Fundamental Drivers of International Price and Consumption Disparities* †

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Abstract

In this study, we firstly propose a way to measure the tradability of 120+ commodities based on price dispersion across countries. This approach is used to construct price indices of tradables and non-tradables for 150+ countries. We document that the share of tradables in terms of total expenditure of consumers is lower for richer countries than poor countries and that the relative price of non-tradables, which plays an important role in the rate determination for real exchange, behaves in accordance with the Balassa-Samuelson effect. Secondly, we propose a common-factor approach (based on principal components) to compress the large volume of information on prices and quantities consumed globally. The ability of the factors to account for the variation in the data depends on the degree to which the original variables co-move, which is stronger for prices and weaker for quantities. We find that income is responsible for 98% of the variation in the first principal component of quantities. For prices, income plays a secondary role: It explains only 24% of the first component, but 85% of the second. These findings are robust to the inclusion of a range of explanatory variables, as well as to the level of data aggregation.

JEL classifications: F40, E01, C43

Keywords: Economic measurement; Price and consumption; Principal component analysis; Tradability

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1. Introduction

It is an empirical regularity that due to productivity differentials, the price of services is systematically higher in productive and developed countries than in less developed ones, while the price of tradable goods tends to be equalised internationally via arbitrage. However, various market frictions and barriers to trade may prevent the equalisation of the prices of tradables. Differences in the prices of comparable goods may adversely affect real consumption and income, and ultimately, the well-being of consumers. Recently, a growing body of literature has focused on examining the volatility of cross-country commodity prices (Feenstra et al., 2015; Inklaar and Rao, 2017) while their co-movement has been studied by other researchers (de Nicola et al., 2016; Bunn et al., 2017). The main objective of this paper is to study, through a unified framework, these two aspects of international pricing patterns, i.e., cross-country dispersion in prices and price co-movements among commodities and commodity groups. We also extend the analysis to consider their implications for global consumption patterns.

If, when converted to a common currency, the price of a product is equal in two countries, domestic-currency prices will respond one-for-one to exchange-rate changes, and thus currency values play no role in the changes in prices. This constitutes the fundamental basis of the concept of the "law of one price" (hereafter, the LOP) which is facilitated by arbitrage – the process of buying in the country where the product is cheap and selling where it is expensive. When prices are not equalised, prima facie there is a dead-weight efficiency loss that could be eliminated by transferring the product from the low-cost location to where it is more highly valued.[1] Due to prohibitive transportation costs, there are goods and services that are not traded internationally. Anderson and van Wincoop (2004) estimate the total trade cost for a rich country to be approximated by a 170-percent ad-valorem tax. This encompasses both domestic (retail and wholesale distribution) and international (transport and border-related trade barriers) costs of about 55% and 74%, respectively. The authors draw upon a mixture of literature that employ direct and indirect (inferences from trade volumes and prices) measures to determine trade costs; however, they emphasise the incompleteness and sparseness of data available across countries. According to these authors, policies that directly affect trade, such as tariffs and quotas, are less important for trade costs than other policies such as those pertaining to transport

[1] The aggregated version of LOP is the purchasing power parity (PPP), according to which the value of the country's currency equals the ratio of some macroeconomic index of prices at home to that abroad. For a recent review of the LOP and its relation to PPP, see Marsh et al. (2012). For reviews of PPP theory, see, among many others, Frenkel (1978), Dornbusch (1988), Rogoff (1996), Taylor and Taylor (2004) and Manzur (2008).

infrastructure, property rights, regulation and language. Thus, developing countries generally have much higher trade costs. Trade costs also vary substantially across products.

An early explanation as to why relative prices of non-tradables co-vary positively with the level of development is the Balassa-Samuelson hypothesis (Balassa, 1964; Samuelson, 1964) that attributes this phenomenon to the cross-country productivity gap in tradable sectors, which is wider than that in non-tradable sectors. Productivity differences might be a result of more productive firms (which focus on producing tradables) crowding out less productive firms (Bergin et al., 2006). They could also, according to Buera et al. (2011), be a result of misallocation of productive resources due to financial frictions. Sposi (2015) examines an exogenous source for this gap, namely the specialisation in production of goods with high trade barriers: Because tradables can be intermediate inputs for other goods, this specialisation increases the measured productivity difference. Productivity bias is not the only source of price dispersion, however, nor are price differentials exclusive to non-tradable items. In fact, there is an accumulation of evidence of higher prices of tradable consumption goods in richer countries, as presented by Alessandria and Kaboski (2011) and Simonovska (2015). Simonovska (2015) argues that non-homothetic preferences lead to high cross-country variation of price elasticities of demand for identical items. When trade barriers exist, monopolistic firms exert their pricing power to charge systematically higher mark-ups to more affluent consumers, whose are less responsive to changes in the prices of tradables. Variable mark-ups, therefore, can also be a contributing factor to the seeming failure of the LOP in its absolute form (see also Froot and Rogoff, 1995). Additionally, as tradables constitute a large share of the consumption basket, studying their price determination is crucial in understanding international disparities in welfare. Given the well-documented international price dispersion of seemingly identical items, our second research focus deals with their consumption. Clements et al. (2006b) find that about 80 percent of the total variation in the consumption patterns of 45 countries can be explained by a simple demand equation system. More importantly, allowing consumption to be proportional to income explains one half of this variation.

Our contributions to the current literature are as follows. First, we make use of an index that measures the degree of commodity tradability on the basis of price dispersion, that is strictly free of the unit of measurement effects. Second, based on this index, we are able to reasonably pinpoint the composition of two stylised baskets of goods: Tradables and non-tradables. As a result, novel formulations of unit-free price and quantity indices can be constructed. These allow us to examine the classic proposition of productivity bias as an important currency value determinant. Third, adopting a principal component analysis (hereafter, PCA) alternative to index number theory, we detect strong co-movement of consumptions that can be represented by a common factor that is highly correlated with income. The corresponding price common factor, though it captures a high proportion of the co-movement among prices, correlates much more weakly with income. Unlike typical outcomes of PCA, these common factors have a clear economic interpretation, which is related to cross-country affluence.

The rest of the paper has the following structure: Section 2 discusses the foundation of the productivity bias effect. Section 3 empirically discusses the price dispersion among commodities. Sections 4 and 5 examine price and consumption co-movements and the outcomes of principal component analyses. Section 6 provides concluding comments.

2. Departures from the law of one price

In this section, we discuss a formal framework linking the relationship between the prices of tradables and non-tradables to movements of nominal exchange rates. Denote the price of tradables as $\mathrm{P_T}$ and of non-tradables as $\mathrm{P_N}$. Define price levels at the home country and abroad as $\mathrm{P = P_N^{\vartheta} P_T^{1-\vartheta}}$ and $\mathrm{P^* = [P_N^*]^{\vartheta^*} [P_T^*]^{1-\vartheta^*}}$, where ϑ and ϑ^* denote the weights of tradables in the two countries, measured by the corresponding shares of expenditures. Rearranging these terms and converting them into the same currency, we have: $\mathrm{P = (P_N/P_T)^{\vartheta} P_T}$ and $\mathrm{SP^* = SP_T^*(P_N^*/P_T^*)^{\vartheta^*}}$ where S denotes the price of one foreign currency unit in local currency units. Then, we log-transform the price ratio $\mathrm{P/(SP^*)} = \dfrac{\mathrm{(P_N/P_T)^{\vartheta} P_T}}{\mathrm{(P_N^*/P_T^*)^{\vartheta^*} SP_T^*}}$ as follows:

$$\log\left(\frac{\mathrm{P}}{\mathrm{SP^*}}\right) = \underbrace{\vartheta(\log \mathrm{P_N} - \log \mathrm{P_T})}_{\substack{\text{Domestic}\\ \text{relative price of non-tradables}}} - \underbrace{\vartheta^*(\log \mathrm{P_N^*} - \log \mathrm{P_T^*})}_{\substack{\text{Foreign}\\ \text{relative price of non-tradables}}} + \underbrace{(\log \mathrm{P_T} - \log \mathrm{P_T^*} - \log \mathrm{S})}_{\substack{\text{Real appreciation}\\ \text{of tradables}}}. \tag{1}$$

In other words, the deviation from PPP (for all goods) can be decomposed into three terms: (i) The domestic relative price of non-tradables; (ii) the foreign relative price of non-tradables and; (iii) the real appreciation of tradables. If, for example, the productivity in the tradable sector is much higher in a foreign country ($\mathrm{P_T^*} << \mathrm{P_T}$), while the productivity gap is small for non-tradables ($\mathrm{P_N^*} \approx \mathrm{P_N}$), the combination of the first two components will be large and positive. This phenomenon is referred to as the "Harrod-Balassa-Samuelson effect" (hereafter, HBS). We can make two further assumptions in this model: First, the weight of non-tradables is the same in both countries ($\vartheta = \vartheta^*$) and second, that PPP holds for tradables only ($\log P_T - \log P_T^* - \log S = 0$). Then, (1) simplifies to: $\log P - \log P^* - \log S = \vartheta \left[\log (P_N/P_T) - \log (P_N^*/P_T^*)\right]$. In this specification, real exchange rate is proportional only to difference in the relative price of non-tradables.

More formally, let us consider a two-country, two-sector setting: The Rich country and the Poor country, both produce tradables and non-tradables. This setting is illustrated in Figure 1. Rich is absolutely more productive in making both goods (which, in essence, is the reason the country is rich), but relatively more so for tradables. This implies that $(P_N/P_T)_{Rich} > (P_N/P_T)_{Poor}$. Without loss of generality, let P^* now refer to the price level of a third, numeraire country, or a "world" price level. In order to isolate the impact of differing relative price structures, we assume that the monetary side of the economy is the same in the 2 countries in the sense that the overall price level is the same in Rich and Poor, that is, both countries share the same absolute price schedule (AA).

If PPP applies to the overall price levels, the exchange rates (ERs) of Poor and Rich are determined by the corresponding price ratio between them and the numeraire country, i.e., $S_{Poor} = P_{Poor}/P^*$ and $S_{Rich} = P_{Rich}/P^*$. Along the AA, price level is unchanged ($P_{Poor} = P_{Rich}$), so that the ERs of these countries are equalised. If, however, the actual ERs are determined in the market for traded goods, the productivity differential, i.e., a lower OR_{Rich} slope than that of OR_{Poor} (as shown on the right-hand side of Figure 1), will lead to $S_{Rich} > S_{Poor}$ (as shown on the left-hand side), so that Rich's currency is worth more.[2] This is the foundation of the HBS effect as discussed earlier. In summary, differences in real exchange rates are attributed to fluctuations in the relative price of non-tradables.[3]

We seek to extend this line of work with estimates of the slope coefficients of price schedules using actual data, and the results are summarised in Figure 2. First, panel A of this

[2] Equivalently, the difference in the slopes of the rays on the right-hand side indicate the magnitude of nominal ER appreciation when Poor becomes Rich.

[3] The empirical validity of the HBS hypothesis is controversial. For example, Lothian and Taylor (2008) observe that the HBS effect explains about 40% of the variation in the level of the sterling-dollar real exchange rate using data spanning over two centuries. In contrast, the seminal paper of Engel (1999) documents that over 90% of the US-EU real ER is explained not by the difference in relative prices of non-tradables, but by the appreciation of tradables' prices. The deviation from the LOP for tradables arises from at least three sources. First, international market segmentation could weaken the force of arbitrage that is crucial for price convergence. For instance, real appreciation of China's agricultural prices was 8.2% per annum from 2005 to 2015, arguably due to substantial trade costs that disconnect China's market from the world's (Imai (2018)). Second, the higher weights of commodities whose price rises fast (such as computers, telecommunication devices and agriculture products) could also lead to an increase in tradables' prices. Third, final products considered as "tradables" may in fact contain a large non-tradable component such as mark-ups to cover costs of local wholesale and retail services, marketing and advertising (see, e.g., Corsetti and Dedola (2005) and Burstein et al. (2006)).

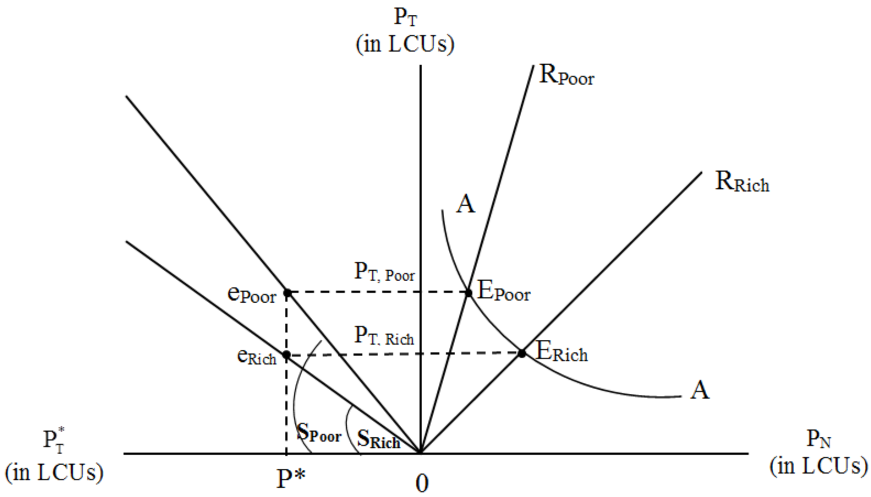

Figure 1. Nominal Exchange Rates and Prices

Notes: There are two countries: Rich and Poor, and two items: Tradable (T) and Non-tradable (N). P_T and P_N denote the price of T and N in local currency units (LCUs). The slopes of the two rays from the origin on the right-hand side are P_T/P_N in the two countries. The curve AA is the absolute price schedule, which represents combinations of the prices of tradables and nontradables consistent with a given price level. In order to isolate the effects of the different relative price structure, both countries are taken to share the same AA schedule. The PPP exchange rates for these countries are represented by the slopes of the two rays on the left-hand side, P_T/P_T^*, where P_T^* denotes the world price of tradable goods. Source: Clements and Lan (2007), p.472.

figure shows the scatter plot of $P_c^T = \exp\left(\log P_c^T\right)$ against P_c^N ($c = 1, \ldots, 155$). These numbers represents the prices (converted to US dollar) of tradables and non-tradables, respectively, in country c. The construction of these price indices is discussed in Section 3 and in Appendix A3. The fitted regression line for the Poor countries, which forms the relative price schedule, takes the form $P_{Poor}^T = C_1 + 0.45\ P_{Poor}^N$, and the corresponding relative schedule for the Rich is $P_{Rich}^T = C_2 + 0.3\ P_{Rich}^N$, where C_1 and C_2 are non-zero intercepts.

To construct panel B of Figure 2, consider the following equation:

$$\log \widetilde{P}_c^T = \left(-w_c^N/w_c^T\right) \log \widetilde{P}_c^N, \tag{2}$$

where $\log \widetilde{P}_c^T = \log P_c^T - \log P_c$ and $\log \widetilde{P}_c^N = \log P_c^N - \log P_c$ are the tradables and non-tradables' prices relative to the price level in country c and w_c^T and w_c^N denote the budget shares of tradables and non-tradables, respectively. Equation (2) implies that each country c potentially lies on a unique absolute price schedule, for which the slope is determined by the ratio of non-tradables and tradables' budget shares. The derivation of Equation (2), as well as these budget shares, shall be discussed in Section 3. To obtain a universal absolute price schedule (as illustrated in Figure 1), we further assume that the expenditure structure is the same across countries,

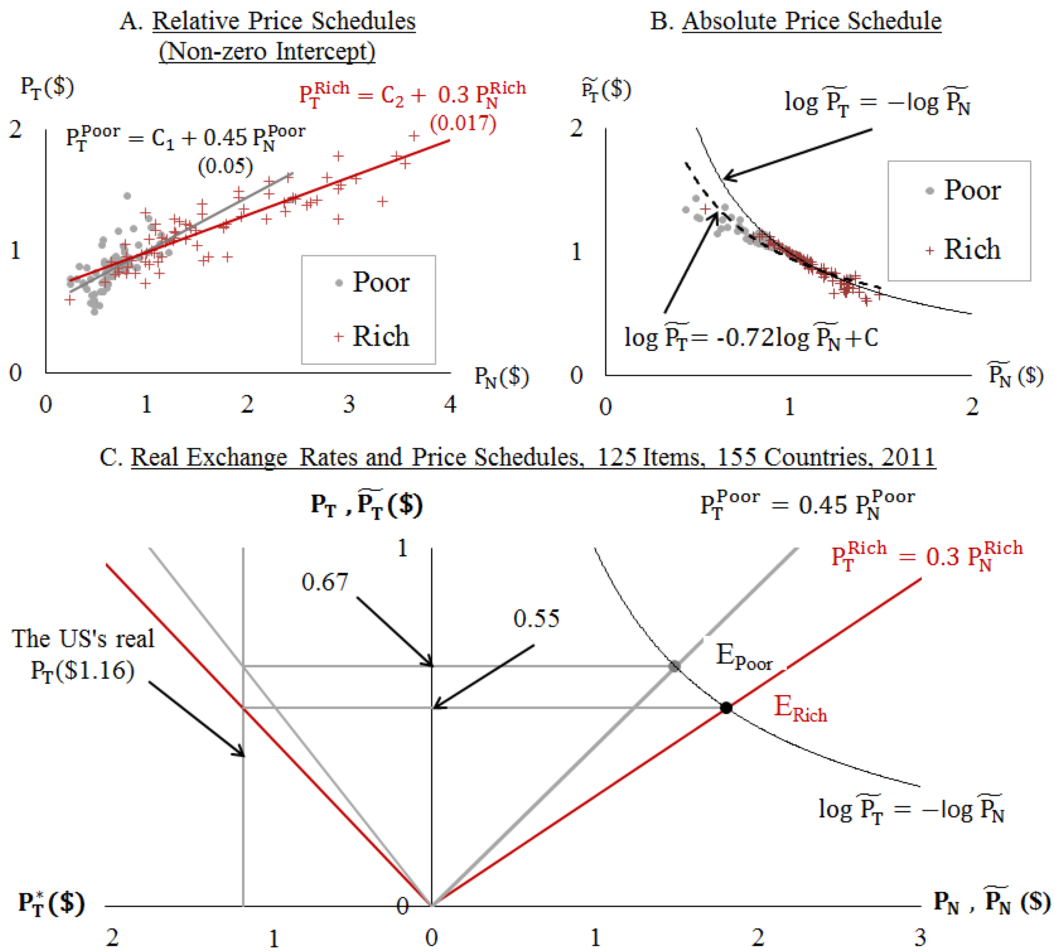

Figure 2. Real Exchange Rates and Prices

Notes: There are two groups of countries (Rich and Poor) and two groups of items (Tradables and Non-tradables). Here, P_T and P_N denote the budget share-weighted average prices of tradables and non-tradables. Tradables are items that have a (cross-country) standard deviation of real exchange rates not higher than 70%. All values are indices of real relative prices. Panel C combines panels A and B: The right-hand side presents the fitted relative price schedules (with intercepts forced to be zeroes) of Rich and Poor. In the left-hand side of C, the slopes of the rays indicate the ratio of the dollar prices of tradable in Rich and Poor, relative to that in the US. The Poor and Rich rays cut the vertical line of the US price ($P_T^* = 1.16$) at points with $P_T = 0.67$ and $P_T = 0.55$, respectively. These are solutions of the simultaneous equations representing the corresponding relative schedules and absolute schedule. See text for details.

so that $w^T = w^N = 0.5$[4], and thus (2) simplifies to: $\log \widetilde{P}_c^T = -\log \widetilde{P}_c^N$. Equivalently, the price level of tradables can be expressed as a power function of the form: $\widetilde{P}_c^T = a\left[\widetilde{P}_c^N\right]^b$ with $a = 1; b = -1$. Panel B shows the scatter plot of $\widetilde{P}_c^T$ against $\widetilde{P}_c^N$. We see that the actual absolute price schedule (the dashed curve) assumes the fitted form of $\log \widetilde{P}_c^T = -0.72\log \widetilde{P}_c^N$, rather than $\log \widetilde{P}_c^T = -\log \widetilde{P}_c^N$ (the solid curve). The difference is a direct result of cross-country heterogeneous expenditure structures. For ease of interpretation, we use the latter functional form, i.e., a homothetic preference approximation.

[4] As shown later in Section 3, this assumption does not seem to be too impractical from an empirical viewpoint.

Finally, the right-hand side of panel C combines panels A and B. Here we adjust the two relative price rays described above so that they go through the origin, viz., setting $C_1 = C_2 = 0$. The slopes of these rays imply that on average, compared to the Poor, the relative price of non-tradables is about 1.5 times higher in the Rich countries: Since $P^T_{Rich} = P^T_{Poor}$, it follows that $P^N_{Rich}/P^N_{Poor} = \frac{P^T_{Rich}/0.3}{P^T_{Poor}/0.45} = 1.5$. This condition simplifies the discussion while imposing no loss of generality. To solve for the coordinates of the intersection between absolute price schedule and Rich relative price schedule (the point E_{Rich}), we substitute $P^T = 0.3P^N$ into $\log P^T_c = -\log P^N_c$, which yields $P^T = \sqrt{0.3} = 0.55$ and $P^N = 1.81$. The corresponding coordinates of E_{Poor} are $P^T = 0.67$ and $P^N = 1.5$.

On the left-hand side of panel C, the real tradable price for the numeraire country, the US (indicated by the intersection of a vertical line and the horizontal axis), is 1.16. This line also intersects the Rich and Poor real exchange rays at 0.55$ and 0.67$, respectively. The difference between the slopes of the real exchange rays approximates the scale of exchange rate differential between these two country groups: On average, when countries join the Rich group, their real exchange rate will appreciate by (0.67 – 0.55)/0.55 = 22%. From the conventional PPP point of view, productivity differential introduces a 22% bias in the Rich's purchasing power. Deaton and Aten (2017) succinctly summarise this empirical regularity as (p. 244): *"Non-traded goods are typically cheaper in poorer economies, so that PPPs are typically lower than exchange rates for poor countries, and are more so the poorer the country"*.

3. Tradability and price dispersion

In this section, we move from the parity of price indices as discussed in the previous section, to that corresponding to single items. In logarithmic form, the deviation from this parity is defined via the expression: $k_{i,c} = \log p_{i.c} - \log p^*_i - \log S_c$, where S_c is the market exchange rate between the two countries, $p_{i,c}$ is the local currency price, and p^*_i the foreign price of i. To be consistent with the previous literature, we also refer to $k_{i,c}$ as "goods-level real exchange rates (RER)". These deviations refer to the differences between the domestic price and an international price that is common to all countries. For example, the International Comparison Program (hereafter, ICP) chooses US prices so that $\log p^*_i = 0$ $(\forall i)$ (Cuthbert, 2009). However this choice of numeraire can be considered arbitrary since it "singles out" the US as a special country. An alternative approach is to treat all countries symmetrically, as proposed by Betts and Kehoe (2017), and construct deviations for all possible country pairs, for each item i:

$$k_{c,d} = \log p_c - \log p_d - \log S_c - \log S_d \ (c, d = 1 \ldots C; c < d), \tag{3}$$

where the subscripts c and d refer to countries c and d respectively, and C denotes the total number of countries. Absolute PPP then implies that $k_{c,d} = 0 \ (\forall c, d)$. Positive (negative) values of $k_{c,d}$ indicate over-valuation (under-valuation) of c with respect to d, for the commodity in consideration. A reasonable criterion for PPP to hold, in relative terms, is that the distribution of $k_{c,d}$ $(c, d = 1 \ldots C; c < d)$ tightly centres around zero. The lower the standard deviation, the stronger the degree to which prices coincide, and the more it is tradable.

This approach is applied to the 12 broad household consumption categories, with the PPP rates and market exchange rates as published by the 2011 International Comparison Program (ICP), in 155 countries (World Bank, 2013).[5] This results in a total of $(155 \times 154)/2 = 11,935$ unique bilateral relationships. As can be seen from column (8) of Table 1, items of which composition contains a high proportion of non-tradable inputs, such as services like "Education", "Housing and Utilities" and "Health" generally have a higher deviation from PPP. According to the standard deviation of $k_{c,d}$, "Transport" is ranked as the most tradable, while "Education" is the least tradable.

As pointed out in Section 2, the starting point of the HBS hypothesis hinges upon how we classify an item as "tradable". A simple solution to the tradable and non-tradable identification issue is to rely on the extent to which a good is actually traded, as in Lombardo and Ravenna (2012).[6] Since we use retail prices collected at the last point of consumption, or purchaser prices, price dispersion most likely reflects local distribution margin. To the degree that an item's tradability depends on all its components' tradability, such a proxy seems reasonable.[7] Figure 3 summarises the dispersion of $k_{c,d}$ $(c > d)$ at the disaggregated basic heading level. If an item exhibits a large dispersion of PPP deviations, such as "Medical services", this is an indicator that it has a substantial non-traded component.

But how large should this dispersion be for the item to be classified as effectively "non-tradable"? In Figure 4, we show the cumulative number and total expenditure of the items as

[5]Details about the source, the construction, and adjustments of these data can be found in Appendix A1.

[6]This proposal becomes more relevant, especially as the share of traded goods that previously perceived as non-tradables are documented to rise since the 1990s as documented by Fieleke (1995).

[7]It is important to emphasize that since we are using purchasers' prices which always contain certain non-tradable components, it is difficult to provide clear-cut definition of tradable items and measurements of tradability are *ad-hoc*. Additionally, controlling for quality difference is problematic, especially for housing-, education- and medical-related items.

Table 1. PPP Deviations, 12 Consumption Categories, 155 Countries, 2011

Category Name	% of GDP	% of consumption	PPP deviation Min	Max	Median	Mean	SD
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1. Transport	7.07	10.87	-1.27	1.48	-0.07	-0.05	0.38
2. Food	18.94	29.11	-1.53	1.21	-0.08	-0.09	0.39
3. Furnishings	3.53	5.43	-1.54	1.25	-0.19	-0.19	0.45
4. Recreation & culture	3.19	4.90	-1.56	1.37	-0.16	-0.16	0.46
5. Miscellaneous	5.01	7.70	-1.75	1.42	-0.21	-0.21	0.50
6. Alcohols & tobacco	2.30	3.54	-2.22	1.95	-0.13	-0.13	0.52
7. Restaurants & hotels	3.41	5.24	-2.16	1.65	-0.17	-0.18	0.54
8. Communication	2.19	3.37	-2.08	1.68	-0.13	-0.14	0.55
9. Clothing & footwear	3.37	5.18	-2.20	1.55	-0.28	-0.29	0.57
10. Health	2.60	4.00	-2.74	2.35	-0.37	-0.38	0.73
11. Housing & utilities	11.82	18.16	-3.15	2.50	-0.14	-0.16	0.82
12. Education	1.62	2.49	-3.93	3.76	-0.22	-0.23	1.28
13. All household consumption	65.05	100	-3.93	3.76	-0.17	-0.19	0.65
14. Gross domestic product	100		-1.61	1.37	-0.15	-0.16	0.52

Notes: This table presents the summary statistics of the deviations from PPP for the 12 consumption categories listed under the "Individual Consumption Expenditure By Households" main aggregate. The items are ranked in order of increasing standard deviation. The underlying variable, for a given category, is $k_{c,d} = \log p_c - \log p_d - \log (S_c/S_d)$ $(c, d = 1, \ldots, 155; c < d)$. $\log p_c$ and $\log p_d$ are prices of the item in countries c and d, and S_c and S_d are the corresponding market exchange rates. For each item, there are $[155 \times (155 - 1)]/2 = 11,935$ observations, which are the elements in the upper triangle of the skew-symmetric matrix $\mathbf{K} = [k_{c,d}]$. The second-to-last row refers to all items and the last row refers to the price indices for GDP.

a function of the cut-off value (denoted as ω) measured as the standard deviation of $k_{c,d}$. If we increase this cut-off, the pool of non-tradables shrinks. We opt for increasing ω by a step of 1% in a range from 0% to 130%.[8] According to Lombardo and Ravenna (2012), the share of the tradable sector, in terms of final goods for consumption, has an average value between 49% and 55% across 48 countries. We then choose $\omega = 70\%$, on the basis that this value corresponds to a total share of tradables of 50% (as shown in panel A of Figure 4), which is close to the average threshold reported by Lombardo and Ravenna (2012). This cut-off value shows that trade costs typically manifest into a 70% difference in price of identical items, which is the average trade cost value estimated by Anderson and van Wincoop (2004). This finding implies that an item with a standard deviation of 0.7 lies close to the "dividing border" between tradables and non-tradables which is also an average line.

Next, denote the tradable share in country c as: $w_c^T = (\Sigma_{i \in T} M_{ic})/M_c$ where M_{ic} and M_c are the expenditures that people in c devote to i and all items, respectively. The non-tradable share is $w_c^N = (\Sigma_{i \in N} M_{ic})/M_c = 1 - w_c^T$. Then, a cross-country unweighted average of tradable shares is $\mu_T = (1/C)\Sigma_{c=1}^{C} w_c^T$ and the non-tradable counterpart is $\mu_N = 1 - \mu_T$. According to our computations, $\mu_T = 57\%$ while $\mu_N = 43\%$. How do these measures relate to the

[8]The upper bound corresponds to the maximum actual standard deviation.

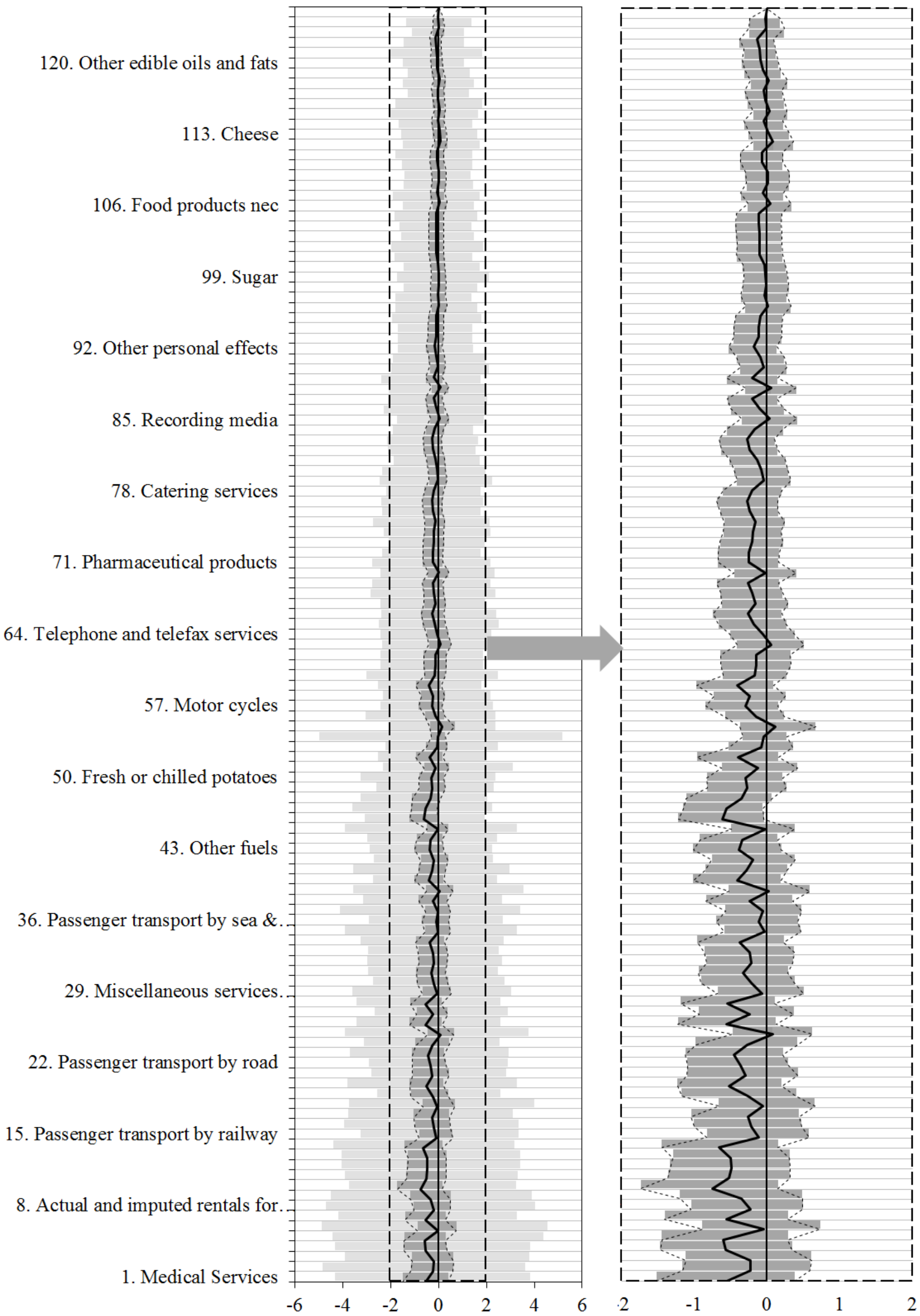

Figure 3. Distribution of PPP Deviations, 125 Basic Headings, 155 Countries, 2011

Notes: This figure presents the distributions of real exchange rates of 125 basic headings across 155 countries. For item i, this variable is defined as: $k_{c,d} = \log p_c - \log p_d - \log (S_c/S_d)$ $(c, d = 1, \ldots 155; c < d)$, so that there is 11,935 pair-wise observations in each row. In the left panel, the solid middle line is the median. The dark shaded area indicates the inter-quartile range (IQR). The light shaded area indicates the full range. A selection of item numbers and names are given on the far left. The IQR is also presented separately in the right-hand panel, as a "blow-up" of the left-hand panel.

50% share of tradables across all items and countries (as indicated in panel A, Figure 4)? If we instead use a country-specific budget share-weighted average, we have: $\mu_T^{'} = \Sigma_{c=1}^{C} w_c^T (M_c/M) =$

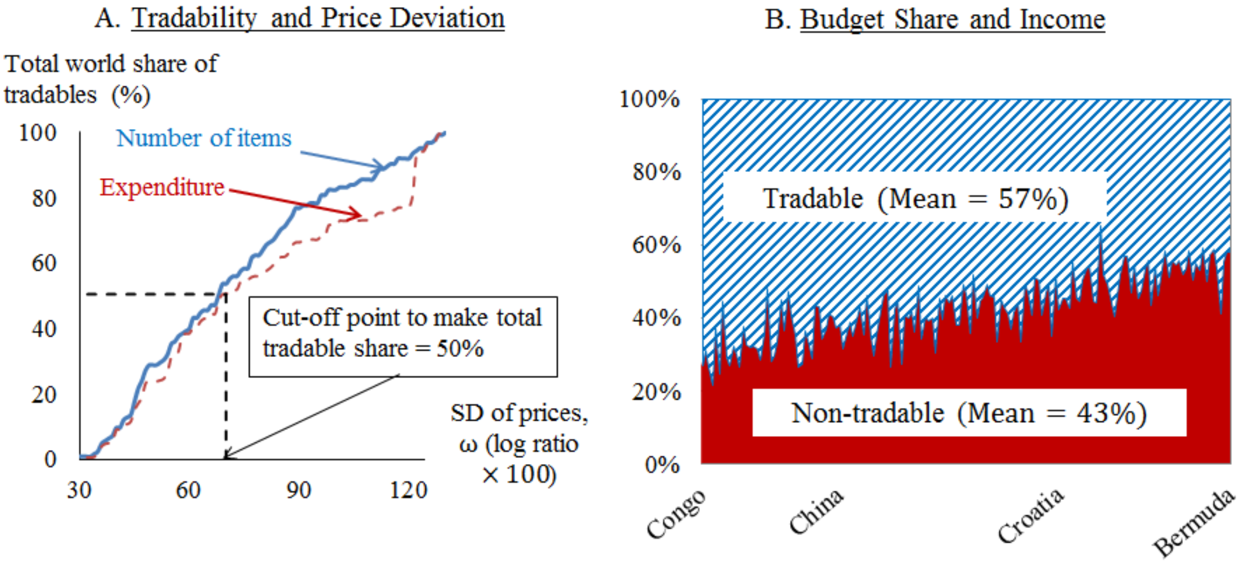

Figure 4. Tradability, Price Dispersion and Income Shares

Note: Items with $\sigma_{k_{c,d}} \leq \omega$, where $k_{c,d}$ = log p_c – log p_d – log (S_c/S_d), are considered "tradable". In panel A we show how the number of tradable items and their cumulative share in total expenditure (in \$US) increases with the wider bandwidth (ω). For each item, total expenditure is computed by summing across countries. "Net purchases abroad" is excluded. The intersection between the vertical dashed line and the x-axis shows that the value of $\omega = 70\%$ implies a 50% share of tradables in the total budget of the world as a whole. This value can also be expressed as a cross-country weighted average of tradable shares (w_c^T): $\mu_T^{'} = \Sigma_{c=1}^{C} w_c^T (M_c/M)$ where M_c and M denote the total expenditures of country c and the world. Panel B shows the shares of tradables and non-tradables across countries, as the sum of expenditures of items belonging to these baskets, on the basis of choosing $\omega = 70\%$. The unweighted mean tradable share is $\mu_T = (1/C)\Sigma_{c=1}^{C} w_c^T$ $(c = 1, \ldots, 155)$, where w_c^T is the share of tradables in c. $\mu_N = 1 - \mu_T$ denotes the mean share of non-tradables.

$\Sigma_{c=1}^{C} \frac{\Sigma_{i=1}^{T} M_{ic}}{M_c} \frac{M_c}{M} = \Sigma_{c=1}^{C} \Sigma_{i=1}^{T} M_{ic}/M = 50\%$. Similarly it can be shown that $\mu_N^{'} = 1 - \mu_T^{'} = 50\%$.

Having defined what individual items are "tradable", we proceed to derive the prices of tradable and non-tradable as baskets. When comparing prices internationally, we cannot directly use the individual items' local prices ($p_{i,c}$), since these contain not only different currency units, but also different quantities of consumption (i.e, 1 kg of rice, 1 gallon of water etc.). To account for these confounding factors, we show in Appendix A3 how multilateral indices of tradables and non-tradables' prices, denoted as log P_c^T and log P_c^N can be constructed from bilateral average budget-share weighted indices.[9] Nevertheless, we kept the same notations for the sake of simplification. From these component indices, we can also reconstruct the price level of

[9]An important difference between these newly constructed price indices and the "raw" indices discussed in the beginning of Section 2 is that both consumption units and currency units drop out of the new indices, thus we can use them directly in a cross-country comparison.

country c as:

$$\begin{aligned} w_c^T \log P_c^T + w_c^N \log P_c^N &= \log P_c, \quad \text{or equivalently,} \\ w_c^T \log \widetilde{P}_c^T + w_c^N \log \widetilde{P}_c^N &= \log \widetilde{P}_c = 0, \end{aligned} \tag{4}$$

where $\log \widetilde{P}_c^T = \log P_c^T - \log P_c$ and $\log \widetilde{P}_c^N = \log P_c^N - \log P_c$ are the tradables and non-tradables' prices relative to the price level. We term these new relative prices the "adjusted prices". By construction, the adjusted prices lead to overall price level being equalised across countries ($\log \widetilde{P}_c = 0\ \forall c$) and this validates the PPP.[10] These price indices are presented in panels B and C of Figure 2.

4. A common factor of consumption and price

In this section, we study the cross-country disparities of relative prices and consumption. In Section 3, for each country c, the relative price of item i can be defined as the average of bilateral price differences between c and other countries:

$$\bar{k}_{i,c} = \frac{1}{155}\sum_{d=1}^{155} k_{c,d} = \frac{1}{155}\sum_{d=1}^{155}\left[\log\frac{p_{i,c}}{S_c} - \log\frac{p_{i,d}}{S_d}\right] = \log\frac{p_{i,c}}{S_c} - \frac{1}{155}\sum_{d=1}^{155}\log\frac{p_{i,d}}{S_d}.$$

That is, the relative price of item i in country c is the deviation from the "world" average price, $P_i = \frac{1}{155}\sum_{d=1}^{155}\log\frac{p_{i,d}}{S_d}$. This approach takes into account the differences in currencies and consumption units. It is also a democratic approach since each country receives equal importance in the construction of the average price.[11] We can then compute the average price matrix as

$$\mathbf{K} = \left[\bar{\mathbf{k}}_1, \bar{\mathbf{k}}_2, \ldots, \bar{\mathbf{k}}_{125}\right] \text{ where } \bar{\mathbf{k}}_i = \left[\bar{k}_{i,c}\right] \ (i = 1, \ldots, 125).$$

Note that $n = 125$ is the number of consumption basic headings, the most disaggregated level that provides expenditure data, from the International Comparison Program.

By construction, a positive (negative) $\bar{k}_{i,c}$ represents over-(under-) valuation of item i in country c, relative to the rest of the world. It can be shown that the grand mean of **K** over all i and c is zero, and the grand standard deviation is one.[12] An analogous matrix can be

[10] This is also equivalent to setting the price of the full consumption basket as equal to \$1 (or $\widetilde{P}_c = 1\ \forall c$).

[11] Alternatively, we can use a weighted average world price, i.e. $P_i = w_{i,d}\log(P_{i,d}/S_d)$ where the weights are the countries' budget shares in total world expenditure for i. That is, $\Sigma_{d=1}^{155} w_{i,d} = 1$. But such a specification would introduce a Gershrenkron bias toward large countries, which consume more and thus their prices have a bigger impact on the world price (Gerschenkron, 1947).

[12] See Appendix A3.

constructed for consumption

$$\mathbf{Q} = \left[\overline{\mathbf{q}}_1, \overline{\mathbf{q}}_2, \ldots, \overline{\mathbf{q}}_{125}\right] \text{ where } \overline{\mathbf{q}}_i = \left[\overline{q}_{i,c}\right] = \left[\frac{1}{155}\sum_{d=1}^{155}\left(\log q_{i,c} - \log q_{i,d}\right)\right],$$

where $q_{i,c} = M_{i,c}/p_{i,c}$ is the real per capita consumption of i in c.[13] Total income is defined as the sum of per capita expenditure by households, non-profits serving households and individual government on consumption and is deflated by the cost of living. That is, income is $Y_c = M_c/P_c$ where $M_c = \sum_{i=1}^{125} M_{i,c}$ denotes the total per capita consumption and P_c is country c's cost-of-living index, or price level, as discussed in Section 2. Finally, we construct the relative real income as $\log(Y_c/\overline{Y})$, where $\overline{Y}$ denotes the cross-country geometric mean of Y_c. For simplicity, hereafter we refer to the real income measure as "income". Note that this aggregate measure of real expenditure differs from the volumes of consumption mentioned above, which is an item-specific measure.

We now consider in a more formal fashion the underlying structure of international prices and consumption. We can imagine the various item-specific measurements of price and consumption to be arranged in a $155 \times n$ matrix, such as **K** and **Q**, with the rows referring to countries and columns to items. n varies at different aggregation levels. The number of pairwise correlations derived from such a large matrix may be costly to examine individually. Instead, we employ a principal component analysis (hereafter PCA) that exploits the co-movement tendency among these variables.Specifically, following Theil (1971), we derive a new set of uncorrelated variables (the "principal components"), each of which is a linear combination of the original variables.[14] We focus on the first two principal components, computed as

$$\mathbf{pc}_1 = \mathbf{X}\mathbf{a}_1 \text{ and } \mathbf{pc}_2 = \mathbf{X}\mathbf{a}_2,$$

where $\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, \ldots \mathbf{x}_{125}]$ is the matrix of which each column is the standardised corresponding column of the underlying variable (**K** or **Q**). $\mathbf{a}_1$ and $\mathbf{a}_2$ are the first and second eigenvectors of the correlation matrix of **X**. An important result from PCA is that the sum of the variances of original variables equals that of the PCs. Additionally, $\mathbf{pc}_1$ accounts for the largest amount of variation, followed by $\mathbf{pc}_2$ and the rest of the PCs. $\mathbf{pc}_1$ and $\mathbf{pc}_2$, also termed "component scores", are projections of the original variables onto the first two principal directions, and are linear combinations of the original variables given the coefficients contained in $\mathbf{a}_1$ and $\mathbf{a}_2$

[13]The volumes ($q_{i,c}$) cannot be directly measured, and in most applications, they are inferred (Rao, 2013). It can be seen that the accuracy of volume estimate depends on that of price estimates.

[14]A description of this approach and an example study is provided in Appendix A6.

(Campbell and Atchley, 1981). For example, given the consumption of the US in terms of all items as $x_1, x_2, \ldots, x_{125}$, the US will be given a score of $pc_1 = \Sigma_{i=1}^{125} a_{1,i} x_i$ of the first PC. If the coefficients are all unity ($a_{1,i} = 1 \forall i$), pc_1 becomes the sum of all consumption, whereas if $\sum_{i=1}^{125} a_{1,i} = 1$, pc_1 coincides with the weighted average consumption. In the latter special case, the unit of pc_1 is the same as that of the original variables, but in general, PCs cannot be compared directly with these variables due to the different units of measurement. This is why in empirical applications, we are more interested in the direction/sign of the relationship between the PCs and other variables, than in the magnitude of such relationships.

5. Results

We apply the above method to the 125 basic consumption headings from the International Comparison Program.[15] The PCA results for consumption are summarised in panel A of Table 2. Column (2) indicates the number of items at each aggregation level. Column (3) indicates that the first PC associated with this variable accounts from more than 40% of total data variation (at the basic heading level) to up to 96% (at the main aggregate level). Not surprisingly, data exhibit less variation among aggregated variables, which leads to higher explanatory power of PC1.[16] We can see that in this respect, the aggregation procedure serves as a complementary dimension reduction technique to PCA. Columns (5) and (6) display the coefficients and t-statistics of the correlations between the PC1s and real income. Overall, income is significantly and strongly related to consumption PC1, exhibiting an almost perfect correlation, and this is robust to aggregation. In contrast, PC2s have almost no correlation with income, and contribute little to the total data variation.

Figure 5 plots consumption PC1 against its PC2 at the basic heading level. These values are the "factor scores" assigned to each country along the first and second principal dimensions. Note that by construction, these two series are orthogonal. In panel A, where all data points are included, we can see a broad arrangement along the horizontal axis that is in agreement with

[15] Details about the source, the construction, and adjustments of these data can be found in Appendix A1.

[16] In unreported results, the number of PCs required to explain at least 80% of the basic heading-specific variation is 24. This means that we can reasonably "compress" our 125-column database into a 24-column one, a mere 19% of the original number of items. However, the contributions of PC2 to PC24 to total variation (not reported) are all very small, ranging from 5% to 0.7%. The value of dimension reduction decreases substantially as we move to higher aggregation levels: At the main aggregate level, there are only two "broad items" left, and the first PC contributes 96% to total variation. At the category level, if we include all possible categories (n = 16), we obtain a contribution of 71% from PC1 to total variation. If we only use the 12 household-related categories (these are analysed in Section 3) and exclude the government consumptions, PC1 explains about 80% of total variation. This estimate coincides with that of Clements et al. (2006a), who present a panel analysis on essentially the same items, across 45 countries.

Table 2. PCA, Prices and Consumption

Aggregation level	No. of variables/items	Variation contribution (%)		Correlation with income				SD	
		PC1	PC2	PC1		PC2		PC1	PC2
				Coef.	t-stat	Coef.	t-stat		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
			A. Consumption						
Basic heading	125	42.3	5.2	0.98	61.1	0.06	0.76	7.3	2.6
Class	101	46.1	5.3	0.98	61.6	0.03	0.43	6.8	2.3
Group	48	59.4	5.3	0.98	67.1	0.08	1.03	5.4	1.6
Category	16	71.2	7.2	0.99	74.8	0.04	0.55	3.4	1.1
Main Aggregate	2	95.8	4.2	0.98	68.8	0.16	2.01	1.4	0.3
			B. Relative price						
Basic heading	125	86.5	4	0.24	3.11	0.85	19.7	10.4	2.2
Class	101	86.6	3.7	0.26	3.35	0.84	18.9	9.4	1.9
Group	48	88.8	3.3	0.26	3.39	0.74	13.8	6.5	1.3
Category	16	91.4	3.4	0.28	3.56	0.58	8.74	3.8	0.7
Main Aggregate	2	98	2	0.25	3.23	0.42	5.72	1.4	0.2

Notes: This table summarises the dimension-reduction results of the principal component analyses. Column 2: The number of items at each aggregation level. Columns 3 and 4: The contributions (in percent) of the first and second principal components to the total data variation. Columns 5 - 8: The coefficients and t-stats of the correlation between income and PCs. Income refers to the real per capita consumption relative to the cross-country geometric mean income. Columns 9 - 10: The standard deviations of PC1 and PC2.

the degree of affluence. In other words, richer countries generally have higher scores/PC1. This reconfirms the result of panel A of Table 2. If we omit the 10 countries that are located the farthest from the zero line, the emerging pattern is much clearer in panel B.

The above discussion deals with the question of whether it is feasible to describe a large number of variables by compressing them into a single PC. A striking conclusion from Table 2 is that regardless of the variables used, there is almost invariably a strong association between the first PC and income [as shown in column (5)]. In this section we correlate the constructed principal component with income and possible drivers capable of generating the observed cross-country pricing and consumption patterns. This exercise addresses the question: Which determinant is this common factor related to the most? Specifically we use the following model:

$$\log \mathrm{PC}_{i,c} = \alpha + (\beta + \lambda \mathrm{D}_c)\log(\mathrm{Y}_c/\overline{\mathrm{Y}}) + \Theta' \mathbf{Z}_c + \varepsilon_c, \tag{5}$$

where $\log \mathrm{PC}_{i,c}$ $(i = 1, 2)$ denotes the logarithm of first and second component scores obtained from the PCA using either relative prices or quantities. Note that, since these PCs are linear combinations of the original variables, they do not retain the units of relative prices or quantities.[17] $\log(\mathrm{Y}_c/\overline{\mathrm{Y}})$ is the per capita income of country c relative to the cross-country geometric

[17] In fact, by construction, PCs exhibit the largest possible data variations (see Section 4). Therefore, we are more interested in the significance of the relationship between PCs and other determinants than in the magnitude of such a relationship.

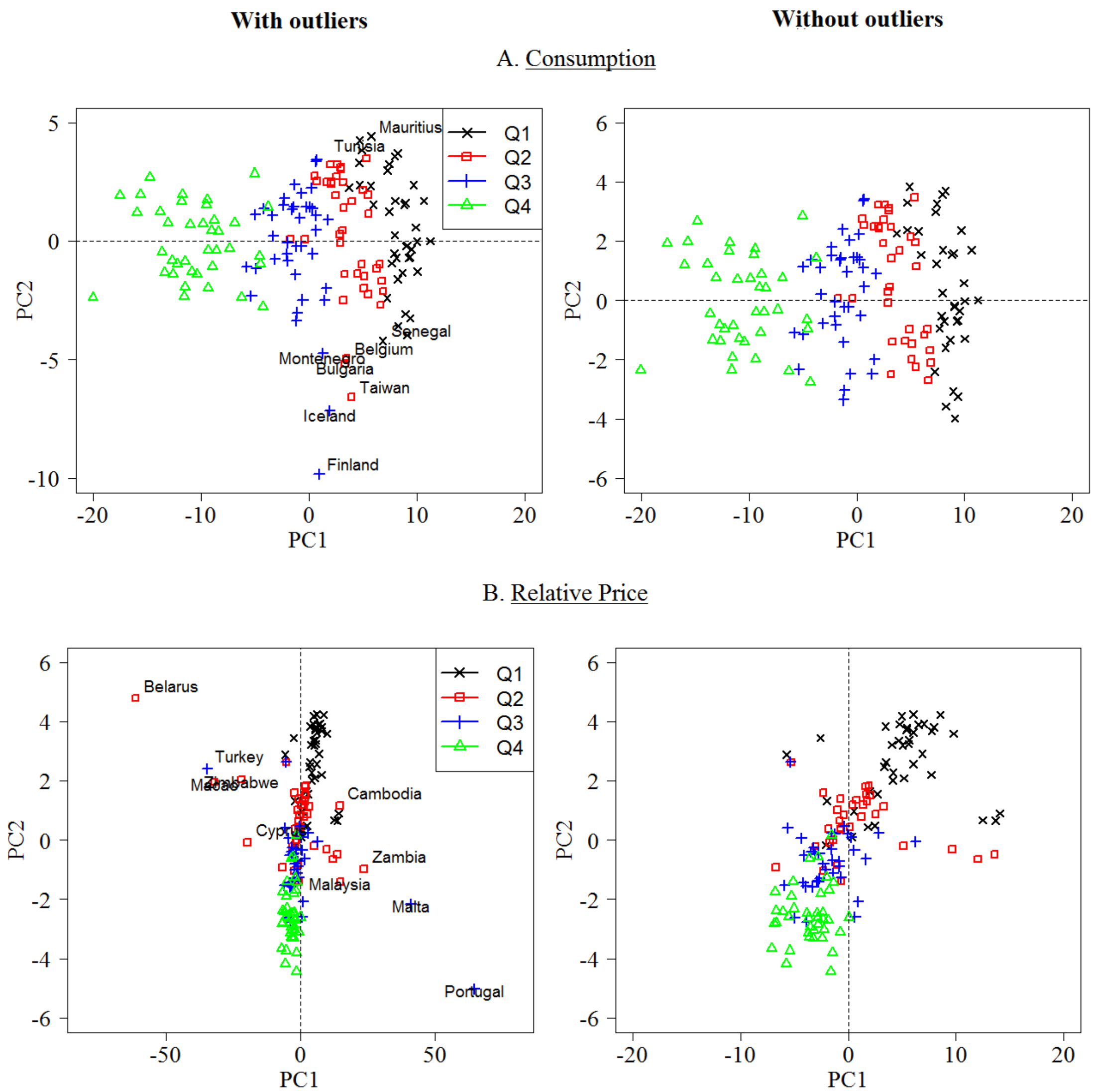

Figure 5. First and Second Principal Component Scores

Notes:
Panel A of this figure presents the scatterplot of the first two principal components for consumption in two settings: (i) With outliers (left) and (ii) without outliers (right). Income is defined as the per capita real consumption relative to the cross-country geometric mean. The "outliers" constitute the ten countries exhibiting the greatest absolute vertical distances from zero. Panel B shows analogous values for relative prices, where outliers are those with greatest horizontal distance to zero. In both panels, countries are color-coded based on income quartiles, with Q1 being the richest and Q4 the poorest.

mean; D_c is a dummy variable that takes the value of 1 if c belongs to the "Poor" group (the third and fourth income quartiles) and is 0 otherwise; $\mathbf{Z}_c$ denotes a vector of explanatory variables for country c (other than income)[18] and ε_c is a disturbance term. We also include the real exchange

[18]In particular, $\mathbf{Z}$ includes: (i) Macroeconomic indicators such as logarithmic nominal exchange rates and populations, the share of food expenditure in GDP (as a proxy for development) and (ii) potentially important generators of deviations from law of one price such as consumption tax rates, landlockness and openness to trade. Finally, there is a dummy variable that equals 1 if the country is in the Eurozone. The choice of these variables follows that of Cavallo et al. (2014).

rates as a regressor, but only when studying relative prices and quantities.

From panel B of Table 2, we can see that compared with consumption, it is much easier to describe the cross-country pricing patterns with just the first PC. Specifically, column (3) shows that PC1 contributes from 86% to 98% of total data variation.[19] However, this PC is only weakly related to income, with a correlation coefficient of only 0.26 on average. However, there is a caveat to the interpretation of the role of PC1. To see this, we plot the relative price PC1 against PC2 in panel B of Figure 5. We highlight the income groups by different colours. It can be seen that if we consider all points together, no linear relationship between these two components is visible. More importantly, the greater dispersion of PC1 (which leads to an 86.5% contribution to the total variation), appears primarily driven by outliers. We then list 10 of the possible outlying countries, defined as those that have the greatest horizontal distances to a fitted regression between PC2 and PC1 based on the rest of the data. In panel B, we redo this plot with outliers omitted, in which case the relative dispersion along the second dimension is greater. It also appears that PC2, though originally only contributing 4% to total variation, actually has a very high correlation with income: The arrangement of the countries along PC2 coincides almost completely with the order of the income groups. From columns (7) and (8) of panel B of Table 2, the correlation coefficient between basic heading PC2 and income is 0.85, which is significant at the 1% level. However, the strength of this relationship decreases the more we aggregate our data.

Table 3 presents the results for regression (5). For the purposes of illustration, we only present the results for basic heading level.[20] There are several important observations: First, regardless of the model specifications, real income plays an important role in explaining the cross-country variation of consumption. However, the rich-poor difference has a significant impact only in model (6) where all control variables are added. The adjusted R^2 values across different specifications are in agreement with column (5) of Table 2. By comparing columns (4) to (6) it can be seen that the cross-country variation of consumption is almost solely explained by income. In stark contrast, except for exchange rate-related factors, no variable has a significant impact on the PC1 of relative prices. Additionally, the "currency union effect" is strong and is observed for all variables at the basic heading level, while it is only preserved for quantity at the category level.

[19] As a result, we only need the first PCs to reasonably summarise the information contained in price correlation at all levels of aggregation.

[20] Results at other levels do not differ significantly, and are available upon request.

Table 3. Determinants of Price and Consumption, 155 Countries, 125 Basic Headings

	A. Consumption				B. Relative Price				
Dep.var	PC1		PC2		PC1		PC2		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
$\log(Y_c/\overline{Y})$	5.420***	5.552***	-0.317		-1.511		0.704***	0.845***	1.004***
	(0.36)	(0.30)	(0.58)		(2.61)		(0.26)	(0.11)	(0.09)
icinteract	1.270***	1.263***	-0.97		2.408		0.186		
	(0.41)	(0.32)	(0.70)		(3.17)		(0.31)		
logPop	0.183***	0.179***	-0.203**	-0.172**	0.374		0.051		
	(0.05)	(0.04)	(0.09)	(0.08)	(0.51)		(0.04)		
Real ER	4.412***	3.853***	-0.0306		22.30**	19.72***	5.552***	5.187***	5.452***
	(1.24)	(1.06)	(2.18)		(8.60)	(3.24)	(0.86)	(0.60)	(0.56)
Food share	0.0394**	0.0387**	-0.106***	-0.0495***	-0.00226		-0.0184*	-0.016	
	(0.02)	(0.02)	(0.03)	(0.02)	(0.08)		(0.01)	(0.01)	
EUR	0.903***	1.085***	-2.213**	-2.173**	0.288		0.785***	0.811***	0.787***
	(0.33)	(0.34)	(1.07)	(1.01)	(0.86)		(0.14)	(0.15)	(0.15)
Llock	0.505*	0.600**	-0.956**	-0.863**	-0.598		0.22		
	(0.29)	(0.30)	(0.41)	(0.43)	(0.86)		(0.15)		
Topen	0.00257		0.00144		0.00812		-0.00134		
	(0.00)		(0.00)		-(0.01)		(0.00)		
VAT	0.0334		-0.0822*	-0.0999**	-0.0707		0.014		
	(0.02)		(0.04)	(0.05)	(0.11)		(0.01)		
Constant	-3.192***	-2.750***	6.540***	5.230***	-0.0824	4.084***	0.813	1.399***	1.019***
	(0.76)	(0.75)	(1.19)	(1.06)	(9.13)	(0.84)	(0.80)	(0.26)	(0.14)
F-test	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Partial F-test	0.05		0.09		0.92		0.00	0.11	
Adjusted R^2	0.97	0.97	0.28	0.24	0.06	0.09	0.83	0.82	0.82

Notes: This table reports the results of the regression model: $PC1_c = \alpha + (\beta + \lambda D_c)\log(Y_c/\overline{Y}) + \Theta'\mathbf{Z}_c + \varepsilon_c$ where $PC1_c$ is the first component extracted from the variance-covariance matrix of the underlying variables (real exchange rates, relative prices and quantities). $\log(Y_c/\overline{Y})$ denotes the per capita income of country c relative to the cross-country geometric mean; D_c is a dummy variable that takes the value of 1 if c belongs to the "Poor" group (the third and fourth income quartiles) and is 0 otherwise; ε_c is a disturbance term. $\mathbf{Z}_c$ denotes a vector of explanatory variables for country c in 2011; including: the budget share of Food items ("Food share", in percentage), the logarithm of price differential between c and the US ("real ER"), the logarithm of population ("Pop"), a dummy variable that indicates whether the country is in the Eurozone or not ("EUR"), a dummy variable that indicates whether the country is entirely enclosed by land ("Llock") and the 2011 value-added tax rate for each country ("VAT"). "F-test" reports the p-values of the test of the null that all estimates are jointly insignificant, while "partial F-test" reports the p-values of the test of the null that all individual insignificant estimates are jointly insignificant. Data are obtained from the ICP and the World Development Indicators. Heteroskedasticity-robust standard errors are in parentheses. Significance levels: *: $p < 0.1$, **: $p < 0.05$, ***: $p < 0.01$.

As a robustness check of the strong effect of income on consumption, we redo regression (5) with the first PC from PCA on two variables: (i) the difference between quantity and income, $\log q_{ic} - \log(Y_c/\overline{Y})$ and (ii) a different formulation of relative volume: $\log q_{ic} - \sum_{i=1}^{n} w_{ic} \log q_{ic}$. As such, we "strip out" income effects on consumption, and the PC1 of these residual measures should have a smaller correlation with income. In Appendix A7, we show that this is indeed the case: The explanatory power of income to relative consumption drops significantly, to a level comparable with relative price.[21]

[21] In unreported results, we perform two additional robustness checks: First, using PC1 at other levels do not alter our main results significantly; second, we add a dummy variable that equals one if the country is in a currency arrangement that is similar to the Eurozone (i.e., using common currencies) and zero otherwise. There are in

6. Conclusions

The International Comparison Program (ICP), arguably the largest and most comprehensive joint venture ever conducted by international statistical agencies, collects retail prices in almost all countries. These prices are at the heart of Purchasing Power Parity exchange rates and measures of cross-country living standards and poverty (Deaton and Aten, 2017). The Penn World Table, a major data source for applied macroeconomics, is based on interpolation of data using benchmarks from ICP rounds (Feenstra et al., 2015). Significant improvements in terms of cooperation and statistical methodologies have contributed to the enhanced representativeness and usefulness of the ICP data. Yet, substantial heterogeneity of cross-country consumption behaviour and pricing patterns can make the interpretation of ICP results difficult.

In this paper, we first derive a measure of tradability based on all possible differentials in prices among 150+ countries. According to this measure, the higher the price dispersion, the lower the tradability. Then, we are able to classify the goods in the ICP consumption basket into tradables and non-tradables with reasonable confidence. We further found that the pricing and consumption patterns of these components accord with the predictions of the Balassa-Samuelson hypothesis, in that richer countries tend to exhibit higher prices of non-tradables, and that a fall in tradable prices can lead to a real appreciation of currencies.

Secondly, we propose an alternative approach to traditional index-number theory, which compresses the large volume of information by identifying common factors underlying the heterogeneous patterns. This approach utilises principal component analysis (PCA) which transforms the original data into uncorrelated linear combinations. The original variables, whose information we wish to compress, are the consumption and relative prices of 125 items, across 155 countries. We make two important observations: (i) The first component explains 42% of total consumption variation and 86% of total price variation. The reason for this difference is that the ability of principal components to account for the variation in the data depends on the degree to which original variables co-move, which is stronger for prices, weaker for consumption; (ii) income is responsible for 98% (24%) of the variation in the first principal component of consumption (prices).

However, there is a secondary effect of income to price: 85% of the variation in the second

total 28 such countries: 13 of them use the CFA Franc (XOF or XAF), 8 adopt the East Caribbean Dollar (XCD), 5 use the US dollar and 2 use the Netherlands Antillean Guilder (ANG). In all specifications, the coefficient for this dummy is not significant. This implies that the Eurozone effect is unique, in that it is not similar to being in any other currency arrangements. A similar finding is made by Cavallo et al. (2014).

PC of prices is explained by income. These findings are robust to the inclusion of a wide range of explanatory variables, as well as to the level of aggregation. Ours is a novel application of PCA to cross-sectional data that provides an alternative to the traditional index-number approach to summarising patterns among the 125 ICP variables. In contrast to many applications of PCA, the results have a clear economic interpretation.

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Supplementary Materials for

Fundamental Drivers of International Price and Consumption Disparities

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A1. Dissecting the ICP data

This section first gives a brief overview of the construction of PPP and expenditure measures at multiple aggregation levels of the ICP data. Generally speaking, there are two main sources of ICP data: Price (from global surveys) and expenditure (from national accounts). A reasonable degree of consistency in terms of item coverage must be maintained between these two sources. For example, when considering a certain aggregate, such as "Food", the items identified for surveys must relate to the types of food that are used in deriving that expenditure aggregate in the national account. To reduce the length of a global list of products to be priced by all participating countries, the ICP adopts a regionalised approach which at first determines regional-specific lists.[1] This takes advantage of the homogeneity of preferences among participants within a region, to ensure that items included are both comparable and representative. Subsequently, cross-regional compilation is done via a "linking" process that uses the global item list. In total the ICP data have 7 main levels of aggregation: disaggregated "Global core list" and "Regional list" products, basic headings[2], classes, groups, categories, main aggregations and GDP.

The hierarchical structure of these levels is illustrated via Figure A1.1. At the second level from the top, GDP can be decomposed into 7 main aggregates, namely: "Individual consumption expenditure by households", "Individual consumption expenditure by non-profit institutions serving households (NPISH)", "Individual consumption expenditure by government", "Collective consumption expenditure by government", "Gross fixed capital formation", "Changes in inventories and net acquisitions of valuables" and "Balance of exports and imports". For the purposes outlined in World Bank (2013a), GDP is then divided into 26 major categories, which are further sub-divided into 61 groups and then into 126 classes. An example of a category is "Food and non-alcoholic beverages," which is divided further into two groups: "Food" and "Non-alcoholic beverages". The category "Clothing and footwear" is similarly split into two groups. Groups are then broken into classes - for example, the food group contains 11 classes that include bread and cereals, meat, fish and seafood, and so forth. Each of these classes is

[1] Despite considerable effort to ensure regional homogeneity of consumption baskets, diverse regions such as Asia-Pacific and Africa include sub-regions with markedly different baskets. This is a subject of intensive research among regional coordinators (Rao, 2013).

[2] These headings are the lowest levels of aggregation where nominal expenditure data are available in national accounts.

then divided into basic headings - for example, rice is a basic heading in the bread and cereals class.

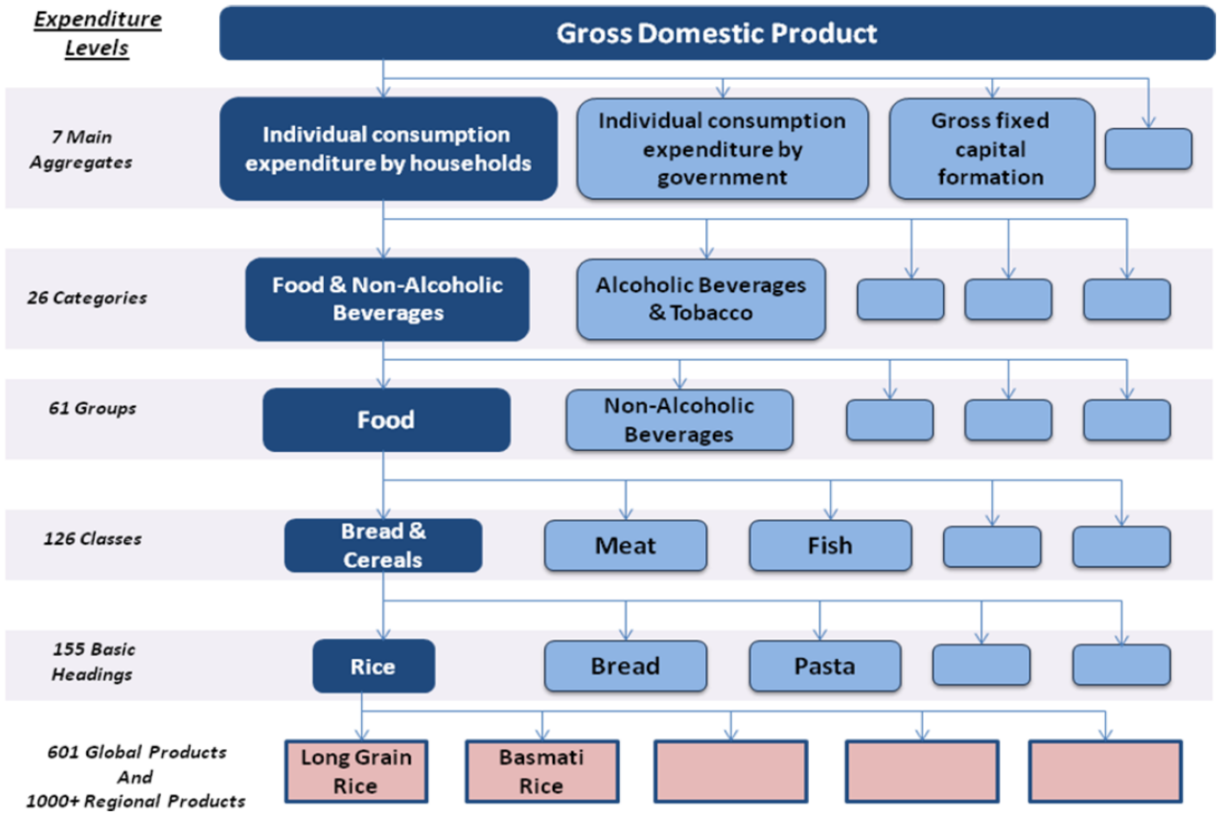

Figure A1.1. ICP Classification of Final Expenditure on GDP

Notes: This figure is extracted from Annex 1 of ICP (2011, p. 6).

The data structure across 155 countries can also be visualized by Figure A1.2. In this paper, we exclude all non-consumption categories, and only study the 131 consumption headings. The main justifications are the pattern of pricing may differ significantly between consumption and non-consumption items, and the former occupy a larger portion of individuals and households' budget. After computing the Purchasing Power Parities (PPPs) and expenditures at all levels, we perform another data treatment. At each level, items associated with receipts from sales of government services, value of imports, value of inventories, and final consumption of non-residents are excluded. These are effectively "balancing items" for which expenditures can be negative for accounting purposes (Cuthbert, 2009). Details about the omitted items are presented in Table A1.3.[3] A diagram outlining all primary steps in the aggregation and construction of

[3]Additionally, there are no nominal expenditure data available for "Individual Consumption Expenditure by Non-profit Institution Serving Households", for any country, at any level.

the three main variables is provided in Figure A1.3.

A1.1. ICP 2011 main results

The purpose of this Section is to provide a "snapshot" of the main results of the latest round of the ICP. Panel A of Figure A1.4 presents a bubble scatter plot. The bubble sizes are proportional to the countries' GDP in PPP terms. The vertical axis is price level index, computed as the ratio between PPP rates and exchange rates, with the world average level equals 100. As such, the bubbles lying above the world average line represent countries that have a price level higher than average and vice versa. The horizontal axis is GDP (in $US per capita) in PPP terms. This means the bubbles on the far right represent the most affluent countries. As can be seen, in general there is a strong positive association between price levels and income, which accords with the productivity bias hypothesis. However, there are some exceptions to the rule: some Asia and Western Asia economies (among which are Singapore, Macao, United Arab Emirates and Qatar) with very high income per capita exhibit the same levels of price as countries with much lower income. It turns out that large economies do not always have higher price levels. Following Clements and Lan (2007), in panel B we compare two measures of GDP per capita as published by the ICP: One uses the market exchange rate (MER), the other uses PPP rates. We can see that for the majority of the countries, income as measured in PPP terms is larger than that measured in MER terms. This is because PPPs are typically lower than MER and more so for poor countries, due to lower prices of non-tradables in these countries (Deaton and Aten, 2017). Indeed, the departure of the bubbles from the 45°line is substantial for the least affluent countries. Apparently, using MER (which lacks proper accounting for purchasing power of currencies) could lead to a significant underestimation of real income. Additionally, since we use the $US as the numeraire, the arrangement observed in panel B could also reflect a strong Dollar effect. That is, in 2011 the $US is overvalued compared with most of the currencies.

A1.2. Data cleaning

Next, we provide an overview of the underlying data from the ICP. Sections 2 to 5 in the main text draw upon the PPP and expenditure data at the "basic heading" level. These are all from the end-user's point of view, that is, consumers. The 2011 round of ICP data contains disaggregated expenditures and prices of 155 basic headings for 182 countries. Total consumption refers to the sum of the first 132 basic headings; this follows the ICP's definition of "Actual Household Consumption", which is the total value of the individual consumption expenditures of households, non-profit institutions serving households, and general government at purchasers' prices. Within the 132 basic headings, we consider the first 32 as food items.

We make two adjustments to the data. Firstly, we remove duplicate entries for three countries (Russia, Sudan and Egypt), each of which is a dual participant in the ICP. Next, Cuba and Bonaire do not have complete data and are omitted. Second, we combine some commodities. Many West Asia countries have little to no PPP real expenditure per capita on pork due to religious reasons. We partially solve this by combining the "Pork" and "Lamb, mutton and goat" groups; so food now consists of 31 basic headings.[4] Using a minimum cut-off of per capita consumption of $0.01, the following 22 countries are omitted: Algeria, Angola, Bangladesh, Brunei Darussalam, Burundi, Egypt, Ethiopia, Iran, Kuwait, Lao PDR, Malawi, Maldives, Mauritania, Myanmar, Nicaragua, Pakistan, Palestinian Territory, Saudi Arabia, Sudan, Tanzania, Togo and Yemen. Our final sample thus contains 182 (the starting number of countries) - 3 (duplicates) - 2 (Cuba and Bonaire) - 22 (small consumption) = 155 countries. One limitation of the ICP data is that the 31 basic headings exclude food consumed away from home, which is important in some high-income countries.

A1.3. Notes on the "Category" level

In the unpublished ICP data that we have access to, both PPP rates and nominal expenditures (in local currency unit, hereafter LCU) are only available at the second most disaggregated level (basic headings), plus a level that is not included in the hierarchical structure as illustrated in Figure A1.1. The latter level is known as the "analytical categories". Some of these analytical categories are aggregations of other analytical categories (which are themselves components of GDP), so that sum of all categories is greater than GDP, thus making themselves "non-hiarchical". This system of accounting appears in public World Bank documents.

It is interesting to contrast the "analytical categories" against the unpublished version that corresponds to the third level from the top (namely "categories"), which is a hierarchical system.[5] The exposition of these systems and the relationship between their components, which

[4]To maintain internal consistency when we combine, expenditures (in both domestic currency units and in US dollars, that is, real expenditures) are summed over the sub-components, whilst the purchasing power parity of the combination is the ratio of nominal to real expenditures. For an extensive study of the consumption pattern of these 31 headings, see Clements and Si (2017).

[5]At a casual investigation, the difference between the two systems is somewhat dubious due to the fact that, incidentally, the number of analytical categories and that of categories for the 2011 round are the same (both at 26 items). In the 2005 round, the number of major categories is just 13, while that of analytical categories is 24. The analytical categories serve as the building blocks of GDP and therefore their number changes very little in 2011: The two newly added types are "Domestic absorption" and "Individual consumption expenditure by households without housing".

has not been discussed hitherto in the literature, is a useful benchmark for future research using ICP data. The definitions of the analytical categories are provided in Table A1.1 which is provided by the ICP. Table A1.2 shows how the two systems are linked. This Table provides an essential guidance for researchers to navigate the ICP database and consistently construct data series at the intermediate aggregation levels.

Table A1.1. ICP Analytical Categories

Code	Definition
(1)	**Gross domestic product:** Actual individual consumption at purchasers' prices-plus collective consumption expenditure by government at purchasers' prices plus gross capital formation at purchasers' prices plus the f.o.b. (free on board) value of exports of goods and services less the f.o.b. value of imports of goods and services.
(2)	**Actual individual consumption:** Total value of the individual consumption expenditures of households, non-profit institutions serving households (NPISHs), and general government at purchasers' prices.
(3)	**Food and non-alcoholic beverages:** Household expenditure onfood products and non-alcoholic beverages purchased for consumption at home (excludes food products and non-alcoholic beverages sold for immediate consumption away from home by hotels, restaurants, cafés, bars, kiosks, street vendors, automatic vending machines, etc.; cooked dishes prepared by restaurants for consumption off their premises; cooked dishes prepared by catering contractors whether collected by the customer or delivered to the customer's home; and products sold specifically as pet foods).
(4)	**Alcoholic beverages, tobacco, and narcotics:** Household expenditure on alcoholic beverages purchased for consumption at home (includes low or non-alcoholic beverages that are generally alcoholic such as non-alcoholic beer, and excludes alcoholic beverages sold for immediate consumption away from the home by hotels, restaurants, cafés, bars, kiosks, street vendors, automatic vending machines, etc.) and household expenditure on tobacco (covers all purchases of tobacco, including purchases of tobacco in cafés, bars, restaurants, service stations, etc.).
(5)	**Clothing and footwear:** Household expenditure on clothing materials; garments for men, women, children, and infants; other articles of clothing and clothing accessories; cleaning, repair, and hire of clothing; all footwear for men, women, children, and infants; and repair and hire of footwear.
(6)	**Housing, water, electricity, gas and other fuels:** Household expenditure on actual and imputed rentals for housing; maintenance and repair of the dwelling; water supply and services related to the dwelling; and electricity and gas and other fuels-plus expenditure of NPISHs on housing plus general government expenditure on housing services provided to individuals.
(7)	**Furnishings, household equipment and maintenance:** Household expenditure on furniture and furnishings; carpets and other floor coverings; household textiles; household appliances; glassware, tableware, and household utensils; tools and equipment for house and garden; and goods and services for routine household maintenance.

(continued on next page)

Table A1.1: ICP Analytical Categories (continued)

Code	Definition
(8)	**Health:** Household expenditure on pharmaceuticals; medical products, appliances, and equipment; outpatient services; and hospital services-plus expenditure of NPISHs on health plus general government expenditure on health benefits and reimbursements and the production of health services.
(9)	**Transport:** Household expenditure on purchase of vehicles, operation of personal transport equipment, and transport services.
(10)	**Communication:** Household expenditure on postal services, telephone and telefax equipment, and telephone and telefax services.
(11)	**Recreation and culture:** Household expenditure on audio-visuals, photographic, and information processing equipment; other major durables for recreation and culture; other recreational items and equipment; gardens and pets; recreational and cultural services; newspapers, books, and stationery; and package holidays-plus expenditure of NPISHs on recreation and culture plus general government expenditure on recreation and culture.
(12)	**Education:** Household expenditure on pre-primary, primary, secondary, postsecondary, and tertiary education-plus expenditure of NPISHs on education plus general government expenditure on educational benefits and reimbursements and the production of educational services.
(13)	**Restaurants and hotels:** Household expenditure on food products and beverages sold for immediate consumption away from the home by hotels, restaurants, cafés, bars, kiosks, street vendors, automatic vending machines, etc. (includes cooked dishes prepared by restaurants for consumption off their premises; cooked dishes prepared by catering contractors, whether collected by the customer or delivered to the customer's home); and household expenditure on accommodation services provided by hotels and similar establishments.
(14)	**Miscellaneous goods and services:** Household expenditure on personal care, personal effects, social protection, insurance, and financial and other services-plus expenditure by NPISHs on social protection and other services plus general government expenditure on social protection.
(15)	**Net purchases abroad:** Purchases by resident households outside the economic territory of the economy-less purchases by non-residential households in the economic territory of the economy.
(16)	**Individual consumption expenditure by households:** Total value of actual and imputed final consumption expenditures incurred by households on individual goods and services; also includes expenditure on individual goods and services sold at prices that are not economically significant.
(17)	**Individual consumption expenditure by government:** Total value of actual and imputed final consumption expenditures incurred by general government on individual goods and services.
(18)	**Collective consumption expenditure by government:** Final consumption expenditure of general government on collective services
(19)	**Gross fixed capital formation:** Total value of acquisitions-less disposals of fixed assets by resident institutional units during the accounting period plus additions to the value of the non-produced assets realized by the productive activity of resident institutional units.
(20)	**Machinery and equipment:** Capital expenditure on fabricated metal products, general-purpose machinery, special-purpose machinery, electrical and optical equipment, transport equipment, and other manufactured goods.

(continued on next page)

Table A1.1: ICP Analytical Categories (continued)

Code	Definition
(21)	**Construction:** Capital expenditure on the construction of new structures and the renovation of existing structures. Structures include residential buildings, non-residential buildings, and civil engineering works.
(22)	**Other products:** Capital expenditure on plantation, orchard, and vineyard development; change in stocks of breeding stock, draft animals, dairy cattle, animals raised for wool clippings, etc.; computer software that a producer expects to use in production for more than one year; land improvement, including dams and dikes that are part of flood control and irrigation projects; mineral exploration; acquisition of entertainment, literary, or artistic originals; and other intangible fixed assets.
(23)	**Changes in inventories and valuables:** The acquisition, less disposals, of stocks of raw materials, semi-finished goods, and finished goods that are held by producer units prior to their being further processed or sold or otherwise used; and the acquisition, less disposals, of valuables (produced assets that are not used primarily for production or consumption but are purchased and held as stores of value).
(24)	**Balance of exports and imports:** The f.o.b. value of exports of goods and services minus the f.o.b. value of imports of goods and services.
(25)	**Domestic absorption:** Actual individual consumption at purchasers' prices-plus collective consumption expenditure by government at purchasers' prices plus gross capital formation at purchasers' prices.
(26)	**Individual consumption expenditure by households without housing:** Individual consumption expenditure by households in column (16) without the actual and imputed rentals included in column (06).

Notes: The shaded categories in the list below are "non-hierarchical" – that is, they combine items outside of the hierarchical ICP classification (e.g., "Education" combines the household expenditures on education with those of non-profit institutions serving households and of the government). Categories not shaded are hierarchical. Data source: ICP 2011 published results. Available at http://siteresources.worldbank.org/ICPEXT/Resources/ICP_2011.html

In the main text, we focus on only the (first) 131 consumption basic headings. The main justification is that the pattern of pricing and consumption may differ significantly between consumption and non-consumption items. At each level, we opt for omitting items that have either negative expenditure for some countries or zero expenditure for all countries. This is to ensure that our consumption measure, $\log q_{ic}$, is valid. Table A1.3 lists the omitted items that are excluded from subsequent analyses.

A2. Aggregation to levels higher than basic headings

With the data structure as presented in Section A1, we proceed to construct the PPPs at levels higher than basic headings using the celebrated Gini-Eltetö- Köves-Szulc (hereafter

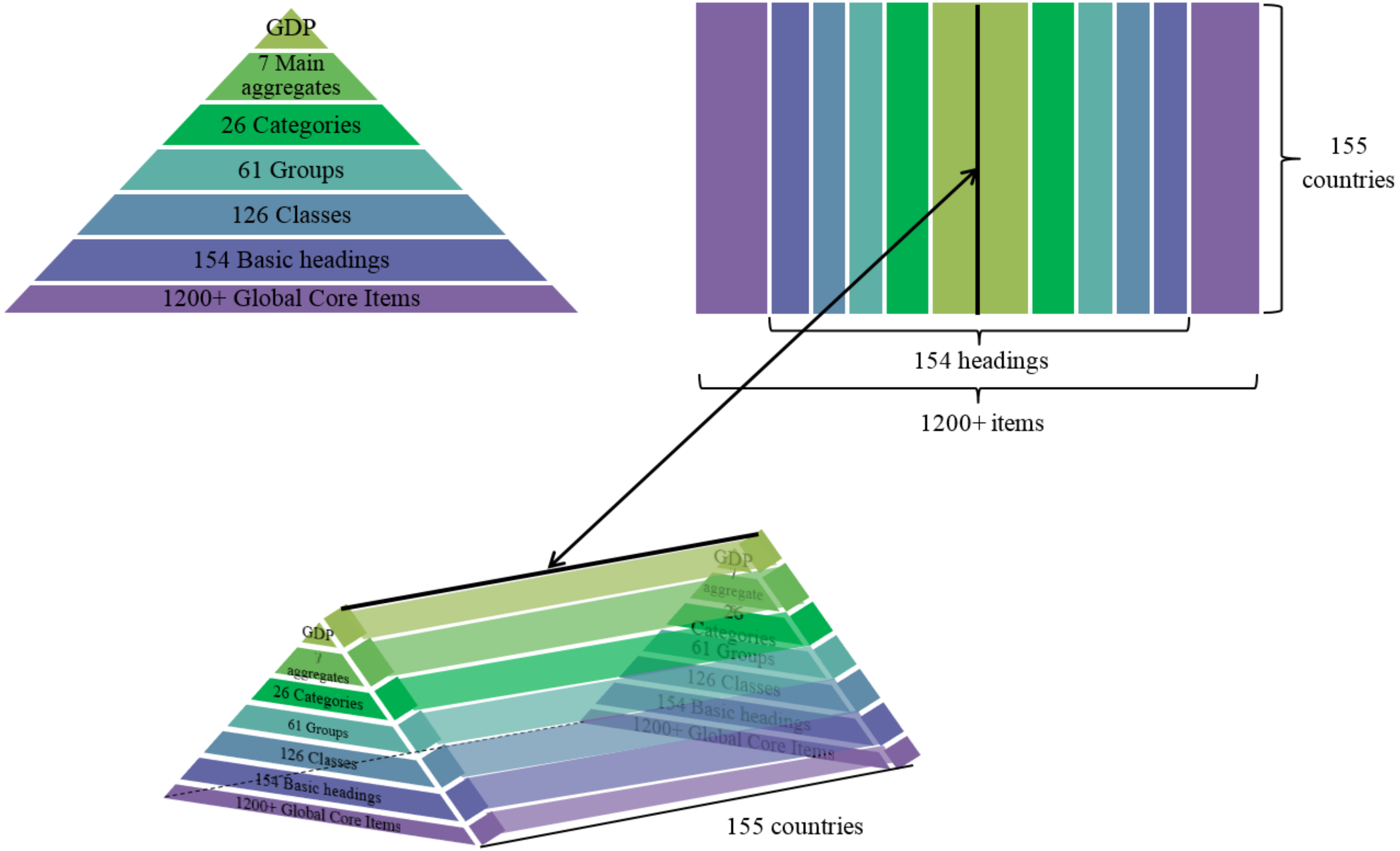

Figure A1.2. Different Perspectives of ICP Data Structure

Notes: Compared to Figure A1.1, here the number of basic headings is reduced by one, as we combine “Pork” and “Lamb, mutton and goat”.

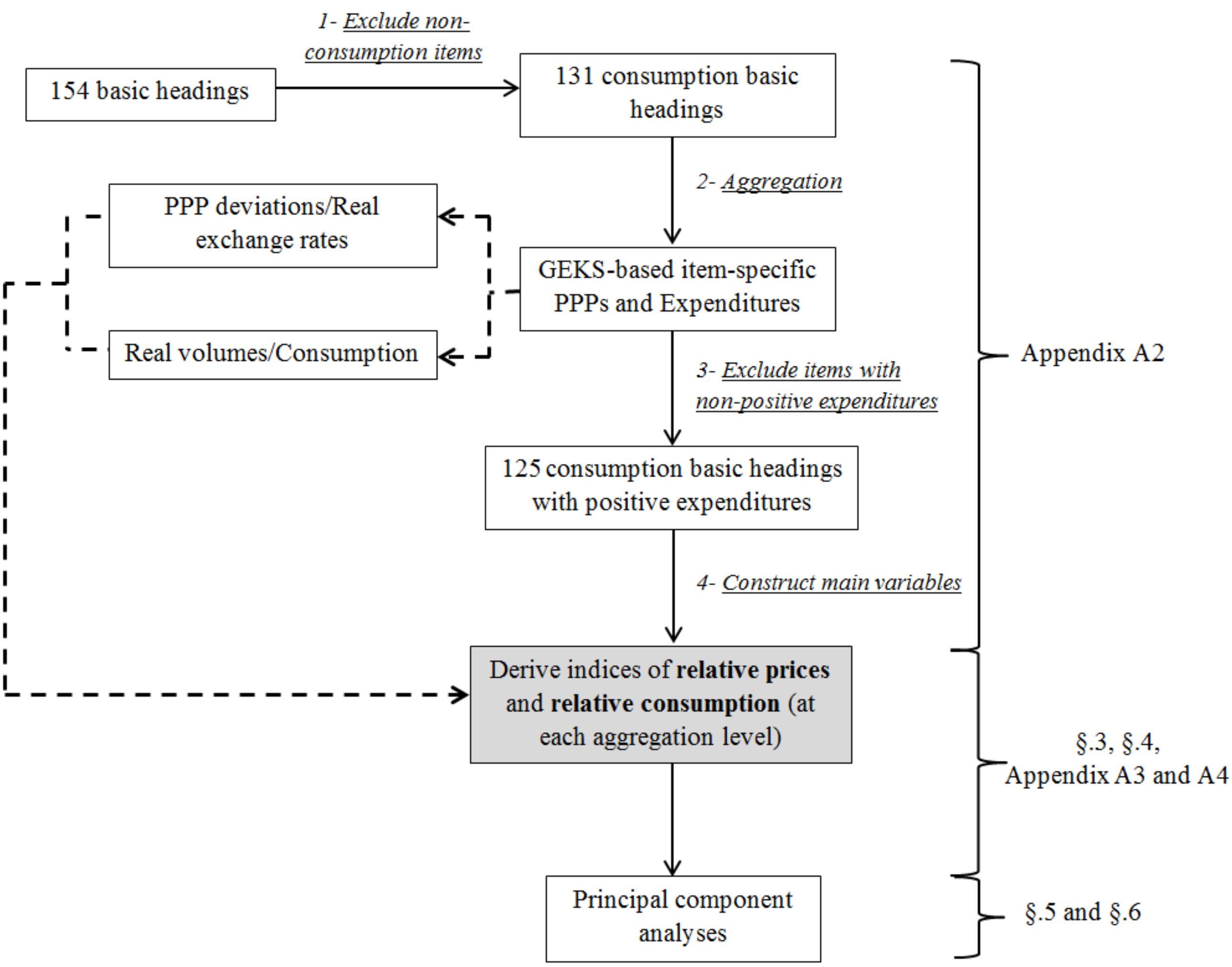

Figure A1.3. Diagram of Data Treatment

Notes: This figure presents a flow of data treatment throughout the paper. The four primary treatment steps are enumerated. The section numbers indicate where corresponding data are constructed and/or examined. Emboldened texts in the shaded box indicate our main variables of interest.

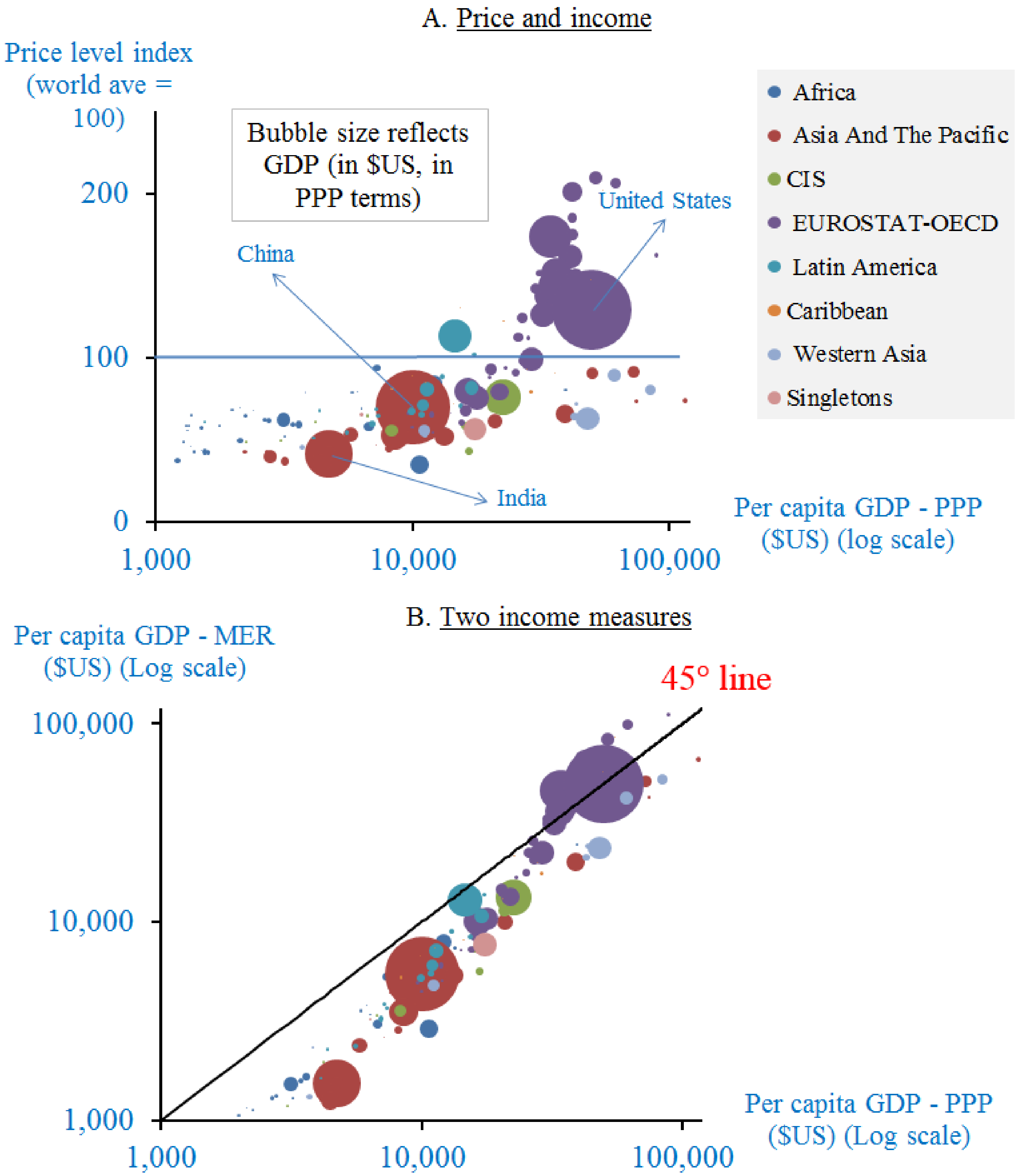

Figure A1.4. 2011 ICP Primary Results, 177 Countries

Notes: "MER" = "Market exchange rate"; "CIS" = "Commonwealth of Independent Countries".

1. There are originally 182 economies in the published ICP database (three of which are dual-participants, meaning they take part in surveys of two regions at the same time: Egypt, Sudan and Russian Federation). After omitting the duplicates, we further exclude Cuba and Bonaire because of incomplete data (as a result of methodological comparability issues).

2. Panel A: This is a replication of the figure used in a presentation by the World Bank's ICP, which is available at http://www.worldbank.org/en/news/video/2017/03/28/icp-tutorial-video-2. The vertical axis represents the ratio between PPP rates and MER.

3. Panel B: The numeraire country (US) lies on the 45°axis, since the two measures are equal for the US.

Table A1.2. Two National Accounting Systems of GDP

C. Code (1)	A. 26 Categories (C.) (Unpublished) C. Name (2)	No. of BH (3)	AC. Code (4)	B. 26 Analytical categories (AC.) (Published) AC. Name (5)	Formula with ACs (6)	Formula with Cs (7)
C1	Food And Non-Alcoholic Beverages (Ind. HH)	28	AC3	Food and non-alcoholic beverages		= C1
C2	Alcohol Beverages, Tobacco And Narcotics (Ind. HH)	5	AC4	Alcoholic beverages, tobacco, and narcotics		= C2
C3	Clothing And Footwear (Ind. HH)	5	AC5	Clothing and footwear		= C3
C4	Housing, Water, Electricity, Gas, And Other Fuels (Ind. HH)	7	AC6	Housing, water, electricity, gas and other fuels		
C5	Furnishing, Household Equipment And Routine Maintenance Of The House (Ind. HH)	13	AC7	Furnishings, household equipment and maintenance		= C5
C6	Health (Ind. HH)	7	AC8	Health		
C7	Transport (Ind. HH)	13	AC9	Transport		= C7
C8	Communication (Ind. HH)	3	AC10	Communication		= C8
C9	Recreation And Culture (Ind. HH)	13	AC11	Recreation and culture		
C10	Education (Ind. HH)	1	AC12	Education		
C11	Restaurants And Hotels (Ind. HH)	2	AC13	Restaurants and hotels		= C11
C12	Miscellaneous Goods And Services (Ind. HH)	10	AC14	Miscellaneous goods and services		
C13	Balance Of Expenditures Of Residents Abroad And Expenditures Of Non-Residents In The Economic Territory (Ind. HH)	2	AC15	Net purchases abroad		= C13
C14	Individual Consumption Expenditure By NPISHS (Ind. NPI)	1	AC26	Individual consumption expenditure by households without housing		= C1
C15	Housing (Ind. Govt)	1	AC16	Individual consumption expenditure by households		= sum [C1:C13]
C16	Health (Health Ind.Govt)	12	AC17	Individual consumption expenditure by government	= AC2 - AC16	= sum [C15:C19]
C17	Recreation And Culture (Ind. Govt)	1	**AC2**	**Actual individual consumption**	= sum [AC3:AC15]	
C18	Education (Edu. Ind. Govt)	6	AC20	Machinery and equipment		= C21
C19	Social Protection (Ind. Govt)	1	AC21	Construction		= C22
C20	Collective Services (Col. Govt)	5	AC22	Other products		= C23
C21	Machinery And Equipment (GFCF)	8	**AC19**	**Gross fixed capital formation**	= sum [AC20:AC22]	= sum [C21:C23]
C22	Construction (GFCF)	3	AC18	Collective consumption expenditure by government		= C20
C23	Other Products (GFCF)	1	**AC23**	**Changes in inventories and valuables**		= C24 + C25
C24	Changes In Inventories (IADV)	2	**AC25**	**Domestic absorption**	= **AC2 + AC18 + AC19 + AC23**	
C25	Acquisitions Less Disposals Of Valuables (IADV)	2	**AC24**	**Balance of exports and imports**		= C26
C26	Balance Of Exports And Imports (BXI)	2	**AC1**	**GROSS DOMESTIC PRODUCT**	= **AC24 + AC25**	= sum [C1:C26]

Notes: This table links two systems of the ICP accounting for GDP: Published and unpublished. The former is described in Table A1.1. The total number of basic headings is 154.

1. The category names' suffixes are grouped into 7 "main aggregates", namely: Ind. HH = "Individual consumption by household", Ind. NPI = "Individual Consumption By Non-profit Institutions Serving Households", Ind. Govt = "Individual consumption by government", Col. Govt = "Collective consumption by government", GFCF = "Gross Fixed Capital Formation", IADV = "Changes In Inventories And Acquisitions Less Disposals Of Valuables", and BXI = "Balance Of Exports And Imports". These constitute the second level in the data structure (see Figure A1.2.).

2. The numerical order of the analytical categories' codes is that presented in the ICP published data. Emboldened cells indicate analytical categories that are the sum of other analytical categories. Shaded cells indicate the categories that are "non-hierarchical" - they combine items outside of the hierarchical ICP classification.

3. To interpret the formulae in columns (6) and (7), note that: (i) "=" indicates exact equalization, (ii) "sum" indicates the summation of all cells (of the corresponding columns) numbered in between the presented numbers.

Data source: World Bank (2013b), World Bank (2011) and authors' investigation and validation.

Table A1.3. Omitted Items at Each Aggregation Level

	Basic heading		Class		Group		Category		Main Aggregate	
	ICP Code	Name	ICP Code	Name	ICP Code	Name	ICP Code	Name	ICP Code	Name
1	1113112	Final Consumption Expenditure Of Non-Resident Households In The Economic Territory (Ind. HH)	111311	Balance Of Expenditures Of Residents Abroad And Expenditures Of Non-Residents In The Economic Territory (Ind. HH)	111310	Balance Of Expenditures Of Residents Abroad And Expenditures Of Non-Residents In The Economic Territory (Ind. HH)	111300	Balance Of Expenditures Of Residents Abroad And Expenditures Of Non-Residents In The Economic Territory (Ind. HH)	120000	Individual Consumption Expenditure By Npishs (Ind. NPI)
2	1201111	Individual Consumption Expenditure By Npishs (Ind. NPI)	120111	Individual Consumption Expenditure By Npishs (Ind. NPI)	120110	Individual Consumption Expenditure By Npishs (Ind. NPI)	120100	Individual Consumption Expenditure By Npishs (Ind. NPI)		
3	1302241	Net Taxes On Production (Health Ind.Govt)	130224	Net Taxes On Production (Health Ind.Govt)	130220	Production Of Health Services (Health Ind.Govt)	130200	Health (Health Ind.Govt)		
4	1302251	Receipts From Sales (Health Ind.Govt)	130225	Receipts From Sales (Health Ind.Govt)						
5	1304241	Net Taxes On Production (Edu. Ind.Govt)	130424	Net Taxes On Production (Edu. Ind.Govt)						
6	1304251	Receipt From Sales (Edu. Ind.Govt)	130425	Receipts From Sales (Edu. Ind.Govt)						
No. of types remain		131 – 6 = 125		107 – 6 = 101		51 – 3 = 48		19 – 3 = 16		3 – 1 = 2

Notes: This Table presents the items either have (i) negative expenditure for some countries or (ii) zero/missing expenditure for all countries, at each aggregation level.Expenditures of types at higher levels are the sum of items at lower levels belonging to those types.

1. All items considered are consumption-related only.
2. The first term in the LHS of the last row's equations is the number of consumption items at the corresponding aggregation levels.
3. In all "Name" columns, the suffixes are: Ind. HH = "Individual consumption by household", Ind. NPI = "Individual Consumption by Non-profit Institutions Serving Households", Ind. Govt = "Individual consumption by government". The last one includes Education and Health services.

GEKS)[6] method (Diewert, 2013). To illustrate, we first describe the application of this method to aggregate our basic heading PPP and real expenditure. Note that our starting point is the processed "global" basic heading PPPs and the corresponding nominal expenditure (in local currency unit) data compiled from national accounts.

For simplicity, we use as an example the 5×155 matrix $\mathbf{P}$ which refers solely to the 5 headings in the first broad food group - "Bread and Cereals". $\mathbf{P}$ can be expressed as:

$$\mathbf{P} = \begin{array}{c} \\ \text{Rice} \\ \text{Other cereals} \\ \vdots \\ \text{Pasta products} \end{array} \begin{array}{c} \begin{array}{ccccc} \text{Benin} & \dots & \text{US} & \dots & \text{Georgia} \end{array} \\ \begin{pmatrix} p_1^1 & \dots & 1 & \dots & p_{155}^1 \\ p_1^2 & \dots & 1 & \dots & p_{155}^2 \\ \vdots & \dots & 1 & \dots & \vdots \\ p_1^5 & \dots & 1 & \dots & p_{155}^5 \end{pmatrix} \end{array} = \left[p_{i,c}\right],$$

where $p_{i,c}$ denotes the "global PPP" between country c and the numeraire country (the US) for item i. The matrix of corresponding real volumes (or implicit quantity levels) has the same dimension as $\mathbf{P}$: $\mathbf{Q} = \left[q_{i,c}\right] = \left[M_{i,c}/p_{i,c}\right]$ where $M_{i,c}$ is the nominal expenditure devoted to item i in c. We can then construct the 155×155 Laspeyres-type bilateral index matrix as:

$$\mathbf{P}_{Las} = \left[\frac{\widetilde{\mathbf{p}}_c \cdot \widetilde{\mathbf{q}}_d}{\widetilde{\mathbf{p}}_d \cdot \widetilde{\mathbf{q}}_d}\right] \quad (c, d = 1, \dots, 155),$$

where $\widetilde{\mathbf{p}}_c \cdot \widetilde{\mathbf{q}}_d = \left(\Sigma_{i=1}^5 p_{i,c} q_{i,d}\right) / \left(\Sigma_{i=1}^5 p_{i,d} q_{i,d}\right)$ is the inner product of the c^{th} row of $\mathbf{P}'$ and d^{th} column of $\mathbf{Q}$. Similarly, a Paasche-type index can be constructed as:

$$\mathbf{P}_{Paas} = \left[\frac{\widetilde{\mathbf{p}}_c \cdot \widetilde{\mathbf{q}}_c}{\widetilde{\mathbf{p}}_d \cdot \widetilde{\mathbf{q}}_c}\right] \quad (c, d = 1, \dots, 155).$$

We then combine the two matrices to derive a Fisher-type index matrix:

$$\mathbf{P}_{Fish} = \left[\left(\frac{\widetilde{\mathbf{p}}_c \cdot \widetilde{\mathbf{q}}_d}{\widetilde{\mathbf{p}}_d \cdot \widetilde{\mathbf{q}}_d} \times \frac{\widetilde{\mathbf{p}}_c \cdot \widetilde{\mathbf{q}}_c}{\widetilde{\mathbf{p}}_d \cdot \widetilde{\mathbf{q}}_c}\right)^{1/2}\right] = \left[P_{c,d}\right] \quad (c, d = 1, \dots, 155), \tag{1}$$

of which each element is the geometric mean of the corresponding elements in the Laspeyres and Paasche matrices.[7]

[6]Named after Gini (1931), Eltetö and Köves (1964) and Szulc (1964). Beside this method, there are many alternative multilateral indices suggested by the ICP Technical Advisory Group (TAG). It should be noted that instead of following the "region to global" approach used in the 2011 ICP, without access to regional disaggregated data, we simply use the ICP-given global results and aggregate them up.

[7]As a prerequisite for the construction of these price index matrices, for any given basic heading, the recorded PPPs must be either different from zero for all countries, or equal to zero for all countries. Since we are using the "processed" global PPPs, there are no all-zero rows in $\mathbf{P}$. For expenditure, there are missing data for some items for a given country c (i.e. the c^{th} column of $\mathbf{Q}$ is zero). This makes the Laspeyres-, Paasche- and Fisher-type PPPs between the two countries c and d undefined. In such cases the ICP suggests that these indices be approximated by the ratio of geometric means of the basic heading PPPs. For example, for the class "Bread and Cereals" we could set $(p_{Las})_{c,d} = (p_{Paas})_{c,d} = (p_{Fish})_{c,d} \approx \frac{(\Pi_{i=1}^5 p_{i,c})^{1/5}}{(\Pi_{i=1}^5 p_{i,d})^{1/5}}$. We use this approach to all other items, and at all

A2.1. Justifications for a GEKS system

What are the primary justifications for adopting a GEKS approach to cross-country comparison in the 2011 ICP? In the index number literature, there are typically two approaches to evaluate the making of the multilateral index numbers comparison. As shall be seen, these two approaches, if used independently, amount to the well-known problems of measurement without theory and theory without measurement. The first is known as the "test" or "axiomatic" approach. The work by Diewert (1999) indicates that the GEKS system (using the Fisher ideal index as the basic building block) passes 9 out of the 11 tests for "desirable" properties (three most relevant for cross-sectional comparisons are transitivity, characteristicity and matrix consistency (Neary, 2004). The second approach relies on the critical assumption that consumers have preferences over different bundles of goods depend on their relative prices. That is, this "economic" approach takes into account the (multilateral) substitution bias. Additive index numbers, such as the Geary-Khamis (Geary, 1958 and Khamis, 1972) (GK) index and the Iklé-Dikhanov-Balk (Iklé, 1972; Dikhanov, 1997 and Balk, 1996) (IDB) index are not consistent with substitution effects. In particular, adoption of single reference relative price will give rise to different inferred consumption volumes of countries that are all located on the same indifference curve. To make matter worse, the substitution bias is much larger in cross-sectional (multilateral) context than in intertemporal context, since both price and volume are more variable in the former. In the words of Diewert (1999, p. 50), this example of the classical index number problem is stated as: *"(...) the quest for an additive multilateral method with good economic properties (i.e., a lack of substitution bias) is a doomed venture: Non-linear preferences and production functions cannot be adequately approximated by linear functions."* In contrast, the GEKS with a flexible functional form exhibits a reasonable degree of approximation to the indifference curve (Diewert, 2013). GEKS can be regarded as being consistent with an economic approach.

Notwithstanding the consumer-theory inconsistency of the additive indices, the economic approach to index number itself attracts two important criticisms, namely, the uniform preferences across all final purchasers and homothetic preference.[8] The recent literature on international comparison makes several important discussions regarding the second point. Neary (2004) proposes a multilateral system known as the Geary-Allen International Accounts (GAIA),

higher aggregate levels.

[8]That is, preferences are represented by a homogeneous utility function.

which is consistent with non-homothetic preferences. The GAIA system 's main weakness is its single set of "global" reference relative prices that may be inappropriate considering the cross-country price differences (see, e.g., Deaton and Heston, 2010 and Feenstra, Ma, and Rao, 2009). An econometric generalization of the Neary model is proposed by Barnett, Diewert, and Zellner (2009), namely, to use a set of representative reference prices for each country. The equal-weighted geometric average of the resulting country-specific parities is conjectured to be close to the (point estimate) GEKS index.[9] It is noteworthy that, in any application, the usefulness of the multilateral indices is partially dictated by the users' purpose. With respect to ours, the GEKS proves to be an ideal candidate.

A2.2. Potential modifications to the GEKS index

When comparing prices internationally, we cannot directly use the individual items' local prices $p_{i,c}$, since these implicitly contain not only different currency units, but also different quantities of consumption (i.e, 1 kg of rice, 1 gallon of water etc.). The bilateral index derived by (1), which is a Fisher ideal index, accounts (to some degree) for consumption unit differentials, by allowing the volume unit to be the same in the numerators and denominators. Nevertheless, aggregating across different consumption units is, in practice, a less than ideal approach. To circumvent this issue, we takes an average of the disparities between c's currency and the rest of the world's (Diewert, 2013, p. 123) and construct the price level of c as: $P_c = \left(\prod_{d=1}^{155} P_{c,d}\right)^{1/155}$. However, the different currency units problem remain unresolved: The ratio of the relative prices between a and b and between a and c does not equal to the price between b and c.[10] To account for this issue, we divide the price level of c by the US's:

$$P_c/P_{US} = \left(\prod_{d=1}^{155} \frac{P_{c,d}}{P_{US,d}}\right)^{1/155} \qquad (c, d = 1, \ldots, 155), \tag{2}$$

and thus convert the average price difference to the difference between c and the US. This method of weighting is a democratic one, that is, each country (d) gets the same weight in the geometric mean of the separate country (c) estimates (Barnett et al., 2009). P_c/P_{US} can be interpreted as the number of units of country c's currency required to purchase one Dollar's worth of "Bread and Cereals" and receive an equivalent amount of utility.

As of 2011, the ICP publishes the values of P_c/P_{US} only at the category level, but by

[9] Nevertheless, this econometric approach to index number problem is generally not feasible due to the exhaustive number of parameters to be estimated (Diewert, 2013, p.161).

[10] Mathematically, the matrix $\mathbf{P}_{\text{Fish}}$ is not yet transitive. That is, it does not satisfy the condition that PPP computed between two countries should be the same whether it is computed directly or indirectly via a third one (Rao, 2013).

following the above procedure, we are able to extend compute this ratio for other item classes aside from "Bread and Cereals", and then at all aggregation levels, up to "GDP". The hierarchical structure of these levels is provided in Table A1.2). The corresponding aggregated expenditures are simply the sum of the level-specific component items' expenditures. Note that at each level, we use the derived prices and volumes of the immediate lower level as a starting point, rather than keep aggregating from the basic heading data.[11] The output of this process is a $n \times 155$ matrix of PPPs and a corresponding $n \times 155$ matrix of expenditure where n denotes the number of level-specific item groups.[12]

One could argue that using P_c/P_{US} does not satisfactorily account for the problems of averaging over vastly distinct consumption units and currency units (in both the numerator and denominator). Additionally we still have to rely on selecting a (arguably arbitrage) numeraire. A third disadvantage of this approach is that the weight vector is not a normalized one (i.e., $\sum_i q_i \neq 1$). Nevertheless, to facilitate direct comparison with the ICP results and the previous literature, we follow the above approach to construct aggregated price indices. However, for price comparison at the basic heading level, in the next section we introduce a new budget-share weighted, currency-neutral price measurement that arguably avoids the described setbacks.

A3. Derivations of a real cost-of-living indices

As pointed out in the previous section, we seek to account for the effects of different currencies and consumption units. Recall that from Section 3, we do this by using the difference between the Dollar price in one country (c) and another (d):

$$k_{i,c,d} = k_{i,c} - k_{i,d} = \underbrace{\log\left(p_{i,c}/S_c\right)}_{\$\text{ price of i in c}} - \underbrace{\log\left(p_{i,d}/S_d\right)}_{\$\text{ price of i in d}} = \log\ \left(p_{i,c}/p_{i,d}\right) - \log\ \left(S_c/S_d\right),$$

where S_c and S_d are the exchange rates of c and d, respectively. By construction, the term $\log\left(p_{i,c}/p_{i,d}\right)$ neutralizes the effect of different consumption units, while the term $\log\ \left(S_c/S_d\right)$

[11] We adopt this sequential approach, due to a technical reason. For some basic headings, net expenditures (used as weights) for a number of countries are large and negative, leading to negative values of the Laspeyres and/or Paasche matrices and computation of the Fisher index matrix is not feasible. This issue has not been covered in the ICP methodology documents. Adding up expenditures at the higher levels resolves some of this problem, however, negative values of Laspeyres and Paasche matrices can still exist if the negative expenditure figures are sufficiently large. We follow recommendation by the ICP: For these cases, all indices are replaced by the ratio of geometric means of the basic heading PPPs between each of the two countries and the US. As shown later on, negative expenditures are also omitted when computing relative prices and real volume.

[12] Using this approach, we are able to derive a series of cross-country PPPs at GDP level that has an almost perfect correlation with the corresponding published PPPs (that includes both consumption and non-consumption items). The implied root mean squared errors (RMSE) is 9.5. That is, the average difference between our measure and the published one by the ICP is 9.5 units of local currency.

neutralizes the effect of different currencies. For simplicity, where it is straightforward to do so, we suppress the item subscript i when referring to the bilateral price differences and simply use $k_{c,d}$.

A3.1. A generalized index

For item i, as before define the vector of cross-country Dollar prices as: $\widetilde{\mathbf{k}}_i = \left[k_{i,1}, k_{i,2} \ldots, k_{i,155}\right]'$ and the corresponding weight (budget shares) vector as $\widetilde{\mathbf{w}}_i = \left[w_{i,1}, w_{i,2}, \ldots, w_{i,155}\right]'$. Define the matrix of price differentials as

$$\mathbf{K}_i = \begin{bmatrix} 0 & k_{1,2} & \ldots & k_{1,155} \\ k_{2,1} & 0 & \ldots & k_{2,155} \\ \vdots & \vdots & \ddots & \vdots \\ k_{155,1} & k_{155,2} & \ldots & 0 \end{bmatrix} = \widetilde{\mathbf{k}}_i \widetilde{\iota}' - \widetilde{\iota}\, \widetilde{\mathbf{k}}_i',$$

where $\widetilde{\iota}' = [1, 1, \ldots, 1]$ is a 155-element vector of 1. Since $k_{c,d} = 0$ when $c = d$ and $k_{c,d} = -k_{d,c}$, $\mathbf{K}_i$ is a skew-symmetric matrix.

Next, we compute the multilateral price index of country c as the average of the weighted price differentials between c and the rest of the world (including c):

$$\log P_{i,c} = \frac{1}{155} \sum_{d=1}^{155} \frac{w_{i,c} + w_{i,d}}{2} k_{c,d};\, \Sigma_{i=1}^{125} w_{i,c} = \Sigma_{i=1}^{125} w_{i,d} = 1\ (c, d = 1, \ldots, 155). \tag{3}$$

Then, the vector of price indices can be defined as the sum of two terms:

$$\widetilde{\mathbf{P}}_i = \begin{bmatrix} \log P_{i,1} \\ \log P_{i,2} \\ \vdots \\ \log P_{i,155} \end{bmatrix} = \frac{1}{2}\frac{1}{155} \times \begin{bmatrix} \Sigma_{d=1}^{155} w_{i,1} k_{1,d} \\ \Sigma_{d=1}^{155} w_{i,2} k_{2,d} \\ \vdots \\ \Sigma_{d=1}^{155} w_{i,155} k_{155,d} \end{bmatrix} + \frac{1}{2}\frac{1}{155} \times \underbrace{\begin{bmatrix} \Sigma_{d=1}^{155} w_{i,d} k_{1,d} \\ \Sigma_{d=1}^{155} w_{i,d} k_{2,d} \\ \vdots \\ \Sigma_{d=1}^{155} w_{i,d} k_{155,d} \end{bmatrix}}_{\mathbf{K}_i \widetilde{\mathbf{w}}_i}. \tag{4}$$

In essence, this approach is close to the construction of the Fisher Ideal Index underlying the GEKS methodology (as in (1)), in the sense that we measure the bilateral difference with a "mid-point" weight.

Next, define the matrices $\mathbf{A}_i$ and $\mathbf{B}_i$ as:

$$\mathbf{A}_i = \widetilde{\mathbf{k}}_i \widetilde{\iota}' \widetilde{\iota}\, \widetilde{\mathbf{w}}_i' = 155 \begin{bmatrix} w_{i,1}k_{i,1} & w_{i,2}k_{i,1} & \ldots & w_{i,155}k_{i,1} \\ w_{i,1}k_{i,2} & w_{i,2}k_{i,2} & \ldots & w_{i,155}k_{i,2} \\ \vdots & \vdots & \ldots & \vdots \\ w_{i,1}k_{i,155} & w_{i,2}k_{i,155} & \ldots & w_{i,155}k_{i,155} \end{bmatrix};$$

$$\mathbf{B}_i = \widetilde{\iota}\, \widetilde{\mathbf{k}}_i' \widetilde{\iota}\, \widetilde{\mathbf{w}}_i' = \sum_{d=1}^{155} k_{i,d} \begin{bmatrix} w_{i,1} & w_{i,2} & \ldots & w_{i,155} \\ w_{i,1} & w_{i,2} & \ldots & w_{i,155} \\ \vdots & \vdots & \ldots & \vdots \\ w_{i,1} & w_{i,2} & \ldots & w_{i,155} \end{bmatrix}.$$

It can be shown that the first term of $\widetilde{\mathbf{P}}_i$ is a vector proportional to the main diagonal of the matrix $\mathbf{A}_i - \mathbf{B}_i$. A typical element of this diagonal vector is $\Sigma_{d=1}^{155} w_{i,c}\left(k_{i,c} - k_{i,d}\right)$ $(c, d = 1, \ldots, 155)$.

We can express this difference term by $\widetilde{\eta}_i - \widetilde{\vartheta}_i$, where $\widetilde{\eta}_i$ and $\widetilde{\vartheta}_i$ denote the main diagonal of $\mathbf{A}_i$ and $\mathbf{B}_i$, respectively. Then, $\widetilde{\mathbf{P}}_i = \frac{1}{2 \times 155}\left(\mathbf{K}_i \widetilde{\mathbf{w}}_i + \left[\widetilde{\eta}_i - \widetilde{\vartheta}_i\right]\right)$. How do we interpret $\widetilde{\eta}_i$ and $\widetilde{\vartheta}_i$? As shown in the subsequent discussions, $\widetilde{\eta}_i$ is proportional to the vector cost-of-living indices defined in a conventional way: That is, by weighting the local prices with local budget shares rather than weighting average prices with average shares. As for $\widetilde{\vartheta}_i$, it is the vector of price indices that weights all countries' prices by each country's budget shares, one at a time.

Finally, aggregating the item-specific price vectors gives us a formulation of the vector of cost-of-living index:

$$\widetilde{\mathbf{P}} = \sum_{i=1}^{125} \widetilde{\mathbf{P}}_i = \begin{bmatrix} \Sigma_{i=1}^{125} \log P_{i,1} \\ \Sigma_{i=1}^{125} \log P_{i,2} \\ \vdots \\ \Sigma_{i=1}^{125} \log P_{i,155} \end{bmatrix} = \frac{1}{2 \times 155} \sum_{i=1}^{125} \left(\mathbf{K}_i \widetilde{\mathbf{w}}_i + \left[\widetilde{\eta}_i - \widetilde{\vartheta}_i\right]\right). \tag{5}$$

Since both consumption units and currency units drop out of these indices, we can use them in cross-country comparisons. Another major advantage of this approach is that rather than relying on a single numeraire, as normally is the case in PPP computations, all bilateral differences are taken into account.

A3.2. An intermediate index

If, when computing the price for country c, we do not use the bilateral average weights as above, and instead only use the weights corresponding to c, the multilateral price index (3) becomes: $\log P^*_{i,c} = \frac{1}{155} \sum_{d=1}^{155} w_{i,c} k_{c,d}$ $(c, d = 1, \ldots, 155)$, and the individual price index vector is proportional to the difference between the main diagonals of $\mathbf{A}_i$ and $\mathbf{B}_i$:

$$\widetilde{\mathbf{P}}^*_i = \begin{bmatrix} \log P^*_{i,1} \\ \log P^*_{i,2} \\ \vdots \\ \log P^*_{i,155} \end{bmatrix} = \frac{1}{155} \begin{bmatrix} \Sigma_{d=1}^{155} w_{i,1} k_{1,d} \\ \Sigma_{d=1}^{155} w_{i,2} k_{2,d} \\ \vdots \\ \Sigma_{d=1}^{155} w_{i,155} k_{155,d} \end{bmatrix} = \frac{1}{155}\left(\widetilde{\eta}_i - \widetilde{\vartheta}_i\right).$$

The corresponding cost-of-living indices are:

$$\widetilde{\mathbf{P}}^* = \sum_{i=1}^{125} \widetilde{\mathbf{P}}^*_i = \frac{1}{155} \sum_{i=1}^{125} \left(\widetilde{\eta}_i - \widetilde{\vartheta}_i\right). \tag{6}$$

Subtract both sides of (6) from (5) we have:

$$\widetilde{\mathbf{P}} - \widetilde{\mathbf{P}}^* = \frac{1}{2 \times 155} \sum_{i=1}^{125} \left(\mathbf{K}_i \widetilde{\mathbf{w}}_i - \left[\widetilde{\eta}_i - \widetilde{\vartheta}_i\right]\right) \therefore 2\widetilde{\mathbf{P}} - \widetilde{\mathbf{P}}^* = \frac{1}{155} \sum_{i=1}^{125} \mathbf{K}_i \widetilde{\mathbf{w}}_i.$$

A3.3. The conventional index

We can further simplify the generalized index by replacing price differentials in (3) with the local prices at c and weight them by the local budget shares, so that: $\log P^{**}_{i,c} =$

$w_{i,c} \log \frac{p_{i,c}}{S_c}$ and $\Sigma_{i=1}^{125} w_{i,c} = 1$ $(c = 1, \ldots, 155)$. Then, the individual price index vector is proportional to the main diagonal of $\mathbf{A}_i$:

$$\widetilde{\mathbf{P}}_i^{**} = \begin{bmatrix} \log P_{i,1}^{**} \\ \log P_{i,2}^{**} \\ \vdots \\ \log P_{i,155}^{**} \end{bmatrix} = \frac{1}{155} \begin{bmatrix} 155\, w_{i,1} k_{1,c} \\ 155\, w_{i,2} k_{2,c} \\ \vdots \\ 155\, w_{i,155} k_{155,c} \end{bmatrix} = \frac{1}{155} \left(\widetilde{\eta}_i\right) .$$

The corresponding cost-of-living indices are:

$$\widetilde{\mathbf{P}}^{**} = \sum_{i=1}^{125} \widetilde{\mathbf{P}}_i^{**} = \frac{1}{155} \sum_{i=1}^{125} \left(\widetilde{\eta}_i\right) . \tag{7}$$

The indices $\widetilde{\mathbf{P}}^{**}$ are popularly used to (i) compute a "real" consumption measure (by deflating international consumptions with them) and (ii) compute a relative price measure (by subtracting them from the domestic prices). The difference between (5) and (7) can be expressed as:

$$\widetilde{\mathbf{P}} - \widetilde{\mathbf{P}}^{**} = \frac{1}{2 \times 155} \sum_{i=1}^{125} \left(\mathbf{K}_i \widetilde{\mathbf{w}}_i - \left[\widetilde{\eta}_i + \widetilde{\vartheta}_i\right] \right) \therefore 2\widetilde{\mathbf{P}} - \widetilde{\mathbf{P}}^{**} = \frac{1}{155} \sum_{i=1}^{125} \left(\mathbf{K}_i \widetilde{\mathbf{w}}_i - \widetilde{\vartheta}_i \right) .$$

And the difference between (6) and (7) is:

$$\widetilde{\mathbf{P}}^{*} - \widetilde{\mathbf{P}}^{**} = -\frac{1}{155} \widetilde{\vartheta}_i .$$

A3.4. Comparison of the three indices

On the left-hand side (LHS) of Figure A3.5 we present the scatterplots of $\widetilde{\mathbf{P}}^{*}$ and $\widetilde{\mathbf{P}}^{**}$ against $\widetilde{\mathbf{P}}$. It can be seen that the intermediate indices track the generalized indices much better than the conventional indices. Specifically, panel B reveals that the generalized index is systematically lower than the conventional index. Intuitively, when we use the generalized indices as a benchmark, not accounting for differences in consumption unit introduces substantial biases. We label 10 countries with the greatest absolute vertical distance from the 45° line, in descending order: Iraq, Antigua and Barbuda , Jamaica, Suriname, Oman, Anguilla, Bahrain, Jordan, UAE and Qatar. Of these, 6 are Middle East and OPEC countries.

When we plot the differences between these indices ($\widetilde{\mathbf{P}} - \widetilde{\mathbf{P}}^{*}$ and $\widetilde{\mathbf{P}} - \widetilde{\mathbf{P}}^{**}$, respectively) against income (on the right-hand side of Figure A3.5), we can see that the biases tend to be significantly larger for poorer countries. The magnitude of the biases are much larger when using the conventional indices than when using the intermediate indices, and in some cases the difference is tenfold. The same "outliers" are labelled.

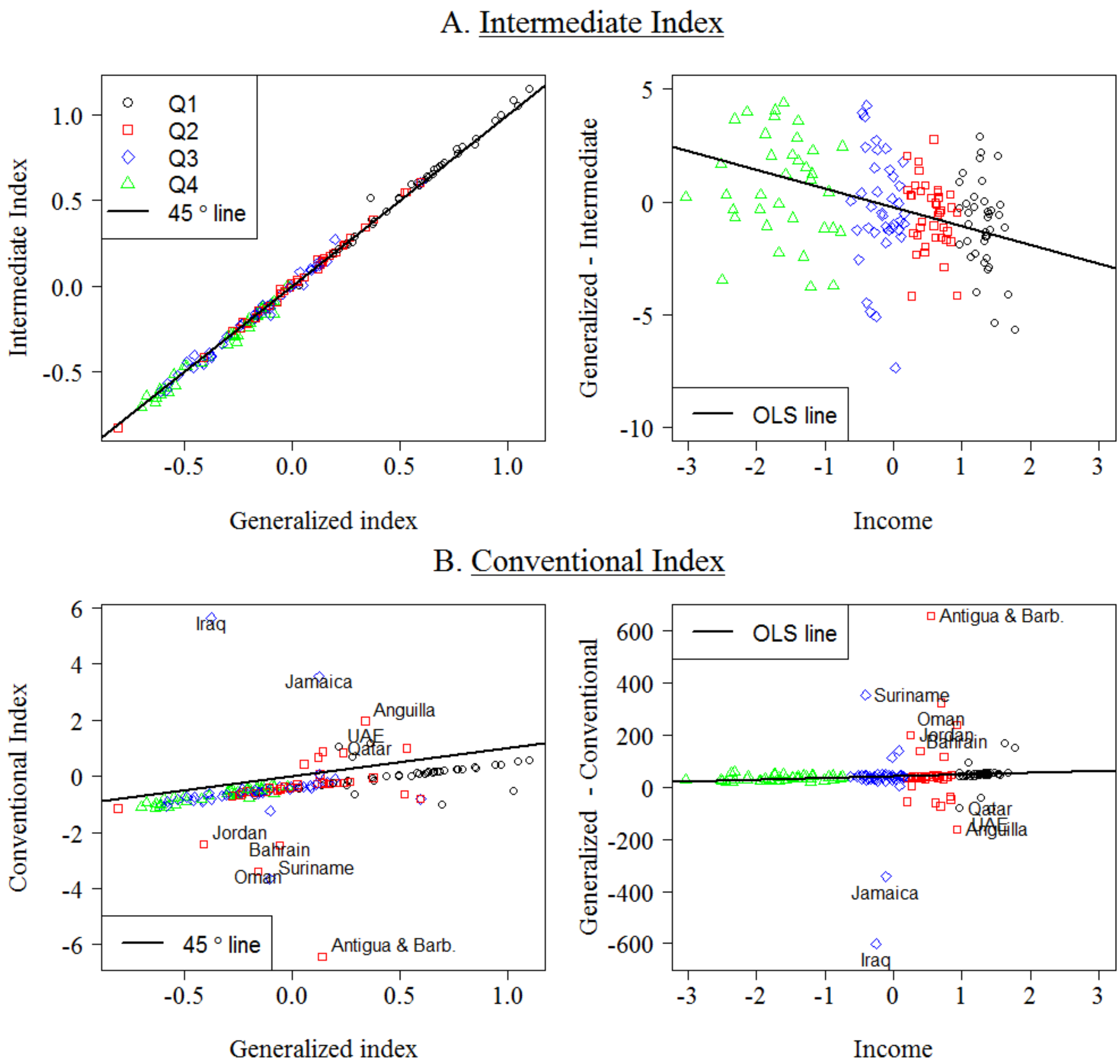

Figure A3.5. Price Indices and Income, 155 Countries, 2011

Notes: This figure compares the generalized index with the intermediate index (panel A) and with the conventional one (panel B).

1. Each point represents one country. These are color-coded based on income quartiles: Q4 the poorest and Q1 the richest. Income is defined as log $(Y/\overline{Y})$ where Y denotes the real per capita consumption and $\overline{Y}$ is the cross-country geometric mean of Y.

2. The left-hand side figures give the scatter plot of the intermediate and conventional indices against the generalized index.

3. The right-hand side figures plot the differences between the two indices and the generalized index against income.

A3.5. Generalized indices for tradables and nontradables

Now, we restrict the construction of the generalized price index to the basket of tradables: That is, $i \in T$ where T denotes the subset of tradable items.[13] There are $m < 125$ items in this

[13]Definition of tradability is discussed in Section 3 of the main text.

subset. Define the price of a tradable item in c as:

$$\log P^{T}_{i,c} = \frac{1}{155}\sum_{d=1}^{155}\frac{w^{T}_{i,c}+w^{T}_{i,d}}{2}\, k^{T}_{c,d}\ (c,d=1,\ldots,155)\,, \tag{8}$$

where $w^{T}_{i,c} = M^{T}_{i,c}/\left(\sum_{i=1}^{m} M^{T}_{i,c}\right)$ and $w^{T}_{i,d} = M^{T}_{i,d}/(\sum_{i=1}^{m} M^{T}_{i,d})$ are the tradable-conditional budget shares of i ($i \in T$) in c and d. $M^{T}_{i,c}$ and $M^{T}_{i,d}$ are the expenditure devoted to i in c and d. The budget shares satisfy $\sum_{i=1}^{m} w^{T}_{i,c} = \sum_{i=1}^{m} w^{T}_{i,d} = 1$. $k^{T}_{c,d}$ denotes the difference of i's Dollar prices in c and d. Then, (5) is modified to arrive at a tradable price index vector:

$$\widetilde{\mathbf{P}}^{T} = \left[\sum_{i=1}^{m}\log P^{T}_{i,c}\right] = \sum_{i=1}^{m}\widetilde{\mathbf{P}}^{T}_{i} = \frac{1}{2\times 155}\sum_{i=1}^{m}\left(\mathbf{K}^{T}_{i}\widetilde{\mathbf{w}}^{T}_{i} + \left[\widetilde{\eta}^{T}_{i} - \widetilde{\vartheta}^{T}_{i}\right]\right)\ (c=1,\ldots,155),$$

where $\mathbf{K}^{T}_{i}, \widetilde{\mathbf{w}}^{T}_{i}, \widetilde{\eta}^{T}_{i}$ and $\widetilde{\vartheta}^{T}_{i}$ are the tradable counterparts of $\mathbf{K}_{i}, \widetilde{\mathbf{w}}_{i}, \widetilde{\eta}_{i}$ and $\widetilde{\vartheta}_{i}$. Similar indices can be constructed for the $125-m$ non-tradables items as

$$\widetilde{\mathbf{P}}^{N} = \left[\sum_{i=m+1}^{125}\log P^{N}_{i,c}\right] = \sum_{i=m+1}^{125}\widetilde{\mathbf{P}}^{N}_{i} = \frac{1}{2\times 155}\sum_{i=m+1}^{125}\left(\mathbf{K}^{N}_{i}\widetilde{\mathbf{w}}^{N}_{i} + \left[\widetilde{\eta}^{N}_{i} - \widetilde{\vartheta}^{N}_{i}\right]\right)\ (c=1,\ldots,155),$$

where N denotes the subset of non-tradables. $\widetilde{\mathbf{P}}^{T}$ and $\widetilde{\mathbf{P}}^{N}$ form the basis of the analyses in Section 2.

How can $\widetilde{\mathbf{P}}^{T}$ and $\widetilde{\mathbf{P}}^{N}$ be related to $\widetilde{\mathbf{P}}$? To answer this question, consider a typical elements of each of these three vectors (which pertains to the price index in a single country):

$$\begin{aligned}
\log P_{c} &= \sum_{i}\log P_{i,c} = \frac{1}{2\times 155}\sum_{i}\sum_{d}\left(w_{i,c}+w_{i,d}\right)\ k_{c,d} \\
&= \frac{1}{2\times 155}\sum_{i\in T}\sum_{d}\left(w_{i,c}+w_{i,d}\right)k^{T}_{c,d} + \frac{1}{2\times 155}\sum_{i\in N}\sum_{d}\left(w_{i,c}+w_{i,d}\right)k^{N}_{c,d}\,; \\
\log P^{T}_{c} &= \sum_{i\in T}\log P^{T}_{i,c} = \frac{1}{2\times 155}\sum_{i\in T}\sum_{d}\left(w^{T}_{i,c}+w^{T}_{i,d}\right)\ k^{T}_{c,d}\,; \\
\log P^{N}_{c} &= \sum_{i\in N}\log P^{N}_{i,c} = \frac{1}{2\times 155}\sum_{i\in N}\sum_{d}\left(w^{N}_{i,c}+w^{N}_{i,d}\right)\ k^{N}_{c,d}\,.
\end{aligned} \tag{9}$$

Define the total weight of tradables in c as $w^{T}_{c} = \left(\sum_{i\in T} M^{T}_{i,c}\right)/\left(\sum_{i} M_{i,c}\right)$ and that of non-tradables as $w^{N}_{c} = 1 - w^{T}_{c}$. It follows that $w^{T}_{c}\times w^{T}_{i,c} = \frac{\sum_{i\in T} M^{T}_{i,c}}{\sum_{i} M_{i,c}} \times \frac{M_{i,c}}{\sum_{i\in T} M^{T}_{i,c}} = \frac{M_{i,c}}{\sum_{i} M_{i,c}} = w_{i,c}$ and similarly, $w^{T}_{d}\times w^{T}_{i,d} = w_{i,d}$. Then, multiplying both sides of the third line in (9) by w^{T}_{c}

yields:

$$w_c^T \sum_{i\in T} \log P_{i,c}^T = \frac{1}{2\times 155} \sum_{i\in T} \sum_{d} w_c^T \left(w_{i,c}^T + w_{i,d}^T \right) k_{c,d}^T$$
$$= \frac{1}{2\times 155} \sum_{i\in T} \sum_{d} \left(w_{i,c} + \frac{w_c^T}{w_d^T} w_{i,d} \right) k_{c,d}^T . \tag{10}$$

Multiplying both sides of the last line in (9) by w_c^N then add them to each side of (10) gives:

$$w_c^T \sum_{i\in T} \log P_{i,c}^T + w_c^N \sum_{i\in N} \log P_{i,c}^N = \frac{1}{2\times 155} \sum_{i\in T} \sum_{d} \left(w_{i,c} + \frac{w_c^T}{w_d^T} w_{i,d} \right) k_{c,d}^T$$
$$+ \frac{1}{2\times 155} \sum_{i\in N} \sum_{d} \left(w_{i,c} + \frac{w_c^N}{w_d^N} w_{i,d} \right) k_{c,d}^N . \tag{11}$$

Finally, subtract both sides of (11) from each side of the second line in (10) gives:

$$\log P_c - \left(w_c^T \log P_c^T + w_c^N \log P_c^N \right) = \frac{1}{2\times 155} \left(\sum_{i\in T} \sum_{d} \left[1 - \frac{w_c^T}{w_d^T} \right] w_{i,d} k_{c,d}^T + \sum_{i\in N} \sum_{d} \left[1 - \frac{w_c^N}{w_d^N} \right] w_{i,d} k_{c,d}^N \right) . \tag{12}$$

Now, define new tradable and non-tradable price indices for c as:

$$\log \Pi_c^T = \frac{1}{155} \sum_{i\in T} \sum_{d} w_{i,d} k_{c,d}^T ; \ \log \Pi_c^N = \frac{1}{155} \sum_{i\in T} \sum_{d} w_{i,d} k_{c,d}^N .$$

These new indices are similar to the intermediate index introduced earlier. The only difference is that they use the weights specific to country d, rather than to c. Substituting them into (12) allows us to write:

$$\Delta P_c = \log P_c - \left(w_c^T \log P_c^T + w_c^N \log P_c^N \right) = \log \Pi_c^T \left(\frac{1}{2} \sum_{d} \left[1 - \frac{w_c^T}{w_d^T} \right] \right) + \log \Pi_c^N \left(\frac{1}{2} \sum_{d} \left[1 - \frac{w_c^N}{w_d^N} \right] \right) .$$

It can be seen that the difference between the weighted price index and the generalized index is attributable to the overall weights (w_c^T and w_c^N) being specific to country c, as opposed to bilateral weights that are averaged when constructing generalized index. Nevertheless, the difference is not significant: The cross-country mean of ΔP_c is -3%. Following the conventional practice in the literature, we use $\left(w_c^T \log P_c^T + w_c^N \log P_c^N \right)$ as the price level index.

A4. Decomposition of the real exchange rates

As seen in Section 2, the validity of the HBS model rests on two crucial assumptions: (i) homothetic preference of consumers (that is, the shares of tradables/non-tradables are the same across countries) and (ii) LOP holds for tradables. But as shown in Figures 4 and 2, these assumptions are not practical: Budget share of tradable falls while its price rises as income rises. Recall that in Section 2 we defined the real exchange rate, at the price level, between a domestic country (c) and a foreign country (d), as $P_c/P_d = \frac{(P_c^N/P_c^T)^{\vartheta^c} P_c^T}{(P_d^N/P_d^T)^{\vartheta^d} P_d^T}$. Since all the new "generalized" indices used here are unit-free, we can omit S. We can write the LHS of the above expression as $P_c/P_d = \left[\left(P_c/P_d\right) \times \left(P_d^T/P_c^T\right) \right] \times \left(P_c^T/P_d^T\right)$. The term in the squared bracket can thus be interpreted as the differential in relative prices of non-tradables in c and d (or the HBS effect). We denote this term as $K_{c,d}^N = \left(P_c/P_d\right) \times \left(P_d^T/P_c^T\right) = \left(P_c^N/P_c^T\right)^{\vartheta^c} \left(P_d^T/P_d^N\right)^{\vartheta^d}$.[14] The second term, denoted as $K_{c,d}^T = P_c^T/P_d^T$, is the differential in tradable prices. Since there are only two goods in this context, the decomposition takes a simple form of an identity. In logarithmic form

$$\log P_c - \log P_d = \log K_{c,d}^N + \log K_{c,d}^T, \quad (c, d = 1, \ldots, 155), \tag{13}$$

If the HBS hypothesis holds, we expect to observe a large value of $\log K_{c,d}^N$ with respect to $\log K_{c,d}^T$.

To implement this decomposition, we first consider the case of a single base country, the United States. That is, we set c = US and derive the corresponding vectors of $[\log K_{US,d}^N]$ and $[\log K_{US,d}^N]$. Table A4.4 documents the elements of these vectors, together with their respective contributions in total changes in the US-based bilateral real exchange rates. We partition the countries into income quartiles. On average, the contribution of the changes in non-tradable price is comparable to those in tradable price (both are about 15%, albeit with opposite signs) for the richest quartile.

[14]We circumvent the need to use a specific functional form of the price level (i.e. ignoring ϑ^c and ϑ^d) and only need bilateral information on the price level indices and tradable price indices to infer the real exchange rate (Betts and Kehoe, 2017). This is, in essence, a cross-sectional counterpart to the conventional time series "PPP-accounting" exercise (see e.g. Engel, 1999).

Table A4.4. Decomposition of Real Exchange Rate Changes (Base = US)

Country	PPP	Traded	Non-traded	Country	PPP	Traded	Non-traded	Country	PPP	Traded	Non-traded	Country	PPP	Traded	Non-traded
A. First quartile				B. Second quartile				C. Third quartile				D. Fourth quartile			
1. Bermuda	-0.47	-0.52	0.04	39. Anguilla	0.21	-0.12	0.33	78. St. Vin. & Gren.	0.44	-0.03	0.47	117. Bolivia	0.97	0.46	0.51
2. Cayman Isnds	-0.14	-0.33	0.19	40. Bahrain	0.61	0.35	0.26	79. Macedonia	0.86	0.39	0.47	118. Honduras	0.70	0.27	0.42
3. Hong Kong	0.41	0.23	0.19	41. Czech	0.31	0.10	0.21	80. Thailand	1.01	0.55	0.46	119. Kyrgyzstan	1.23	0.47	0.76
4. Norway	-0.49	-0.56	0.06	42. Bahamas	0.02	-0.26	0.28	81. South Africa	0.47	0.16	0.31	120. Vietnam	1.17	0.68	0.49
5. Luxembourg	-0.30	-0.20	-0.10	43. Trini. & Tob.	0.50	0.06	0.44	82. Colombia	0.56	0.10	0.46	121. India	1.25	0.84	0.41
6. Switzerland	-0.55	-0.49	-0.06	44. Poland	0.61	0.26	0.34	83. St. Lucia	0.43	-0.03	0.46	122. São Tomé & P.	0.69	0.19	0.50
7. UAE	0.34	0.19	0.15	45. Slovakia	0.41	0.07	0.34	84. Bosnia & Herz.	0.65	0.16	0.49	123. Cambodia	1.16	0.70	0.46
8. Sweden	-0.30	-0.32	0.02	46. Barbados	0.03	-0.22	0.25	85. Turks & Caicos	-0.04	-0.21	0.16	124. Ghana	0.82	0.26	0.56
9. Germany	-0.03	-0.13	0.10	47. Lithuania	0.53	0.14	0.39	86. Venezuela	0.35	-0.23	0.58	125. Lesotho	0.72	0.28	0.44
10. Australia	-0.39	-0.40	0.01	48. Oman	0.71	0.34	0.37	87. Ukraine	1.13	0.48	0.65	126. Tajikistan	1.10	0.41	0.70
11. Austria	-0.08	-0.22	0.14	49. St. Kitts & Nevis	0.41	-0.13	0.54	88. Tunisia	0.88	0.37	0.51	127. Nigeria	0.82	0.36	0.47
12. Denmark	-0.41	-0.43	0.02	50. Croatia	0.36	0.00	0.36	89. Peru	0.70	0.24	0.46	128. Kenya	1.04	0.56	0.48
13. Canada	-0.21	-0.33	0.12	51. Hungary	0.53	0.16	0.36	90. Azerbaijan	1.08	0.45	0.63	129. Djibouti	0.70	0.24	0.46
14. Iceland	-0.07	-0.30	0.23	52. Russia	0.76	0.24	0.52	91. Belize	0.66	0.11	0.54	130. Cameroon	0.80	0.24	0.57
15. Finland	-0.24	-0.29	0.05	53. Chile	0.38	0.04	0.34	92. El Salvador	0.74	0.22	0.52	131. CÃťe d'Ivoire	0.82	0.27	0.55
16. France	-0.12	-0.19	0.07	54. Estonia	0.35	0.05	0.30	93. Ecuador	0.71	0.25	0.46	132. Senegal	0.75	0.28	0.47
17. Belgium	-0.15	-0.21	0.05	55. Turkey	0.56	0.16	0.40	94. Jamaica	0.43	0.03	0.40	133. Nepal	1.18	0.74	0.44
18. United Kingdom	-0.10	-0.09	-0.02	56. Montserrat	0.32	-0.09	0.40	95. Sri Lanka	1.13	0.60	0.53	134. Zambia	0.76	0.32	0.44
19. Netherlands	-0.13	-0.17	0.04	57. Uruguay	0.29	-0.06	0.34	96. Albania	0.78	0.28	0.50	135. Uganda	1.13	0.59	0.53
20. Singapore	0.17	-0.03	0.20	58. Seychelles	0.62	0.10	0.52	97. Namibia	0.47	0.09	0.38	136. Congo, Rep.	0.57	0.00	0.57
21. Taiwan	0.73	0.46	0.27	59. Latvia	0.42	0.05	0.37	98. Botswana	0.56	0.16	0.39	137. Haiti	0.76	0.28	0.48
22. Aruba	0.28	-0.03	0.30	60. Antigua & Barb.	0.41	-0.05	0.46	99. Armenia	0.94	0.29	0.65	138. Gambia	1.19	0.71	0.48
23. Macao	0.52	0.20	0.32	61. Montenegro	0.66	0.20	0.46	100. Mongolia	0.97	0.44	0.53	139. Sierra Leone	1.09	0.53	0.56
24. Japan	-0.29	-0.43	0.14	62. Kazakhstan	0.83	0.36	0.47	101. Iraq	0.93	0.41	0.52	140. Chad	0.75	0.24	0.51
25. Ireland	-0.21	-0.27	0.06	63. Mexico	0.56	0.23	0.32	102. Georgia	1.01	0.29	0.71	141. Benin	0.84	0.33	0.51
26. Italy	-0.03	-0.19	0.16	64. Mauritius	0.59	0.17	0.42	103. Gabon	0.39	-0.09	0.48	142. Rwanda	0.94	0.35	0.59
27. Cyprus	0.12	-0.11	0.22	65. Malaysia	0.79	0.33	0.46	104. Guatemala	0.79	0.35	0.45	143. Zimbabwe	0.71	0.27	0.45
28. New Zealand	-0.10	-0.21	0.11	66. Virgin Islands	-0.04	-0.21	0.17	105. Swaziland	0.68	0.29	0.39	144. Madagascar	1.15	0.69	0.46
29. Spain	0.06	-0.09	0.15	67. Panama	0.72	0.29	0.43	106. Fiji	0.52	0.19	0.33	145. Guinea-B.	0.81	0.29	0.52
30. Israel	-0.06	-0.24	0.17	68. Belarus	1.36	0.66	0.70	107. Paraguay	0.73	0.28	0.44	146. Mali	0.85	0.38	0.47
31. Sint Maarten	0.26	-0.01	0.27	69. Romania	0.61	0.24	0.37	108. Moldova	1.04	0.41	0.63	147. Mozambique	0.70	0.23	0.47
32. Greece	0.06	-0.15	0.21	70. Bulgaria	0.79	0.30	0.49	109. Eq. Guinea	0.50	-0.02	0.52	148. Liberia	0.75	0.33	0.42
33. Curaçao	0.36	0.01	0.35	71. Serbia	0.67	0.23	0.44	110. Suriname	0.66	0.21	0.45	149. Burkina Faso	0.83	0.35	0.48
34. Malta	0.29	-0.05	0.34	72. Brazil	0.18	-0.18	0.36	111. Indonesia	0.93	0.46	0.46	150. Comoros	0.64	0.14	0.50
35. Portugal	0.18	-0.05	0.23	73. Costa Rica	0.44	0.08	0.36	112. Philippines	0.97	0.52	0.45	151. C. Africa	0.64	0.10	0.54
36. Slovenia	0.18	-0.04	0.22	74. Grenada	0.42	-0.01	0.44	113. Cape Verde	0.61	0.20	0.41	152. Guinea	1.05	0.41	0.64
37. South Korea	0.30	-0.07	0.38	75. Jordan	0.96	0.44	0.52	114. China	0.69	0.30	0.39	153. Niger	0.83	0.32	0.50
38. Qatar	0.19	0.20	-0.01	76. Dominican Rep.	0.76	0.35	0.41	115. Morocco	0.78	0.27	0.52	154. Congo, D.R.	0.64	0.06	0.57
				77. Dominica	0.43	0.05	0.39	116. Bhutan	1.16	0.75	0.41				
Mean	-0.01	**-0.15**	**0.14**	Mean	0.52	**0.12**	**0.39**	Mean	0.72	**0.25**	**0.48**	Mean	0.88	**0.37**	**0.51**
Median	-0.07	-0.18	0.14	Median	0.53	0.10	0.39	Median	0.71	0.27	0.46	Median	0.82	0.33	0.49
SD	0.29	0.22	0.12	SD	0.26	0.20	0.10	SD	0.26	0.21	0.10	SD	0.20	0.20	0.07
Min	-0.55	-0.56	-0.10	Min	-0.04	-0.26	0.17	Min	-0.04	-0.23	0.16	Min	0.57	0.00	0.41
Max	0.73	0.46	0.38	Max	1.36	0.66	0.70	Max	1.16	0.75	0.71	Max	1.25	0.84	0.76

Notes: For the US against the rest of the world, this table presents the values of the real exchange rate (PPP), which is decomposed into the effect of real change in tradable prices (Traded) and in non-tradable prices (Non-Traded), as specified by Equation (13). Income is defined as the log difference between the 2011 per capita GDP in each country and the geometric mean of the world incomes.

Table A4.5. Decomposition of Real Exchange Rate Changes (All Countries as Base)

Country	PPP	Traded	Non-traded	Country	PPP	Traded	Non-traded	Country	PPP	Traded	Non-traded	Country	PPP	Traded	Non-traded
A. First quartile				B. Second quartile				C. Third quartile				D. Fourth quartile			
1. Bermuda	1.01	0.67	0.34	40. Anguilla	0.31	0.27	0.05	79. St. Vin. & Gren.	0.09	0.18	-0.09	118. Bolivia	-0.45	-0.32	-0.13
2. United States	0.53	0.15	0.38	41. Bahrain	-0.09	-0.20	0.11	80. Macedonia	-0.33	-0.24	-0.09	119. Honduras	-0.17	-0.13	-0.05
3. Cayman Isnds	0.67	0.48	0.19	42. Czech	0.22	0.05	0.16	81. Thailand	-0.49	-0.41	-0.08	120. Kyrgyzstan	-0.71	-0.32	-0.39
4. Hong Kong	0.11	-0.08	0.19	43. Bahamas	0.51	0.41	0.10	82. South Africa	0.06	-0.01	0.07	121. Vietnam	-0.64	-0.54	-0.11
5. Norway	1.03	0.71	0.32	44. Trini. & Tob.	0.03	0.09	-0.06	83. Colombia	-0.04	0.04	-0.08	122. India	-0.73	-0.70	-0.03
6. Luxembourg	0.83	0.35	0.48	45. Poland	-0.08	-0.12	0.04	84. St. Lucia	0.10	0.18	-0.08	123. São Tomé & P.	-0.16	-0.05	-0.12
7. Switzerland	1.08	0.64	0.44	46. Slovakia	0.12	0.08	0.04	85. Bosnia & Herz.	-0.13	-0.02	-0.11	124. Cambodia	-0.64	-0.56	-0.08
8. UAE	0.19	-0.04	0.23	47. Barbados	0.50	0.37	0.13	86. Turks & Caicos	0.57	0.36	0.22	125. Ghana	-0.30	-0.12	-0.18
9. Sweden	0.83	0.47	0.36	48. Lithuania	0.00	0.01	-0.01	87. Venezuela	0.17	0.38	-0.20	126. Lesotho	-0.19	-0.13	-0.06
10. Germany	0.56	0.28	0.28	49. Oman	-0.19	-0.20	0.01	88. Ukraine	-0.61	-0.34	-0.27	127. Tajikistan	-0.58	-0.26	-0.32
11. Australia	0.92	0.55	0.37	50. St. Kitts & Nevis	0.12	0.28	-0.16	89. Tunisia	-0.35	-0.22	-0.13	128. Nigeria	-0.30	-0.21	-0.09
12. Austria	0.61	0.37	0.24	51. Croatia	0.16	0.15	0.01	90. Peru	-0.18	-0.10	-0.08	129. Kenya	-0.52	-0.41	-0.11
13. Denmark	0.95	0.58	0.36	52. Hungary	0.00	-0.02	0.02	91. Azerbaijan	-0.56	-0.30	-0.26	130. Djibouti	-0.17	-0.09	-0.08
14. Canada	0.74	0.48	0.26	53. Russia	-0.24	-0.10	-0.14	92. Belize	-0.13	0.03	-0.16	131. Cameroon	-0.28	-0.09	-0.19
15. Iceland	0.60	0.45	0.15	54. Chile	0.15	0.11	0.04	93. El Salvador	-0.21	-0.07	-0.14	132. Côte d’Ivoire	-0.30	-0.13	-0.17
16. Finland	0.77	0.44	0.33	55. Estonia	0.18	0.10	0.08	94. Ecuador	-0.18	-0.10	-0.08	133. Senegal	-0.23	-0.13	-0.09
17. France	0.65	0.34	0.31	56. Turkey	-0.04	-0.01	-0.02	95. Jamaica	0.10	0.12	-0.02	134. Nepal	-0.66	-0.60	-0.06
18. Belgium	0.68	0.36	0.33	57. Montserrat	0.21	0.24	-0.02	96. Sri Lanka	-0.61	-0.46	-0.15	135. Zambia	-0.24	-0.18	-0.06
19. United Kingdom	0.63	0.24	0.40	58. Uruguay	0.24	0.21	0.04	97. Albania	-0.26	-0.14	-0.12	136. Uganda	-0.60	-0.45	-0.16
20. Netherlands	0.66	0.32	0.34	59. Seychelles	-0.09	0.05	-0.15	98. Namibia	0.06	0.06	0.00	137. Congo, Rep.	-0.05	0.15	-0.20
21. Singapore	0.36	0.17	0.18	60. Latvia	0.11	0.10	0.01	99. Botswana	-0.03	-0.02	-0.02	138. Haiti	-0.24	-0.14	-0.10
22. Taiwan	-0.20	-0.32	0.11	61. Antigua & Barb.	0.11	0.20	-0.09	100. Armenia	-0.42	-0.14	-0.28	139. Gambia	-0.67	-0.57	-0.10
23. Aruba	0.25	0.18	0.08	62. Montenegro	-0.13	-0.05	-0.09	101. Mongolia	-0.45	-0.30	-0.15	140. Sierra Leone	-0.57	-0.39	-0.18
24. Macao	0.01	-0.05	0.06	63. Kazakhstan	-0.31	-0.21	-0.10	102. Iraq	-0.40	-0.26	-0.14	141. Chad	-0.22	-0.09	-0.13
25. Japan	0.83	0.58	0.24	64. Mexico	-0.03	-0.09	0.05	103. Georgia	-0.48	-0.15	-0.34	142. Benin	-0.32	-0.19	-0.13
26. Ireland	0.74	0.42	0.32	65. Mauritius	-0.06	-0.03	-0.04	104. Gabon	0.14	0.23	-0.10	143. Rwanda	-0.42	-0.20	-0.22
27. Italy	0.56	0.33	0.22	66. Malaysia	-0.27	-0.19	-0.08	105. Guatemala	-0.27	-0.20	-0.07	144. Zimbabwe	-0.19	-0.12	-0.07
28. Cyprus	0.41	0.25	0.16	67. Virgin Islands	0.57	0.36	0.21	106. Swaziland	-0.15	-0.15	-0.01	145. Madagascar	-0.63	-0.55	-0.08
29. New Zealand	0.63	0.36	0.27	68. Panama	-0.20	-0.14	-0.05	107. Fiji	0.01	-0.04	0.05	146. Guinea-B.	-0.28	-0.15	-0.14
30. Spain	0.47	0.24	0.23	69. Belarus	-0.84	-0.51	-0.33	108. Paraguay	-0.20	-0.13	-0.07	147. Mali	-0.33	-0.24	-0.09
31. Israel	0.59	0.39	0.21	70. Romania	-0.08	-0.09	0.01	109. Moldova	-0.52	-0.26	-0.26	148. Mozambique	-0.17	-0.08	-0.09
32. Sint Maarten	0.26	0.15	0.11	71. Bulgaria	-0.26	-0.15	-0.11	110. Eq. Guinea	0.02	0.17	-0.15	149. Liberia	-0.23	-0.19	-0.04
33. Greece	0.47	0.30	0.17	72. Serbia	-0.15	-0.09	-0.06	111. Suriname	-0.13	-0.06	-0.07	150. Burkina Faso	-0.31	-0.21	-0.10
34. Curaçao	0.17	0.14	0.03	73. Brazil	0.35	0.33	0.02	112. Indonesia	-0.40	-0.32	-0.08	151. Comoros	-0.11	0.01	-0.12
35. Malta	0.23	0.19	0.04	74. Costa Rica	0.09	0.07	0.02	113. Philippines	-0.44	-0.37	-0.07	152. C. Africa	-0.12	0.04	-0.16
36. Portugal	0.35	0.20	0.15	75. Grenada	0.10	0.16	-0.06	114. Cape Verde	-0.08	-0.05	-0.03	153. Guinea	-0.53	-0.27	-0.26
37. Slovenia	0.35	0.19	0.16	76. Jordan	-0.44	-0.30	-0.14	115. China	-0.16	-0.15	-0.01	154. Niger	-0.30	-0.18	-0.12
38. South Korea	0.22	0.22	0.00	77. Dominican Rep.	-0.24	-0.21	-0.03	116. Morocco	-0.26	-0.12	-0.14	155. Congo, D.R.	-0.11	0.08	-0.20
39. Qatar	0.34	-0.05	0.39	78. Dominica	0.09	0.10	-0.01	117. Bhutan	-0.64	-0.61	-0.03				
Mean	0.54	**0.30**	**0.24**	Mean	-0.02	**0.01**	**-0.03**	Mean	-0.20	**-0.10**	**-0.10**	Mean	-0.34	**-0.22**	**-0.13**
Median	0.59	0.32	0.24	Median	0.00	0.01	-0.02	Median	-0.18	-0.12	-0.08	Median	-0.30	-0.18	-0.11
SD	0.29	0.21	0.12	SD	0.26	0.19	0.09	SD	0.26	0.21	0.10	SD	0.19	0.20	0.06
Min	-0.20	-0.32	0.00	Min	-0.84	-0.51	-0.33	Min	-0.64	-0.61	-0.34	Min	-0.73	-0.70	-0.32
Max	1.08	0.71	0.48	Max	0.57	0.37	0.21	Max	0.57	0.38	0.22	Max	-0.05	0.15	-0.03

Notes: For each country against the rest of the world, this table presents the values of the real exchange rate (PPP), which is decomposed into the effect of real change, between each country and the rest, in tradable prices (Traded) and in non-tradable prices (Non-Traded). Decomposition is specified by Equation (13). Income is defined as the log difference between the 2011 per capita GDP in each country and the geometric mean of the world incomes.

For the other three poorer quartiles, however, the contribution of the HBS effect is much higher, from 2 to 1.4 times that of the changes in tradable price. Productivity differentials between the US and the rich countries are probably smaller than between the US and the poor, hence the smaller HBS effect for the rich. The "grand" mean real exchange rate between the US and the rest of the world is 53%. That is, on average, using the derived price indices, the US Dollar is 53% over-valued compared with other currencies. This number can be decomposed into a differential of 15% in tradable prices, and 38% in non-tradables prices.

We repeat this exercise by consecutively using the rest of the countries as the base of comparison and report the country-specific grand means in Table A4.5. On average, using either the richest or poorest countries as bases leads to the domination of tradable price effect over the HBS effect, while both effects are of equal importance for the middle income countries. Additionally, for the second quartile, since their incomes/productivities are closer to the world average, their deviations from PPP are small. Compared to the rest of the world, these countries exhibit negligible differentials in both tradable and non-tradable prices (1% and 3%, respectively).

A5. Relative prices and consumption

Here we examine closer the price comparison matrix:[15]

$$\mathbf{K}_i = \begin{bmatrix} 0 & k_{1,2} & \cdots & k_{1,155} \\ k_{2,1} & 0 & \cdots & k_{2,155} \\ \vdots & \vdots & \ddots & \vdots \\ k_{155,1} & k_{155,2} & \cdots & 0 \end{bmatrix} = \widetilde{\mathbf{k}}_i \, \widetilde{\iota}' - \widetilde{\iota} \, \widetilde{\mathbf{k}}_i',$$

where $\widetilde{\mathbf{k}}_i = \left[k_{i,1}, k_{i,2} \ldots, k_{i,155}\right]'$ is the vector of cross-country Dollar prices. Consider the vector of row averages of $\mathbf{K}_i$:

$$\overline{\mathbf{k}}_i = \mathbf{M}\widetilde{\mathbf{k}}_i, \text{ with } \mathbf{M} = \mathbf{I} - \frac{1}{155} \widetilde{\iota} \, \widetilde{\iota}', \tag{14}$$

where $\mathbf{I}$ is an identity matrix, $\mathbf{M}$ is a symmetric idempotent matrix (so that $\mathbf{M}'\mathbf{M} = \mathbf{M}$) of order 155×155. A typical element of $\overline{\mathbf{k}}_i$ is $\overline{k}_{i,c} = \frac{1}{155} \sum_{d=1}^{155} \left[\log\left(p_{i,c}/S_c\right) - \log\left(p_{i,d}/S_d\right)\right]$ $(c, d = 1, \ldots, 155)$. In the previous section, we point out that $\overline{k}_{i,c}$ is free of currency units and consumption units. The unilateral price vector $\widetilde{\mathbf{p}}_i$ coincides with the multilateral price vector $\overline{\mathbf{k}}_i$ when $\mathbf{M} = \mathbf{I}$.

The equivalence of a "relative volume" measure can be defined in a similar manner to

[15] This matrix has the same form as the "pay parity" matrix from the paper with the same title of Clements, Lan, and Seah (2012), in which case $\widetilde{\mathbf{p}}$ refers to a vector of executive remuneration.

that of prices:

$$\mathbf{Q}_i = \begin{bmatrix} 0 & (\log q_{i,1} - \log q_{i,2}) & \dots & (\log q_{i,1} - \log q_{i,155}) \\ (\log q_{i,2} - \log q_{i,1}) & 0 & \dots & (\log q_{i,2} - \log q_{i,155}) \\ \vdots & \vdots & \ddots & \vdots \\ (\log q_{i,155} - \log q_{i,1}) & (\log q_{i,155} - \log q_{i,2}) & \dots & 0 \end{bmatrix} = \widetilde{\mathbf{q}}_i\,\widetilde{\iota}' - \widetilde{\iota}\,\widetilde{\mathbf{q}}_i',$$

where $\widetilde{\mathbf{q}}_i = \left[q_{i,1}, q_{i,2} \dots, q_{i,155}\right]'$ is a vector of consumption volumes. A typical element of this vector is the logarithmic real consumption: $\log q_{i,c} = \log\left(M_{i,c}/p_{i,c}\right)$ $(c = 1, \dots, 155)$, where $M_{i,c} = p_{i,c}q_{i,c}$ is the per capita expenditure devoted to item i in country c. It is helpful to again note that even though $q_{i,c}$ is free of currency units, it is not free of consumption units. Therefore, cross-item comparison of consumption necessitates the use of $\mathbf{Q}_i$. Similar to price matrices, we can "compress" the matrix $\mathbf{Q}_i$ by using the averages of the bilateral consumption differentials as:

$$\overline{\mathbf{q}}_i = \mathbf{M}\widetilde{\mathbf{q}}_i. \tag{15}$$

From (14) and (15), it follows that:

$$\overline{\mathbf{k}}_i'\overline{\mathbf{q}}_i = \left(\mathbf{M}\widetilde{\mathbf{k}}_i\right)'\mathbf{M}\widetilde{\mathbf{q}}_i = \widetilde{\mathbf{k}}_i'\,\mathbf{M}\,\widetilde{\mathbf{q}}_i = \widetilde{\mathbf{k}}_i'\overline{\mathbf{q}}_i,$$

since $\mathbf{M}'\mathbf{M} = \mathbf{M}$. This means that the product of the cross-country average Dollar prices and relative volumes is equal to the product of the original Dollar prices and relative volumes. Finally, define the matrices of average relative prices and average relative volumes as:

$$\mathbf{K} = [\overline{\mathbf{k}}_1, \overline{\mathbf{k}}_2, \dots, \overline{\mathbf{k}}_{125}];\ \mathbf{Q} = [\overline{\mathbf{q}}_1, \overline{\mathbf{q}}_2, \dots, \overline{\mathbf{q}}_{125}].$$

The above approach can be applied to data at higher aggregated levels constructed in Section A2. **K** and **Q** form the basis of the analyses in Sections A6 and A7 of the Appendix, as well as Sections 4 and 5 of the main text.

A6. A pilot study using PCA

The use of principal component analysis (hereafter PCA) has a long standing in modern macroeconomics research. PCA applications have been primarily associated with time series data. Stock and Watson (2002a) show that for large sample sizes, using principal components that represent common latent factors extracted from a large number of predictors could provide asymptotically efficient forecasts in the context of stationary time series. In a related paper, Stock and Watson (2002b) apply their dynamic factor model to 215 economic variables as possible predictors of 8 monthly US macroeconomics time series. Notwithstanding the difficulties in exchange rate modelling due to the disconnection of economic fundamentals, in a recent paper, Ponomareva, Sheen, and Wang (2019) propose using PCA to extract a common factor that explains about fifty percent of the cross-sectional variance in the returns of 15 US-based exchange rates. These authors find that the common component is highly correlated with various trade-weighted multilateral US exchange rates, and can be related to US fundamentals and commodity prices. We distinct our paper from the above studies in the data aspect: Rather than using either within-country time series of many variables (e.g. US macroeconomic indicators) or cross-country time series of one variable (e.g. US bilateral exchange rates), we use cross-country data for many variables.[16]

A6.1. The correlation of food prices

In this section, we first discuss the principal component analysis (hereafter PCA) via a simplified example. Assume that there is only two items in the world's food consumption basket: "Bread" and "Rice". As in Section A5, define the relative price for bread in country c as: $\overline{k}_{B,c} = (1/155)\Sigma_{d=1}^{155}\left[k_{B,c} - k_{B,d}\right]$ $(c, d = 1, \ldots, 155)$, where $k_{B,c}$ and $k_{B,d}$ are the logarithmic Dollar prices of bread in c and d, respectively.[17] Similarly, the price of rice is denoted as $\overline{k}_{R,c}$. Together these price vectors constitute the $c \times 2$ matrix $\mathbf{K} = \left[\overline{\mathbf{k}}_B, \overline{\mathbf{k}}_R\right]$. The covariance matrix of these two variables can be computed as $\Sigma = (1/154)\mathbf{K}'\mathbf{K}$.[18] A linear combination of the variables can be expressed as $\mathbf{y}_1 = a_1\overline{\mathbf{k}}_B + a_2\overline{\mathbf{k}}_R = \mathbf{K}\mathbf{a}_1$ where $\mathbf{a}_1$ denotes the coefficient vector: $\mathbf{a}_1 = [a_1, a_2]$. According to Theil (1971), a PCA maximizes the sample variance of $\mathbf{y}$, subject to the constraint $\mathbf{a}_1'\mathbf{a}_1 = 1$. This leads to the characteristic equation (also referred to as an "eigendecomposition"):

$$(\Sigma - \lambda_1\mathbf{I})\mathbf{a}_1 = \mathbf{0}, \tag{16}$$

where $\mathbf{I}$ is a 2×2 identity matrix and $\mathbf{0}$ is a vector of zeroes. It follows that $\mathbf{a}_1 =$.

The second eigenvector $\mathbf{a}_2$ can be solved via $(\Sigma - \lambda_2\mathbf{I})\mathbf{a}_2 = \mathbf{0}$ with λ_2 being the second

[16]The study that is closest in spirit to ours is Cuthbert (2009), who examine and compare the properties of 6 primary PPP aggregation methods used by PPP. Broadly speaking, these indices can be considered as the equivalence of our aggregated measures. This author posits that the underlying traits of ICP data, in particular the large negative price elasticities of demand, are useful for sorting countries into groups represented by consumption behaviour. These data characteristics are then shown to be related to two principal components that strongly describe the cross-country variability of the indices. The purpose and approach of this study is also different from ours: While Cuthbert seeks to explore the similarity, or co-movement, among a small number of aggregated volume indices with the ultimate goal of exposing their strengths and weaknesses in measuring real income, we are more interested in both price and consumption structures of a large number of disaggregated items. Additionally, we allow the aggregation level to vary, and show that the main results do not change substantially with aggregation (see Section 5).

[17]Due to the almost uniformly unitary correlation between cross-country PPP prices (from ICP publications) and market exchange rates (MER), if we use PPP prices as our underlying variable, the first principal component would inevitably account for almost 100% of the data variation and is highly correlated with MER.

[18]This is a result of $\overline{\mathbf{k}}_B$ and $\overline{\mathbf{k}}_B$ both having a zero mean by construction.

eigenvalue.[19] We can then compute the second principal component as $\mathbf{y}_2 = \mathbf{K}\mathbf{a}_2$. it can be shown that $\mathbf{y}_2$ is orthogonal to $\mathbf{y}_1$. An important result is that the sum of the variances of the original variable is equal to that of the new variables:

$$\mathrm{Var}(\mathbf{y}_1) + \mathrm{Var}(\mathbf{y}_2) = \lambda_1 + \lambda_2 = \mathrm{Var}(\bar{\mathbf{k}}_B) + \mathrm{Var}(\bar{\mathbf{k}}_R).$$

Additionally, due to each successive component accounts for the maximum amount of variation "left-over" from the previous eigendecomposition and the orthogonality of the components, we have $\lambda_1 > \lambda_2$ (Campbell and Atchley, 1981). This approach is generalized in the following discussion.[20]

A6.2. More data

Next, we extend the above approach to the 14 food headings that account for about 70% of the world's total food consumption. These includes, in order of increasing consumption, "Fresh or chilled vegetables other than potatoes", "Rice", "Fresh, chilled or frozen fish and seafood", "Food products nec", "Fresh or chilled fruit", "Mineral waters, soft drinks, fruit and vegetable juices", "Bread", "Pork + Lamb, mutton and goat", "Other bakery products", "Confectionery, chocolate and ice cream", "Beer", "Other meats and meat preparations", "Beef and veal" and "Poultry". Following Clements and Si (2017), we also group them into 6 broad groups, namely "Staples", "Meat and seafood", "Fruits and vegetables", "Alcohol", "Sweet things" and "Other food".

Internal consistency when computing relative prices is maintained by using, as a benchmark, a food price index derived solely from these 14 price series. The starting point is the data matrix $\mathbf{K} = [\bar{\mathbf{k}}_1, \bar{\mathbf{k}}_2, \ldots, \bar{\mathbf{k}}_{14}]$ where $\bar{\mathbf{k}}_i$ denotes the vector of average bilateral differences in the prices of i (see Equation (14)). We standardize each column of $\mathbf{K}$ and obtain $\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, \ldots \mathbf{x}_{14}]$. We can think of this matrix as containing 14 variables (corresponding to the 14 food items), each has 155 observations. The basic idea of PCA is to find a way to describe these 14 variables with a smaller number of new variables, called "principal components", that preserves most of the original information.

Similar to the 2 variables case, we derive the 14-element vector $\mathbf{a}_1$ and the corresponding value λ_1 so that they satisfy the characteristic equation: $(\mathbf{X}'\mathbf{X} - \lambda_1\mathbf{I})\mathbf{a}_1 = \mathbf{0}$ where $\mathbf{I}$ is an identity matrix and $\mathbf{0}$ is a 14-element zero vector. That is, $\mathbf{a}_1$ is a characteristic vector of the 14×14

[19] Another constraint is that the two eigenvectors are orthogonal, i.e. $\mathbf{a}_1'\mathbf{a}_2 = 0$. Let $\mathbf{A} = [\mathbf{a}_1, \mathbf{a}_2]$. The eigenvectors satisfy $\mathbf{A}'\mathbf{A} = \mathbf{A}\mathbf{A}' = \mathbf{I}$ (Campbell and Atchley, 1981).

[20] Equivalently, we can say that λ_1 is the largest root of $\det(\Sigma - \lambda\mathbf{I}) = 0$ where det(.) means determinant.

positive definite matrix $\mathbf{X}'\mathbf{X}$, that corresponds to the largest root λ_1.[21] From here, the first principal component is constructed as the linear combination of the original variables with the weights (or loadings) given by $\mathbf{a}_1$: $\mathbf{pc}_1 = \mathbf{X}\mathbf{a}_1$. In columns 2 to 5 of Table A6.6, we present the entries of the first three eigenvectors ($\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$), the variance of the corresponding principal components (hereafter PCs), and the contributions of them in the total variance. It can be seen that the first two principal components account for 30% and 18% of the total relative price variation, respectively. In total the first 3 PCs account for about 60% of data variation. Figure A6.6 shows the percentage contributions of all principal components.

How many components should we use to efficiently represent our data? In other words, what is the optimal number of dimensions should we "compress" our 14–dimension data into? A rule of thumb for data compression is to select the PCs that can explain at least 100/14 = 7.14% of the total variance which is the proportion that a standardized original variable explains.[22] In Figure A6.6, the blue bars represent the percentage of price variance explained for each of the first 10 PCs. This is known as a "Scree plot", a term coined by Cattell (1966). Based on the above rule and the PCA result, the first 4 PCs can be used to sufficiently describe the price data.

We continue the above analysis by replacing relative price with consumption. The starting point here is the matrix $\mathbf{Q}$ from Section A5. The PCA is then performed on the matrix $\mathbf{Y} = [\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_{14}]$ where $\mathbf{y}_i$ is the standardized $\overline{\mathbf{q}}_i$. We present the results in another Scree plot, which is shown by the red bars of Figure A6.6. The accompanied loadings are provided in columns (6) to (9) of Table A6.6. In this case, the first PC alone accounts for 54% of the total data variation, and we can use just the first 2 PCs to represent our data. It seems that while the cross-country pricing is too heterogeneous for us to detect a common driving factor, the cross-country consumption behaviour exhibits a stronger common trend.

A6.3. Eigenvalues and correlations

Given the above discussion, it is natural to ask: How do we interpret the explanatory power of the first PC? As mentioned, this consumption is also measured by the first eigenvalue of the correlation matrix. It indicates the maximum amount of variance of the variables which

[21] As shown in Table A6.8, in stead of the data matrix $\mathbf{X}'\mathbf{X}$, we can perform PCA on the covariance matrix, denoted as $\Sigma = \mathbf{X}'\mathbf{X}/(m-1)$ where $m = 155$. Since we standardized the variables, it is also the correlation matrix. Denote the eigenvalues of this matrix as γ_i $(i = 1 \ldots, 14)$. It can be shown that $\gamma_i = \lambda_i/(m-1)$ and component contributions to total variance from the two approaches are the same: $\lambda_i/\Sigma_i\lambda_i = \gamma_i/\Sigma_i\gamma_i = \gamma_i/14$. That is, PCA is a variance preservation transformation.

[22] Since the sum of the eigenvalues of the correlation matrix equals the number of original variables (14), this is equivalent to choosing PCs that correspond to eigenvalues (of a correlation matrix) that are greater than 1. This rule is also known as the "Kaiser-Guttman rule" (Kaiser, 1960 and Guttman, 1954).

Table A6.6. Loadings On the First Three Principal Dimensions

Basic heading	A. Relative price				B. Consumption			
	PC1	PC2	PC3	Other PCs	PC1	PC2	PC3	Other PCs
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1. Fresh or chilled vegetables other than potatoes	0.02	0.51	0.25		0.26	0.08	0.41	
2. Rice	0	-0.47	-0.11		-0.11	0.65	0.13	
3. Fresh, chilled or frozen fish and seafood	0.13	-0.27	0.02		0.14	0.59	-0.19	
4. Food products nec	-0.42	-0.15	-0.08		0.26	0.23	-0.31	
5. Fresh or chilled fruit	-0.04	0.3	0.43		0.3	0.01	0.24	
6. Mineral waters, soft drinks, fruit and juices	-0.42	0.1	0		0.33	0.02	-0.15	
7. Bread	0	0.14	0.36		0.3	-0.08	0.1	
8. Pork + Lamb, mutton and goat	0.16	-0.21	0.48		0.24	0.06	0.16	
9. Other bakery products	-0.38	-0.01	0.17		0.34	0	-0.07	
10. Confectionery, chocolate and ice cream	-0.34	0.11	-0.18		0.32	-0.03	0.01	
11. Beer	-0.3	0.14	0.07		0.23	-0.28	-0.46	
12. Other meats and meat preparations	-0.34	-0.27	0.18		0.31	-0.15	-0.16	
13. Beef and veal	0.1	-0.35	0.49		0.18	-0.16	0.56	
14. Poultry	-0.36	-0.18	0.17		0.3	0.2	0.01	
Eigenvalues/Component variance (λ_i)	639	383	258		1,166	242	148	
% of explained variance ($\lambda_i/\Sigma_{i=1}^{14}\lambda_i$)	29.8	18	12	40	54	11	7	28
Cumulative % of explained variance	29.8	47.8	60	100	54	66	73	100

Notes: Relative price of item i in country c is defined as $k_{i,c} = (1/155)\Sigma_{d=1}^{155} k_{c,d}$ where $k_{c,d}$ is the logarithmic difference between the Dollar prices of i in c and d. The corresponding relative consumption is $q_{i,c} = (1/155)\Sigma_{d=1}^{155} q_{c,d}$ where $q_{c,d}$ is the logarithmic difference between consumption of i in c and d. Columns 2 to 4 of this Table present the values of the first 3 loading vectors (or eigenvectors, of the data matrix $\mathbf{X}'\mathbf{X}$) for relative prices. Columns 5 to 7 show the loading vectors for consumption. These loadings are coefficients of a linear combination of the original variables to construct new variables (principal components). The last three rows show the corresponding eigenvalues, the contributions of the PCs to total data variation, and the cumulated PC variance contributions.

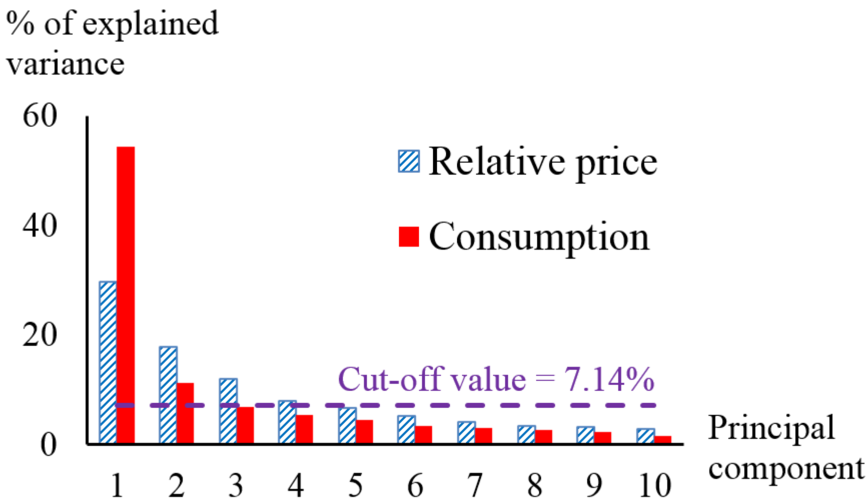

Figure A6.6. Contribution of the First 10 Principal Components to Data Variation

Notes: The cut-off value is based on the Kaiser-Guttman rule (Kaiser, 1960; Guttman, 1954) that a component (extracted from PCA on standardized data) is considered important when it explains at least 1/n of the total data variation (where n is the number of original variables).

can be accounted for with a linear model by a single component. More importantly, according to Friedman and Weisberg (1981), the eigenvalues can be approximated by linear functions of n(n − 1) (off-diagonal) correlation coefficients of the n variables, even when the correct specification is non-linear. Understanding the relationship between the first eigenvalue and the correlations is therefore crucial in relating the underlying data dynamics to the computed percentage of variance explained. Morrison (1967) shows that a linear relationship between the first eigenvalue of the correlation matrix (denoted as γ_1) and the average correlation (denoted as $\bar{\rho}$) would be:[23]

$$\gamma_1 \approx 1 + (n-1)\bar{\rho} \text{ where } \bar{\rho} = \frac{1}{n^2 - n} \sum_{j=1}^{n} \sum_{i=1}^{n} \rho_{i,j} \; (i \neq j). \tag{17}$$

The proportion of variance accounted for by the first PC is $\gamma_1/n = 1/n + (n-1)\bar{\rho}/n$. When n is large, $1/n$ approaches zero and $n/(n-1)$ approaches one, so this proportion approaches $\bar{\rho}$, which is can also be expressed by any function of the central tendency of underlying off-diagonal correlation coefficients.

Let's see how well this approximation rule help explains our data. In the lower (upper) triangle of the matrix in Table A6.7 we present, in the upper triangle, the correlation matrix of the 14 relative price and consumption series. Since each standalone matrix is symmetric, the mean of the off-diagonal elements is equal to the mean of the upper (lower) triangle's elements. The mean correlation is 0.232 for price and 0.48 for consumption. In the following computations, we show only results for prices. The first eigenvalue of the price correlation matrix is 4.18, while that estimated by approximation rule is $1 + (14-1) \times 0.232 = 4.02$. This would yield an underestimation for the actual eigenvalue of $(4.18 - 4.02)/4.18 = 4\%$.[24] The proportion of data variation accounted for by the first principal component can be computed as:[25]

$$\gamma_1/14 = \gamma_1/\left(\sum_{i=1}^{14} \gamma_i\right) = 1/14 + (14-1)\bar{\rho}/14 = 28.7\%. \tag{18}$$

In this example, the actual proportion is 29.8% (Table A6.6). Thus, our approximation of 28.7% likewise underestimates the true value by 4%.

[23] This estimate is exact in the case of all positive correlations. It performs well when there are only a few, small, non-systemic negative correlations. In a general case where there can be negative correlation, an increase in $\bar{\rho}$ will not necessarily lead to an increase in γ_1 We can use the general approximation, given as: $\gamma_1 \approx 1 + \max_m \left[(m-1)\bar{\rho}_{max}\right]$ where $\bar{\rho}_{max}$ is the maximum value of $\bar{\rho}$ among all possible sub-matrices of size m and all possible reversals of the variables with negative correlations. Since the first eigenvalue exhibits the strength of co-movement among variables, its value only depends on the magnitudes of the correlation coefficients, not their signs. Therefore, in (17), using $\bar{\rho}$ as the average of the absolute correlation, gives a reasonable approximation.

[24] The corresponding measures for the first eigenvalue of consumption correlation matrix are 7.62 and 7.25. The resulting underestimation is thus $(7.62 - 7.25)/14 = 2.6\%$.

[25] When $n \to \infty$, we have $1/n \to 0$ and $(n-1)/n \to 1$, so that $\gamma_1/n \to \bar{\rho}$. By definition, we also have $0 \leq \gamma_1/n \leq 1$ and $0 \leq \bar{\rho} \leq 1$. In words, when all items are perfectly correlated (uncorrelated), the first PC captures 100% (0%) of data variation.

Next, the sum of the elements in the correlation/covariance matrix is:

$$\Sigma_{j=1}^{14}\Sigma_{i=1}^{14}|\rho_{i,j}| = 14 + \Sigma_{j=1}^{14}\Sigma_{i=1}^{14}|\rho_{i,j}| \ (i \neq j). \tag{19}$$

Recall from (17) that $\bar{\rho} = \dfrac{1}{14^2 - 14} \sum_{j=1}^{14} \sum_{i=1}^{14} \rho_{i,j} \ (i \neq j) \approx \dfrac{\gamma_1 - 1}{14 - 1}$. Putting this expression into (19) yields:

$$\Sigma_{j=1}^{14}\Sigma_{i=1}^{14}|\rho_{i,j}| = 14 + (14^2 - 14)\,\bar{\rho} \approx 14 + (14^2 - 14)\left(\frac{\gamma_1 - 1}{14 - 1}\right) = 14\gamma_1 = 14^2\left(\frac{\gamma_1}{14}\right). \tag{20}$$

Because of standardization, each variable's contribution to total variation is equal to one. Therefore, the incremental impact of individual variable is made solely through an increase of the absolute covariance. So long as a new variable has at least one non-zero correlation with old variables, the total data variation will increase. Applying (20) gives an estimate of total data variation of $14\gamma_1 = 14 \times 4.02 = 56.3$.

The above expositions are summarized in Table A6.8. This exercise has an important implication: The proportion of explained variance of the first PC is positively associated with the mean of absolute correlation of the variables. Since the estimate of the consumption eigenvalue is greater than that of the price's (Table A6.6), the former variable has a more homogeneous behaviour and it is easier to capture its dynamic with just the first PC.

A6.4. Notes on the decomposition of data variation

Consider again the relative price matrix **K**:

$$\mathbf{K} = \begin{bmatrix} \bar{k}_{1,1} & \bar{k}_{1,2} & \cdots & \bar{k}_{1,125} \\ \bar{k}_{2,1} & \bar{k}_{2,2} & \cdots & \bar{k}_{2,125} \\ \vdots & \vdots & \ddots & \vdots \\ \bar{k}_{155,1} & \bar{k}_{155,2} & \cdots & \bar{k}_{155,125} \end{bmatrix} \text{ where } \bar{k}_{c,i} = \frac{1}{155}\sum_{d=1}^{155}\left[\log\left(p_{i,c}/S_c\right) - \log\left(p_{i,d}/S_d\right)\right].$$

The mean and variance over all items and countries (referred to as "grand" mean and variance) are defined as:

$$\mu_k = \frac{1}{C \times I}\Sigma_{c=1}^{C}\Sigma_{i=1}^{I}\bar{k}_{c,i};\ \sigma_k^2 = \frac{1}{C \times I}\Sigma_{c=1}^{C}\Sigma_{i=1}^{I}\left(\bar{k}_{c,i} - \mu_k\right)^2 \ (C = 155, I = 125).$$

We can rewrite the grand mean as the mean of the column averages:

$$\mu_k = \frac{1}{I}\Sigma_{i=1}^{I}\left(\frac{1}{C}\Sigma_{c=1}^{C}\bar{k}_{c,1} + \cdots + \frac{1}{C}\Sigma_{c=1}^{C}\bar{k}_{c,I}\right).$$

Since each of these averages equal zero, we have $\mu_k = 0$. As a result, the grand variance equals the mean of the column variances, which are simply the average sums of squared column elements: $\sigma_k^2 = \frac{1}{I}\Sigma_{i=1}^{I}\left(\frac{1}{C}\Sigma_{c=1}^{C}\bar{k}_{c,1}^2 + \cdots + \frac{1}{C}\Sigma_{c=1}^{C}\bar{k}_{c,I}^2\right)$. Recall that the i^{th} column of **K** is the row averages of the price comparison matrix $\mathbf{K}_i$. Clements et al. (2012) show that the variance the row averages of K_i and that of each row and column of $\mathbf{K}_i$ all takes the same value, denoted

Table A6.7. Correlation Matrix of Relative Price and Consumption

	Vegetables	Rice	Seafoods	Food products nec	Fruits	Mineral waters and soft drinks	Bread	Pork and Lamb	Bakery products	Chocolate and ice cream	Beer	Other meats	Beef and veal	Poultry
Vegetables	1	-0.16	0.24**	0.45***	0.77***	0.55***	0.56***	0.49***	0.64***	0.61***	0.25**	0.50***	0.33***	0.59***
Rice	-0.50***	1	0.32***	-0.04	-0.27***	-0.26**	-0.32***	-0.16*	-0.29***	-0.31***	-0.42***	-0.39***	-0.14	-0.04
Seafoods	-0.29***	0.37***	1	0.42***	0.33***	0.34***	0.25**	0.31***	0.37***	0.31***	0.1	0.25**	0	0.42***
Food products nec	-0.20*	0.19*	-0.17*	1	0.48***	0.71***	0.49***	0.43***	0.65***	0.60***	0.47***	0.51***	0.21**	0.66***
Fruits	0.60***	-0.18*	0.01	-0.12	1	0.71***	0.64***	0.49***	0.77***	0.72***	0.43***	0.65***	0.41***	0.66***
Min. waters, drinks	0.11	-0.13	-0.20*	0.72***	0.1	1	0.69***	0.54***	0.83***	0.79***	0.61***	0.76***	0.39***	0.74***
Bread	0.18*	-0.27***	-0.05	-0.07	0.11	0.02	1	0.48***	0.72***	0.69***	0.47***	0.72***	0.45***	0.68***
Pork and Lamb	-0.05	0.04	0.21*	-0.24**	0.03	-0.21**	0.06	1	0.53***	0.56***	0.34***	0.57***	0.36***	0.49***
Bakery products	-0.01	-0.04	-0.09	0.61***	0.14	0.63***	0.18*	-0.16*	1	0.86***	0.58***	0.82***	0.40***	0.74***
Chocolate and ice cream	0.04	-0.12	-0.14	0.52***	0.03	0.58***	-0.11	-0.30***	0.47***	1	0.48***	0.76***	0.40***	0.64***
Beer	0.12	-0.06	-0.21**	0.33***	0.27***	0.53***	-0.06	-0.18*	0.39***	0.42***	1	0.68***	0.29***	0.45***
Other meats	-0.26**	0.21**	-0.1	0.65***	-0.02	0.48***	-0.03	-0.01	0.52***	0.27***	0.34***	1	0.38***	0.66***
Beef and veal	-0.22**	0.28***	0.14	-0.09	0.03	-0.29***	0.09	0.57***	-0.02	-0.29***	-0.16	0.24**	1	0.40***
Poultry	-0.19*	0.13	-0.09	0.61***	0.07	0.50***	0.05	-0.06	0.53***	0.36***	0.34***	0.66***	0.08	1

Notes: This Table presents the coefficients of the correlations among the 14 relative price series (lower triangle) and that of corresponding consumption series (upper triangle). Each series covers 154 countries. Significance level: *: p < 0.1, **: p < 0.05, ***: p < 0.01.

Table A6.8. First Eigenvalue and Data Variation

	Exposition
Data matrix:	$\mathbf{X} = \begin{array}{c} \\ \text{Benin} \\ \text{Botswana} \\ \vdots \\ \text{Georgia} \end{array} \begin{array}{c} \begin{array}{cccc} \text{Vegetables} & \text{Rice} & \dots & \text{Poultry} \end{array} \\ \begin{pmatrix} x_{1,1} & x_{1,2} & \dots & x_{1,14} \\ x_{2,1} & x_{2,2} & \dots & x_{2,14} \\ \vdots & & \vdots & \\ x_{154,1} & x_{154,2} & \dots & x_{154,14} \end{pmatrix} \end{array} = \left[x_{c,i}\right]_{[154\times 14]}$
Dispersion matrix:	$\mathbf{XX}' = \begin{bmatrix} x_{1,1} & x_{1,2} & \dots & x_{1,14} \\ x_{2,1} & x_{2,2} & \dots & x_{2,14} \\ \vdots & \vdots & & \vdots \\ x_{154,1} & x_{154,2} & \dots & x_{154,14} \end{bmatrix}_{[154\times 14]} \begin{bmatrix} x_{1,1} & x_{2,1} & \dots & x_{154,1} \\ x_{1,2} & x_{2,2} & \dots & x_{154,2} \\ \vdots & \vdots & & \vdots \\ x_{1,14} & x_{2,14} & \dots & x_{154,14} \end{bmatrix}_{[14\times 154]} = \begin{bmatrix} \Sigma_{i=1}^{14} x_{1,i}^2 & \Sigma_{i=1}^{14}(x_{1,i}x_{2,i}) & \dots & \Sigma_{i=1}^{14}(x_{1,i}x_{154,i}) \\ \Sigma_{i=1}^{14}(x_{2,i}x_{1,i}) & \Sigma_{i=1}^{14} x_{2,i}^2 & \dots & \Sigma_{i=1}^{14}(x_{2,i}x_{154,i}) \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_{i=1}^{14}(x_{154,i}x_{1,i}) & \Sigma_{i=1}^{14}(x_{154,i}x_{2,i}) & \dots & \Sigma_{i=1}^{14} x_{154,i}^2 \end{bmatrix}_{[154\times 154]}$ *Property 1*: Sum of squared elements of **X**: $\Sigma_{c=1}^{154}\Sigma_{i=1}^{14} x_{c,i}^2 = \mathrm{tr}(\mathbf{XX}') = \mathrm{tr}(\mathbf{X}'\mathbf{X}) = 2,142$ where tr(.) denotes the trace operator.*
Variance-covariance / Correlation matrix:	$\mathbf{R} = \frac{1}{154-1}(\mathbf{X}'\mathbf{X}) = \left[\sigma_{j,l}\right]$ $(j, l = 1, \dots, 14)$ (See the lower triangle of Table A4.6).
Sum of (absolute) data variation:	*Property 2*: $\Sigma_{l=1}^{14}\Sigma_{j=1}^{14}\lvert\sigma_{j,l}\rvert = 14 + \Sigma_{l=1}^{14}\Sigma_{j=1}^{14}\lvert\sigma_{j,l}\rvert\ (j \neq l) = 14 + (14^2 - 14)\overline{\rho} = 56.2$, where $\overline{\rho} = \frac{1}{14^2-14}\Sigma_{l=1}^{14}\Sigma_{l=1}^{14}\lvert\sigma_{j,l}\rvert\ (j \neq l) = 0.232$ is the average of (absolute) off-diagonal elements of **R**.
Eigenvalues of correlation matrix:	Sum of eigenvalues: $\Sigma_{j=1}^{14}\gamma_j = \mathrm{trace}(\mathbf{R}) = \mathrm{trace}(\mathbf{XX}')/(154-1) = (2,142)/153 = 14$. *Property 3*: Approximation of the largest eigenvalue: $\widetilde{\gamma}_1 \approx 1 + (14-1)\overline{\rho} = 4.02$, equivalently, $\overline{\rho} = \left(\widetilde{\gamma}_1 - 1\right)/(14-1)$. Actual value is $\gamma_1 = \mathbf{4.18}$.
Combining properties 2 and 3:	$\Sigma_{j=1}^{14}\Sigma_{i=1}^{14}\lvert\sigma_{i,j}\rvert = 14 + \left[(14^2-14)\left(\widetilde{\gamma}_1 - 1\right)\right]/(14-1) = 14\widetilde{\gamma}_1$. Indeed: $56.2 = 14 \times 4.02$. *Property 4*: The proportion of total variance explained by first principal component: $\gamma_1/14 = 28.7\% \approx \left(\Sigma_{j=1}^{14}\Sigma_{i=1}^{14}\lvert\sigma_{i,j}\rvert\right)/14^2 = 29.8\%$.

Notes: This Table derives useful properties involving the relation ship between the proportion of total variance explained by the first eigenvalue (of the correlation matrix) and the average pair-wise correlation. Data used for illustration are standardized relative prices (defied as $\overline{k}$ in Sections A3) of 14 food headings. These headings are listed in Table A6.6. The US is excluded in this sample.

* This result also holds for non-standardized matrix.

as σ_i^2, which is also equal to the variance of the underlying price vector $\widetilde{\mathbf{p}}_i$. Therefore, a more convenient way to express the grand variance is as the mean of the individual items' price variances:

$$\sigma_k^2 = \frac{1}{I}\Sigma_{i=1}^{I}\sigma_i^2. \tag{21}$$

Now consider the commodity-based variance-covariance matrix of $\mathbf{K}$. The sum of $\mathbf{V}$'s elements can be viewed as the total data variation and can be decomposed into two components:

$$\begin{aligned}\Sigma_{c=1}^{C}\Sigma_{i=1}^{I}\sigma_{i,j}^2 &= \underbrace{\Sigma_{i=1}^{I}\sigma_i^2}_{\text{Sum of diagonal elements/trace of } \mathbf{V}} + \underbrace{\Sigma_{c=1}^{C}\Sigma_{i=1}^{I}\sigma_{i,j}^2 \ (i \neq j)}_{\text{Sum of off-diagonal elements of } \mathbf{V}} \\ &= I\,\sigma_k^2 + \Sigma_{c=1}^{C}\Sigma_{i=1}^{I}\sigma_{i,j}^2 \ (i \neq j).\end{aligned} \tag{22}$$

The second line of the above follows from (21). We can see that the first component is related to the degree of individual items' price dispersion: It is approximately twice the sum of the squared tradability indices derived in Section 3 of the main text. The second component, on the other hand, captures the patterns of price co-variability and can be examined via the PCA described in this section. A similar decomposition can be applied to the total variation of consumption.

A7. Visualizing PCA results

In Section A6 we show that the first principal component is constructed as the linear combination of the original variables $\mathbf{x}_i$ with the weights (or loadings) given by $\mathbf{a}_1$: $\mathbf{pc}_1 = \mathbf{X}\mathbf{a}_1$.[26] We can see that $\mathbf{pc}_1$ is a projection of the original data points on the direction of a latent variable. By construction, λ_1 is the largest characteristic root and this projection therefore retains the most variation of the original data points. This also means $\mathbf{pc}_1$ would have the same number of observations (155) as our original variables. In a similar manner, if we redo our decomposition with the residual matrix $(\mathbf{X} - \mathbf{pc}_1\mathbf{a}_1')'(\mathbf{X} - \mathbf{pc}_1\mathbf{a}_1')$, instead of $\mathbf{X}'\mathbf{X}$, we shall get a second component which will further reduce the residual variation by an amount of λ_2, and so on. Continuing with this decomposition gives us a total of 14 vectors p_i and 14 values of λ_i. An important property of λ_i is that it coincides with the variance of the i-th principal component ($\mathbf{pc}_i$). Therefore, the ratio of λ_i and the total variance ($\mathrm{tr}(\mathbf{X}'\mathbf{X})$) can be interpreted as the proportion of variation accounted for by the i-th principal component. Here we further

[26] Note that in Theil (1971)'s formulation, the relationship between $\mathbf{p}_1$ and $\mathbf{a}_1$ can be interchangeably expressed via two equations: $\mathbf{pc}_1 = (1/\lambda_1)\mathbf{X}\mathbf{a}_1$ or $\mathbf{a}_1 = \mathbf{X}'\mathbf{pc}_1$. Here we adopt the notations $\mathbf{pc}_1 = \mathbf{X}\mathbf{a}_1$ and $\mathbf{a}_1 = (1/\lambda_1)\mathbf{X}'\mathbf{pc}_1$ for a more straightforward interpretation of the eigenvector/loading vector $\mathbf{a}_1$. Note that this leaves the product of the two vectors unchanged. It can then be shown that the first principal component reduces the original sum of squares by exactly λ_1, that is: $\mathrm{trace}[(\mathbf{X} - \mathbf{pc}_1\mathbf{a}_1')'(\mathbf{X} - \mathbf{pc}_1\mathbf{a}_1')] = \mathrm{trace}(\mathbf{X}'\mathbf{X}) - \lambda_1$.

extend the discussions above and introduce a number of graphical tools to examine the output of our PCA. In particular, the workings of the "factor map", and how it helps to visualize the relative positions of the original variables/items and observations/countries with respect to the first two principal components is examined.

A7.1. PCA for relative prices

Firstly, we would be interested in visualizing the contributions of original variables to the variation of our first two PCs from the PCA on relative prices. One way to do this is simply looking at the correlation between the first two PCs and these variables: $\rho_{i,1} = \mathrm{Corr}(\mathbf{x}_i, \mathbf{pc}_1)$ and $\rho_{i,2} = \mathrm{Corr}(\mathbf{x}_i, \mathbf{pc}_2)$. In Figure A7.7 we show an arrow map. Each arrow represents one of the 14 variables/items, and the coordinates of the arrow tips are ρ_{i1} and ρ_{i2}, on the horizontal and vertical axes, respectively. Items of the same arrow colour belong to the same broad food group. The radius of the correlation circle is one: If variable i can be perfectly represented by PC1 and PC2, its arrow tip will be positioned exactly on the circle.[27] In this scenario we have: $\rho^2_{i,j} = 0\ (\forall j > 2)$, which means all other PCs have no power in explaining the variation of variable i. However, in our case there is no such ideal situation. The variables that exhibit the strongest association with the first two PCs are "Mineral waters, soft drinks, fruit and vegetable juices", "Food products nec", and "Other bakery products", all have absolute correlation coefficients greater than 0.75. "Bread", "Rice" and "Vegetables" exhibit almost no correlation with the first PC. Along the second dimension, "Rice", "Beef and veal" and "Fresh, chilled or frozen fish and seafood" show the strongest association. Additionally, the closeness of the arrows indicates similar pricing behaviour of corresponding items. These arrows' direction also indicates the pair-wise correlation of the items, and indirectly implies item characteristic (dis)similarity. As can be seen in Figure A7.7, some of the items in the same broad food group exhibit markedly different behaviour. For example, while "Pork and lamb", "Seafood" and "Beef and veal" are quite similar in that they vary mostly with PC1, "Poultry" and "Other meats" seem to belong in a different meat subgroup that exhibits variation mostly along PC2. In the group "Staple", "Rice" shows strong negative correlation with PC2, while "Other bakery products" is strongly negatively correlated with PC1. This exercise points to the fact that grouping items based on broad definition might not be ideal, as their cross-country dynamics can be very different.

Alternatively, we can summarize this two-dimensional information by a "representation quality" measurement for each variable. This measure can be computed as: $\cos^2_i = \rho^2_{i,1} + \rho^2_{i,2}$,

[27] By construction, for each variable i, the sum of squared correlations equals one: $\Sigma_{j=1}^{14} \rho^2_{i,j} = 1\ (i = 1, \dots, 14)$.

which is the squared length of the arrow. $\cos_i^2 = 1$ indicates that the variable can be perfectly represented by a combination of PC1 and PC2. From Figure A7.8 we can see that "Food products nec", "Mineral waters, soft drinks, fruit and vegetable juices" and "Other meats and meat preparations" are the items that seem to be the most appropriately represented by the combination of the first two PCs. In most cases, if an item is well represented by one dimension, it will not be by the other. Additionally, for some other variables, more than two components might be required to perfectly represent the data. These are items that have very low value of $\cos_i^2$. In these cases the arrows are closer to the origin. The shortest arrows coincide with "Pork and Lamb" ($\rho_{i,1} = 0.32, \rho_{i,2} = -0.33, \cos_i^2 = 0.22$) and "Bread" ($\rho_{i,1} = 0.009, \rho_{i,2} = 0.22, \cos_i^2 = 0.05$).

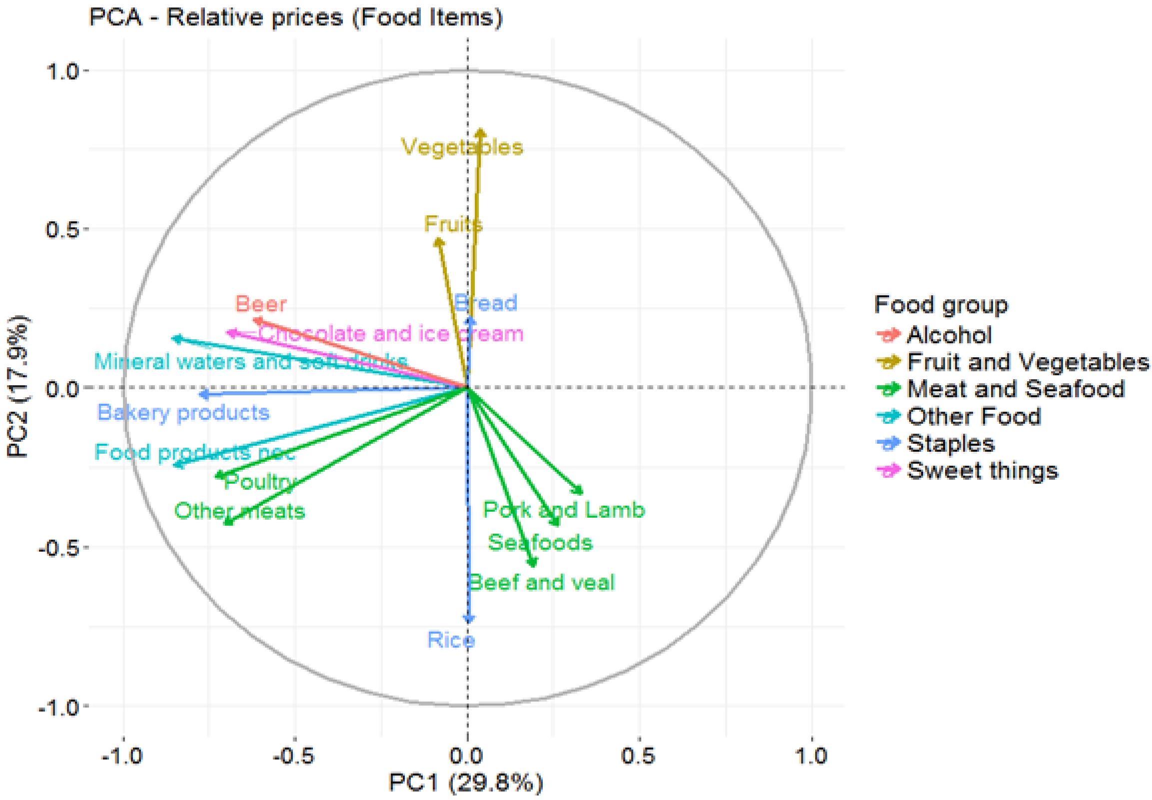

Figure A7.7. Correlation Circle of Relative Price, 14 Items, 154 Countries

Notes: This Figure visualizes the correlation between each of the 14 original variables (14 food prices) and the first two principal components. The coordinates of the arrow on the two axes are the corresponding correlations. The length of the arrows represents the total explanatory power of the PCs for said variables. The percentages in the axes' titles indicate the proportion of variation explained by these PCs.

The second element of a "bi-plot" is the representation of the observations (here, countries) on the factor map, as illustrated in Figure A7.9. Here, the coordinates of each country are the corresponding "scores" of that country on the first two principal directions. In other words, they

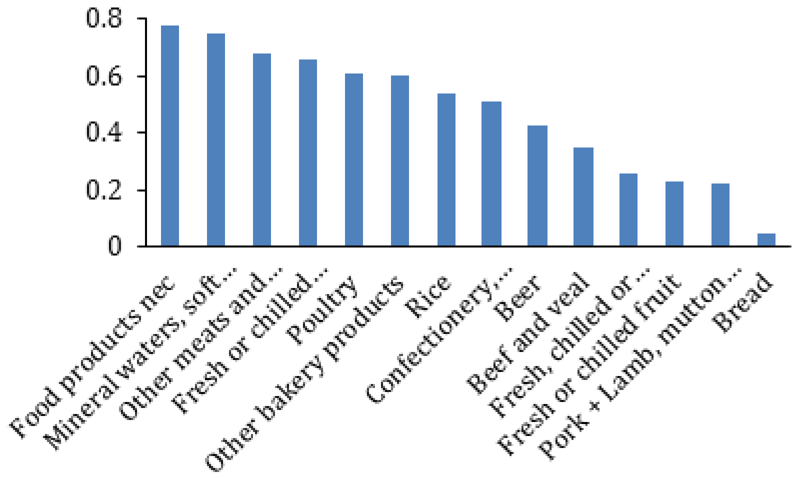

Figure A7.8. Quality of Relative Price Representation, 14 Items, 154 countries

Notes: This Figure presents the quality of PC1 and PC2 in representing each original variable, constructed as $\cos_i^2 = \rho_{i,1}^2 + \rho_{i,2}^2$ where $\rho_{i,1}$ and $\rho_{i,1}$ are the correlations between the corresponding variable and PC1 and PC2, respectively. This measure of quality shows how well a variable can be represented by a combination of PC1 and PC2.

are the realizations of the two new variables: $\mathbf{pc}_1 = \mathbf{Xa}_1$ and $\mathbf{pc}_2 = \mathbf{Xa}_2$. For each country, the new "score" is a linear combination of its original prices for all items. Let's take the country with the highest income, "Bermuda", as an example. The new score of Bermuda along the first principal direction is: $\mathrm{pc}_{1,1} = \Sigma_{i=1}^{14} \mathrm{x}_{i,1} \times \mathrm{a}_{i,1}$. Higher scores imply better representation by the first PC. Along the x-axis of Figure A7.9, we can see that the countries are generally arranged (from left to right) in an order of increasing income. This is expected, since the first PC is shown to be strongly and positively correlated with income. The countries are colour-coded by their income groups (or quartiles). The plot of 154 countries is perhaps too cluttered for identification, however. Cross-country behaviour may be heavily affected by determinants such as geographical and/or cultural "closeness". We examine this possibility by restricting the sample to only some of the highly developed countries, the OECD group. To be specific, these are the countries that are designated as "Eurostat-OECD" in the ICP data. We group these countries into 10 geographical areas, as indicated in Figure A7.10. Compared with the full sample analysis, here the explanatory power of PC1 and PC2 declines slightly – to 28% and 16%, respectively. The points that lie close to each other, such as those of Luxembourg, France and Netherlands, or Austria and Germany, or Australia and New Zealand, can be identified as very similar in terms of their pricing pattern. That is, if an item is relative more expensive in one country, it is likely to be just as expensive in countries that are located close to it in the factor map. In this regard, East Asian countries (Japan and South Korea) are possible outliers of the OECD. The similarity of pricing seems to be driven mostly by geographical factors.

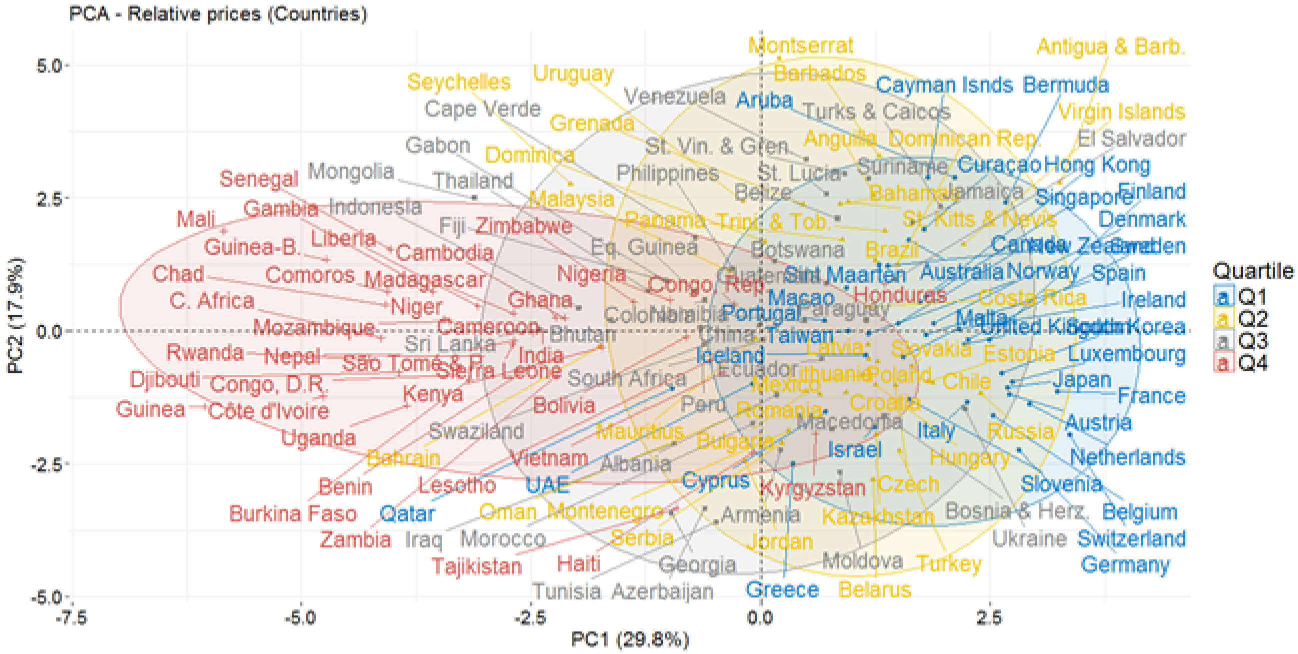

Figure A7.9. Factor Scores for 14 Items, 154 Countries (Relative Price)

Notes: This figure presents the "scores" (positions) of all countries along the first and second principal directions. Countries are grouped and colour-coded by income quartiles (with Q1 is the richest). Concentration ellipses are added to further distinguish country by income clusters. The percentages in the axes' titles indicate the proportion of variation explained by these PCs.

A7.2. PCA for consumption

We have shown that the bi-plots are powerful tools that offer interesting insights about the pattern of global food relative prices. Along the same line, we can explore similar characteristics in terms of consumption behaviour. The underlying variable of this analysis is log q_{ic}. For the consumption/real per capita consumption, the correlations of PCs and the 14 variables are now markedly changed: In Figure A7.11 we can see the majority of the cross-country consumptions exhibit strong correlation with PC1, while "Rice" and "Fresh, chilled or frozen fish and seafood" tilt toward the second PC. Overall these variables are much better represented by the two PCs, as evidence in the arrows' length. In Figure A7.12, similar to the factor map of PCA on relative prices, here the countries arrange along the first principal direction quite well, especially for the poorest economies. The correlations of the first PC with relative income and food budget share are 0.93 and -0.72, respectively. Examining the OECD sample reveals that compared with the full sample analysis, the consumption behaviour among these countries is more diverse, resulting in a large drop of PC1 and PC2's explanatory power (from 54% to 26% and from 18%

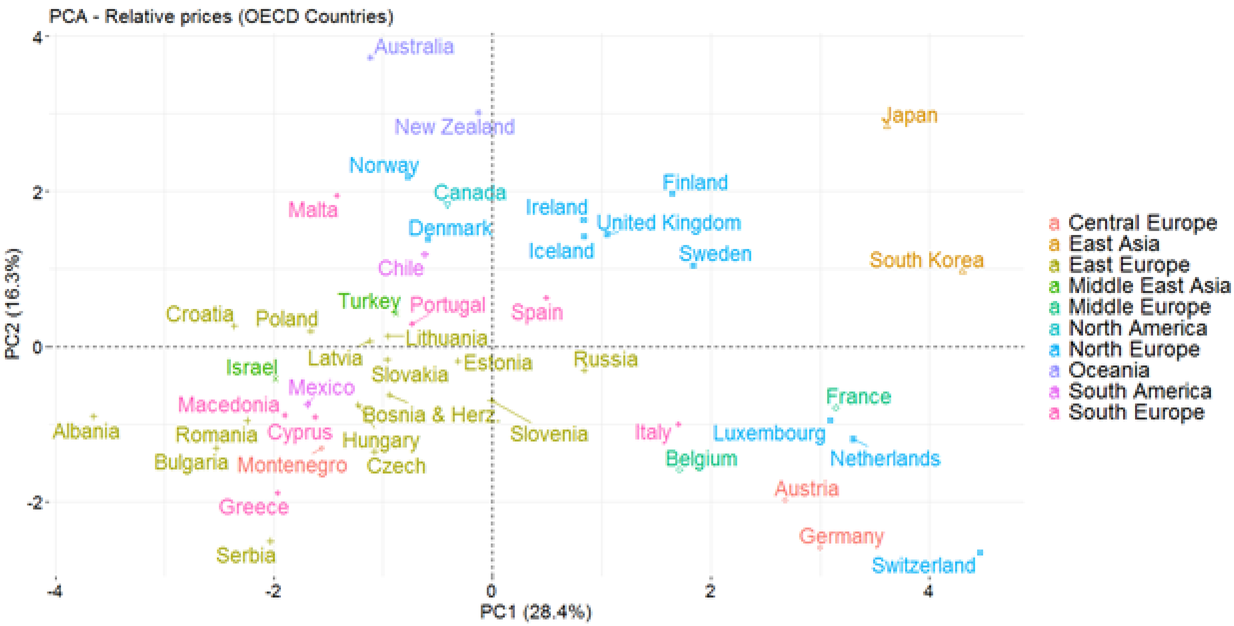

Figure A7.10. Factor Scores for 14 Items, 46 OECD-Eurostat Countries (Relative Price)

Notes: This figure presents the "scores" (positions) of all countries designated as "OECD-Eurostat" along the first and second principal directions. Countries are grouped and color-coded by geographical areas. The percentages in the axes' titles indicate the proportion of variation explained by these PCs.

to 15.6%, respectively). Nevertheless, compared with the result of relative prices (as in Figure A7.9), the countries' scores now form more visible clusters, implying greater similarity among the grouped countries. Japan and South Korea still exhibit features of outliers, together with Albania and Romania.

A7.3. PCA for relative consumption

In Section 5 we show that the explanatory power of income to PC1 of consumption is much higher than the corresponding impact to PC1 of prices. If the common factor driving the cross country dispersion of consumption is essentially income, then a similar analysis on the *difference* between consumption and income should yield lower adjusted R^2. We construct two new measures of "relative consumption" as consumption deflated by (i): Income and (ii): Weighed average consumption. The corresponding measures are $\log q_{ic} - \log (Y_c/\overline{Y})$ and $\log q_{ic} - \Sigma_{i=1}^{n} w_{ic} \log q_{ic} = \log q_{ic} - \log Q_c$ where w_{ic} denotes the expenditure share of i and n is the number of all items at each level. It can be seen that the second measure is analogous to the relative price variable used. Table A7.9 presents the results. As can be seen, the outputs of PCA for the two measures are almost identical, and the magnitudes of correlations between PC1 and income are significantly reduced.

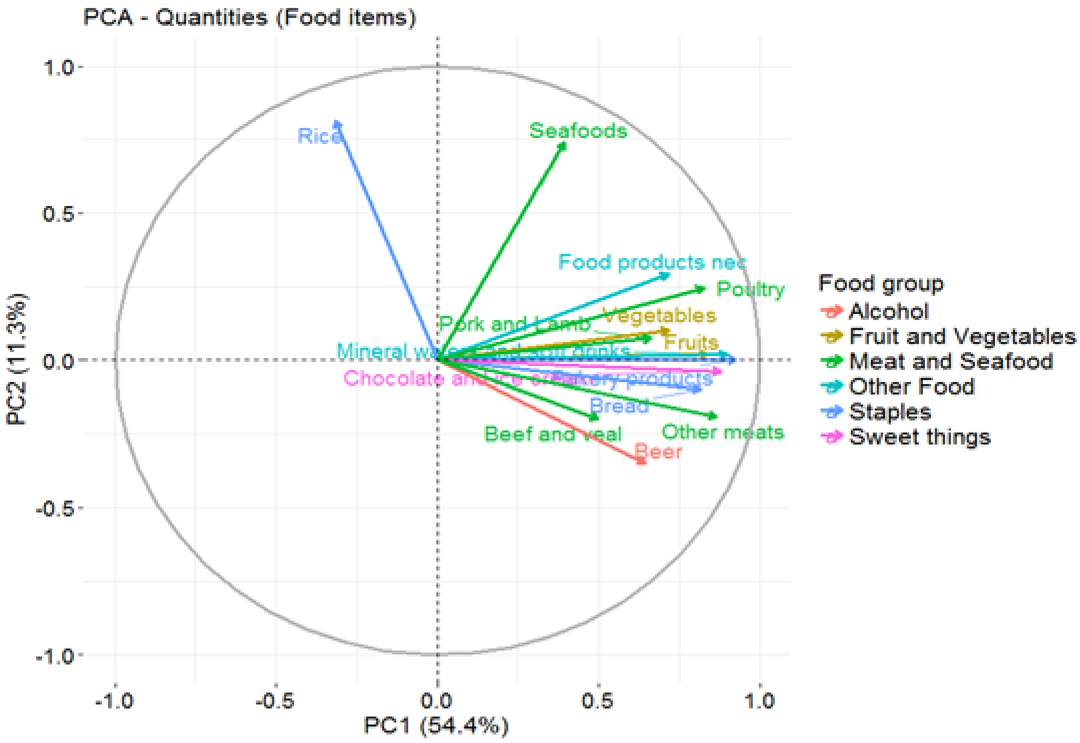

Figure A7.11. Correlation Circles of Consumption, 14 Items, 154 Countries

Notes: See notes to Figure A7.7.

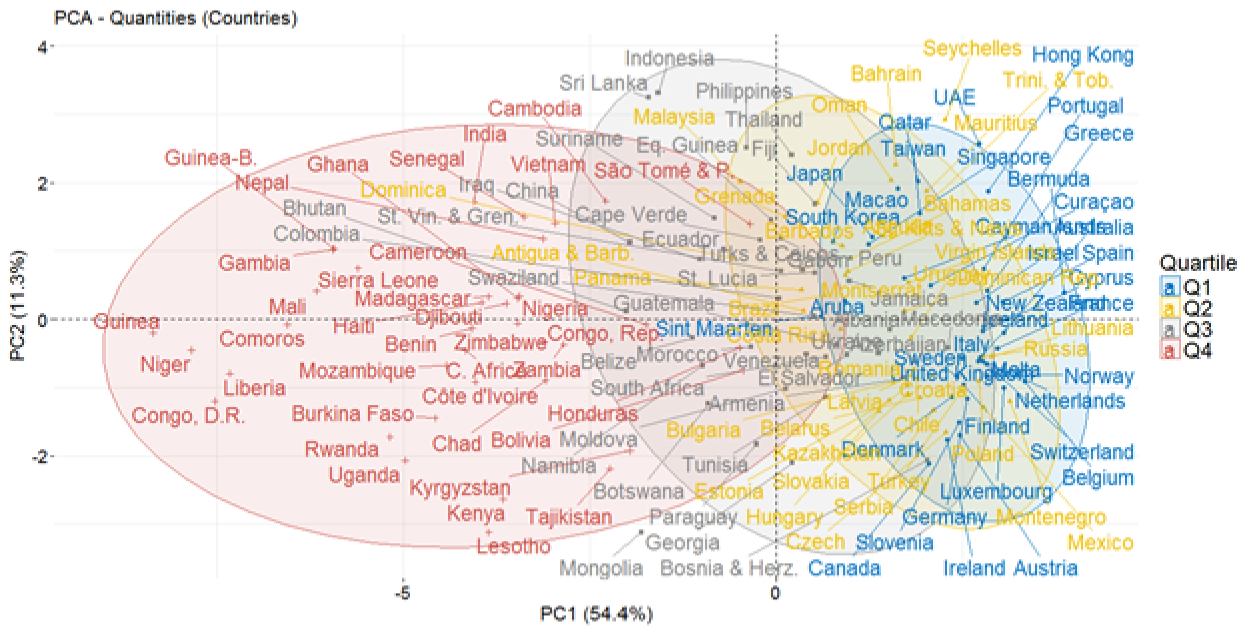

Figure A7.12. Factor Scores for 14 Items, 154 Countries (Consumption)

Notes: See notes to Figure A7.9.

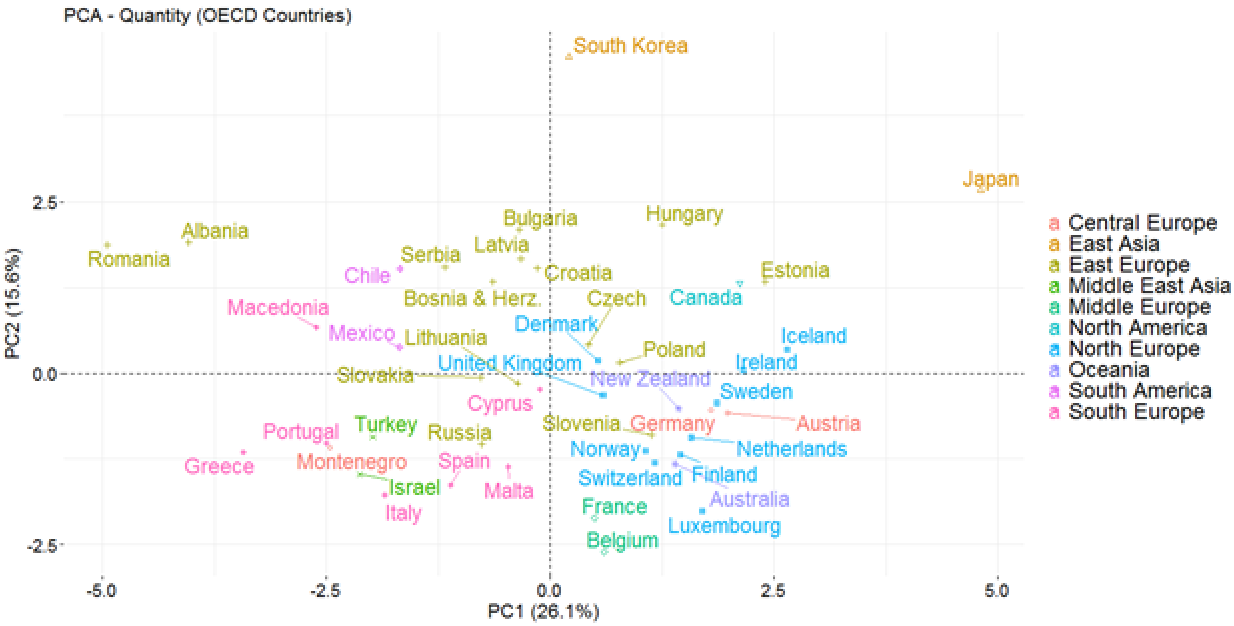

Figure A7.13. Factor Scores for 14 Items, 46 OECD-Eurostat Countries (Consumption)

Notes: See notes to Figure A7.10.

Table A7.9. PCA Results for Relative Consumption

Aggregation level (1)	No. of variables/items (2)	PC1 contribution (%) (3)	No. of PCs required for >80% contribution (4)	Correlation between PC1 and $\log(Y_c/\overline{Y})$ Coefficient (5)	t-stat (6)	SD (7)	SD weighted (8)
A. Deflated consumption $\left[\log q_{ic} - \log(Y_c/\overline{Y})\right]$							
Basic heading	125	26.5	19	0.56	8.39	2.81	
Class	101	30.9	17	0.55	8.2	2.83	
Group	48	46.9	7	0.46	6.47	2.84	
Category	16	71.8	2	0.32	4.22	2.58	
Main Aggregate	2	98.2	1	0.53	7.67	0.93	
B. Relative consumption $\left(\log q_{ic} - \Sigma_{i=1}^{n} w_i \log q_{ic}\right)$							
Basic heading	125	25.5	19	0.57	8.45	2.83	1.4
Class	101	29.8	17	0.56	8.26	2.85	1.32
Group	48	44.5	7	0.48	6.66	2.87	1.26
Category	16	69.2	2	0.31	4.03	2.6	0.82
Main Aggregate	2	99.8	1	0.54	7.98	0.95	0.41

Notes: This table reports the correlation between the first principal component and real income (at each aggregation level) for all consumption items. The underlying variables for PCA are: Deflated consumption (panel A), relative consumption (panel B). Column (2) indicates the number of headings that have positive expenditure (these are documented in Table A1.3). Column (7) shows the standard deviation of the variables (across items and countries) of relative prices, constructed as: $\sqrt{\frac{1}{154 \times n}\Sigma_{j=1}^{154\times n}\left[\log(q/Q)_j - \frac{1}{154 \times n}\Sigma_{j=1}^{154\times n}\log(q/Q)_j\right]^2}$ where n is the number of items at each level. For relative prices, multiply these numbers by 100 gives the percentage difference. Column (8) shows the average of the square root of the weighted variance: $1/154\Sigma_{c=1}^{154}\sqrt{\Pi_c}$ where $\Pi_c = \Sigma_{i=1}^{n} w_{ic}(\log q_{ic} - \log Q_c)^2$. $\log(Y_c/\overline{Y})$ is the per capita income of country c relative to the cross-country geometric mean.

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