,t} &= \alpha_D\left(\pi_{D,t}\right)^{\sigma}Disp_{D,t-1}+(1-\alpha_D)\left(\overline{\pi}_{D,t}\right)^{-\sigma}\left(\pi_{D,t}\right)^{\sigma} \\
&= \left(\pi_{D,t}\right)^{\sigma}\left[\alpha_D Disp_{D,t-1}+(1-\alpha_D)\left(\overline{\pi}_{D,t}\right)^{-\sigma}\right]
\end{aligned}$$

where $Disp_{D,t-1}=\frac{1}{n}\int\limits_0^n\left(\frac{P_{D,t-1}(i)}{P_{D,t-1}}\right)^{-\sigma}di.$

Similarly, in the foreign country labour supply in equilibrium would be,

$$\begin{aligned}
H_{F,t} &= \frac{Y_{F,t}}{A_{F,t}}Disp_{F,t} \\
\text{where } Disp_{F,t} &= \left(\frac{1}{1-n}\right)\int_n^1\left(\frac{P^*_{F,t}(i)}{P^*_{F,t}}\right)^{-\sigma}di
\end{aligned}$$

and $Disp_{F,t}$ can be written recursively as,

$$Disp_{F,t}=\left(\pi^*_{F,t}\right)^{\sigma}\left[\alpha_F Disp_{F,t-1}+(1-\alpha_F)\left(\overline{\pi}_{F,t}\right)^{-\sigma}\right]$$

where $Disp_{F,t-1}=\left(\frac{1}{1-n}\right)\int_n^1\left(\frac{P^*_{F,t-1}(i)}{P^*_{F,t-1}}\right)^{-\sigma}di.$