Understanding Sri Lankan Business Cycles through an estimated DSGE Model

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Abstract

This study estimates a small open economy DSGE model for Sri Lanka to find out the driving forces of business cycles. The model replicates moments of actual data fairly well and it outperforms BVAR models estimated with the same data. The application of the estimated model reveals that domestic supply shocks and external shocks were the main drivers of business cycles in Sri Lanka. The oil price plays a key role in explaining the movements of inflation.

1. Introduction

Short-term fluctuations in key economic aggregates around long-term trend is referred to as business cycles. Explaining the sources of these fluctuations have long been the interest of macroeconomic research. Identifying main drivers of the cycles will help to develop appropriate policies. Early works of macroeconomic models in explaining business cycles, i.e. Keynesian models, were highly criticized by Lucas (1976) for their reliance on backward looking expectations. The seminal work by Kydland and Prescott (1982) as a response to this criticism led to the development of Dynamic Stochastic General Equilibrium (DSGE) models that are based on forward looking rational expectations. DSGE model describes behavior of the model economy as a whole through the interactions and decision making by all the agents in the economy that behave rationally. Real Business Cycle (RBC) model developed by them attempted to explain the business cycles only through a shock to productivity assuming money is neutral. However, RBC model at its original form was not largely supported by the data empirically (see references in Woodford, 2003 and Gali, 2008 for supporting evidences). Later advancements to this model included more shocks to explain business cycles and acknowledged the existence of nominal and real rigidities in the economy.
This class of DSGE models with nominal and real rigidities, known as New Keynesian (NK) models, has been proved to be an appropriate modeling tool for studying business cycle fluctuations (for example, Smets and Wouters, 2007).

There is ample empirical literature on the application of DSGE models in explaining business cycles and as a tool for policy analysis in advanced economies during the last decade. However, as far as emerging and developing countries are concerned empirical evidences are only at the evolutionary stage. The usefulness of the standard DSGE models as a tool for explaining business cycles and as a policy tool in less advanced economies has yet to be proved widely\(^1\). Currently, there is a growing empirical literature on the calibration and estimation of DSGE models for less developed economies and the application of such models to answer several policy related questions. Medina and Soto (2005, 2006 and 2007), Batini et al (2009 and 2010), Gabriel et al. (2010), Haider et al. (2013), Peiris and Saxegaard (2007) and Choudhri and Malik (2012) are a few recent studies on DSGE models for emerging markets. These studies attempted to incorporate a few special features of less advanced economies to the otherwise standard DSGE models. On this basis, the current study is one of the first attempts to estimate an open economy DSGE model incorporating specific features of Sri Lankan economy.

This study has two main contributions to the literature. Firstly, this is one of the first set of fully micro-found DSGE models estimated for Sri Lanka. Therefore, this study is an opening towards DSGE model based policy analysis in Sri Lanka. Second, Sri Lankan business cycle fluctuations and the underlying sources have not been studied yet. It is believed that emerging and developing markets’ business cycles are largely driven by external sources such as oil and commodity price shocks, shocks to the export demand and import price, sudden reversals of current account and sudden stops of fund flows (see Aguiar and Gopinath, 2004). In Sri Lanka also various economic discussions and policy documents have often cited the dominance of external shocks in explaining fluctuations in key economic variables, especially the inflation. However, there were also disagreements to these statements. For example, Duma (2008) claimed based on a VAR based study that external shocks explain only 25% of variation in Sri Lankan inflation. The current study tries to validate these statements quantitatively by measuring the impacts of many shocks that originate domestically and externally with

\(^1\)See Tover (2008) for a discussion on the limitations of application of DSGE models at their standard form to emerging economies.
a special focus on international oil price shock\textsuperscript{2}. Thus, this study serves two purposes. First, estimate a DSGE model with Sri Lankan data to understand the dynamics of the economy and then use the estimated model to identify main drivers of business cycles.

Theoretical foundation of the main features of the DSGE model in this study has been mainly based on an emerging market based model developed by Medina and Soto (2005, 2006 and 2007) for Chile. The model is a medium-scale small open economy DSGE model consisting of multi-sectors and many nominal and real rigidities to match the dynamics commonly observed in economic data. This model includes some specific characters of emerging economies such as financial risk premium depending on country’s net asset position, imported goods entering as an intermediate good, incomplete pass through to import prices and oil as a consumption good and factor of production, in addition to the features of conventional DSGE models such as price and wage rigidities, adjustment cost to investment, habit persistence in consumption. In addition to these characteristics, the households have been disaggregated into two groups: Ricardian and non-Ricardian. This is to recognize the existence of a group of non-optimising households that heavily depend on disposable income for consumption without having access to formal credits or hold assets to smooth consumption. Further, presence of inward worker remittances have been modelled to capture Sri Lankan current account dynamics, since Sri Lanka is one of the largest worker remittance recipient in South Asia.

The model is estimated based on Bayesian approach. A large number of observable variables related to both domestic economy and foreign sector are included in the estimation to capture the dynamics of the economy being studied. Data includes 9 variables observed on a quarterly basis between 1996 and 2014. The model incorporates 11 exogenous shocks emanating from both domestic origin and the rest of the world. Compared to many other studies, both number of observables and number of shocks considered in this study are relatively high. Incorporating more data into the model makes the model more reflective of the economy being investigated and outcome will be more justifiable. At the same time, including a large number of shocks guarantees that many possible shocks hitting the economy are accounted for in explaining the movements of key economic aggregates.

\textsuperscript{2}Though commercially viable oil and gas exploration feasibility in Sri Lanka has been confirmed in 2007 the exploration has not been commenced yet. Until it becomes an oil producing country it has to be treated as an oil importing country that is vulnerable to international oil price changes.
The estimation result is presented and discussed first followed by a detailed analysis on the sources of business cycle. All shocks are grouped into 5 major types, such as domestic supply shocks, domestic demand shocks, monetary policy shocks, international oil price and other external shocks, and their relative importance on the fluctuations in key domestic observable variables are measured to identify the main drivers contributing to Sri Lankan business cycles. This has been done through conditional forecast error variance decomposition. Secondly, historical movements of key variables and the contribution of each class of shocks to these movements are analyzed through historical decomposition. Thirdly, Impulse responses of these shocks are discussed in detail.

The model is successful in replicating stylized facts of business cycles in Sri Lanka, especially consumption volatility in excess of output volatility. Based on estimated parameters oil is found to be less elastic and less substitutable both in consumption and production. Monetary policy reacts more to the movements in exchange rate than the output gap and inflation. This finding is not surprising since Central Bank of Sri Lanka is not yet committed to inflation targeting framework. The model was successful in fairly replicating the second moments of actual data and the model reports higher marginal data density than that of BVAR models estimated using same data set. Moreover, the estimation outcome is robust to different specifications of the model. Historical fluctuations in output growth were mainly driven by the domestic supply shocks and other external shocks, except the recent financial crisis related recession that is driven by demand shocks. Inflation movements were explained by domestic supply and external shocks including the oil price shock. International oil price has a significant influence on inflation. This has been confirmed both by historical decomposition and forecast error variance decomposition analysis. The impulse responses have meaningful economic interpretations.

The rest of the chapter is structured in the following way. Section 2 gives an a brief description about the stylized facts about business cycle fluctuations in Sri Lanka. A brief summary of DSGE based studies for less-advanced economies, including Sri Lanka, is given in Section 3. A detailed explanation of the model economy and derivation of key equations are given in Section 4. In Section 5, estimation methodology, details on calibrated parameters, prior for estimated parameters and data set have been put together. Estimation outcomes are discussed in Section 6 and the application of the estimated model is explored in Section 7. The conclusion and future developments are
2. Stylized facts of business cycles in Sri Lanka

It is appropriate to briefly study the stylized facts of business cycles in Sri Lanka at this juncture before an in-depth analysis of the sources of fluctuations. For this purpose, cycle components have been obtained from Hodrick-Prescott (HP) filtered series with smoothing parameter of 1600 for quarterly data during the sample period. Output, consumption, investment and net export are expressed in real terms converted into logs before applying HP filter. The difference between the original variables and the trend component obtained from HP filtering indicates cycle component for each series. The standard deviations of the cycles of GDP and GDP growth rate along with relative volatility of other key variables in comparison to output volatility provide some insights about the nature of business cycles in Sri Lanka. Detrended cycle components of the variables are plotted in Figure 1, while Table 1 provides variability of these cycles with comparable average values reported for emerging and developed economies by Aguiar and Gopinath (2004).

The table and the figure reveal that Sri Lankan output cycle fluctuation is moderate and resemble that of an advanced economy. This means that output has been fluctuating only slightly around the long-term trend\(^3\). This contradicts with the finding of other developing and emerging economies. Volatility of all other variables are in line with that of emerging markets. Real consumption exhibits relative volatility of 1.98 with output volatility. This denotes that consumption is 1.98 times volatile than output, which is slightly higher than the average reported for emerging economies. Relative volatility of investment is much higher than that of consumption and the average volatility of investment reported for emerging economies. This fact can be attributed to relatively underdeveloped financial markets, fluctuation in foreign direct investment that account for a significant share in total investment in Sri Lanka and government economic and investment policies. Higher relative volatility of investment is a known fact for all emerging markets even though Aguiar and Gopinath (2004) reported only a slightly high ratio for emerging market to their surprise. At the business cycle fre-

\(^3\)This could be partly attributed to the base year of GDP estimation. Base year has been revised recently in 2015. Preliminary estimates of GDP for 2011-2015 based on the new base year suggest that the GDP series becomes more volatile compared to the series based on the old base year (CBSL Annual report, 2015). Once GDP data based on revised series is made available to the future researchers the findings could be different.
Figure 1: Cycles of key macroeconomic variables

Cycles net export is also a highly volatile variable like investment. The fact that relative volatilities of consumption, investment and trade balance for Sri Lanka are much higher than emerging market averages is partly attributed to less volatile output in Sri Lanka. Further, part of high relative volatility in consumption and investment might be due to the data limitation, since these quarterly series are interpolated based on annual series. Details of interpolation is described in the Appendix A under Data description and transformation.

<table>
<thead>
<tr>
<th>Description</th>
<th>$\sigma(Y)$</th>
<th>$\sigma(\Delta Y)$</th>
<th>$\sigma(C)/\sigma(Y)$</th>
<th>$\sigma(I)/\sigma(Y)$</th>
<th>$\sigma(NX)/\sigma(Y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sri Lanka</td>
<td>1.43</td>
<td>0.83</td>
<td>1.98</td>
<td>6.40</td>
<td>5.84</td>
</tr>
<tr>
<td>Emerging economies (Avg)</td>
<td>2.74</td>
<td>1.89</td>
<td>1.45</td>
<td>3.91</td>
<td>3.22</td>
</tr>
<tr>
<td>Advanced economies (Avg)</td>
<td>1.34</td>
<td>0.95</td>
<td>0.94</td>
<td>3.41</td>
<td>1.02</td>
</tr>
</tbody>
</table>

All these facts are visible from Figure 1 as well. The figure has a new evidence that the consumption became volatile only in the recent periods around the financial crisis.
Moreover, the figure gives an evidence that trade balance is counter-cyclical while all other variables are pro-cyclical. In other words, net export moves in opposite direction to the movements in output while all other variables fluctuate in the same direction.

3. DSGE model based studies for less-advanced economies

Empirical works on the use of DSGE models for advanced economies have established empirical evidences. As far as emerging and developing economies are concerned the empirical work on the applicability of DSGE models have been started only recently. There is still disagreement on the applicability of these models to emerging economies (Tover, 2008 and Senbeta, 2011). Emerging economies’ economic structure, sources of shocks and policy initiatives are different from advanced economies. Idiosyncratic structural features such as imperfect financial markets, poor fiscal management with persistent public debt, a large share of credit constrained consumers and firms, existence of non-negligible informal sector, higher macroeconomic volatility, vulnerability to external shocks, much broader scope for monetary policy, absence of explicit inflation targeting framework, incomplete pass-through of exchange rate, lack of high frequency data are a few issues quoted in the literature as the factors challenging the applicability of benchmark DSGE models to the emerging economies.

Regardless of these challenges and criticisms there is currently a growing empirical literature on the use of DSGE models incorporating certain modifications to suit the structure of the emerging economies. Medina and Soto (2005, 2006 & 2007), Gabriel et al. (2010), Haider and Khan (2008), Haider et al. (2013), Peiris and Saxegaard (2007), Choudhri and Malik (2012), Beidas-Strom and Poghosyan (2011) are a few recent studies on DSGE models of emerging markets.

Medina and Soto (2005, 2006 & 2007) estimated medium scale DSGE model that incorporates many features of emerging markets in general and some specific features such as commodity export, oil in consumption and production, taylor-made fiscal policy and monetary policy for Chile. Peiris and Saxegaard (2007) estimated a DSGE model for Mozambique to evaluate monetary policy trade-off in low-income countries. The fiscal and monetary policies were modeled to include features of developing countries such as foreign exchange intervention by monetary authority, role of foreign aid in fiscal deficit management, central bank balance sheet that includes monetary aggregates and international reserves and crawling peg regime for exchange rate. They found that exchange rate peg based monetary policy rule was less successful in stabilizing the
economy compared to inflation targeting based rule.

Haider and Khan (2008) replicated Gali and Monacelli (2005) model for Pakistan that characterized the model economy with standard features of a DSGE model, such as nominal rigidities, habit persistence in consumption. In a later attempt, Haider et al. (2013) calibrated a DSGE model for Pakistan to study business cycles and monetary policy. The model explicitly modeled informal labour and informal production sector that produces non-tradable intermediate goods. Chodhri and Malik (2012) studied different monetary policy rules based on a calibrated DSGE model for Pakistan. The model differentiated households into high-income and low-income groups, incorporated fiscal policy and monetary policy regimes that features seigniorage and fiscal dominance. Batini et al. (2010) calibrated two bloc DSGE model for India and USA with financial friction through financial accelerator mechanism to study welfare implications of fixed, managed float and free float exchange rate regimes for India. They concluded that simple domestic inflation target and exchange rate targeting rule bring lower exchange rate volatility at a significant welfare loss.

A closed economy DSGE model was estimated for India by Gabriel et al. (2010). Complexities related to an emerging market such as existence of credit constrained household, financial friction and informal production sector have been added to the model in stages. The evaluation of the estimated model with these additional features confirmed that the dynamics in the data is well captured in the extended model than the standard model. The estimated model was then used to study business cycles.

Application of DSGE model to Sri Lanka is limited to 3 recent parallel studies, including this study. Other two studies have been carried out around the same time as this study, but they are independent from this study. Karunaratne and Pathberiya (2014) estimated a small open economy DSGE model for Sri Lanka with a sample of 1999-2013. Their model largely followed the benchmark small open economy model proposed by Gali and Monacelli (2005) and extended this model with low exchange rate pass-through as proposed by Liu (2006). The estimated model has been used to study the impacts of 5 shocks on the economy. The estimated model has been validated only based on Brooks and Gelman (1998) convergence diagnostic. The model is relatively a simple model that incorporated only a representative production firm that use only labour as the factor of production. Another point to note about the outcome of this model is that the degree of interest rate smoothing has been estimated to be very low as 0.19, that is highly unlikely.
Ehelepola (2014) calibrated a closed economy DSGE model with fiscal and monetary policy rules for Sri Lanka to conduct welfare analysis. The model resembles many features of the model proposed by Schmitt-Grohe and Uribe (2007). Three scenarios for monetary and fiscal policy stances namely, cashless economy, a monetary economy and an economy with cash and distortionary tax are considered for welfare analysis. The finding suggested that all three policy rules delivered almost the same welfare level as in Ramsey optimal policy. Monetary policy rules confirmed high interest rate inertia, high response to contemporaneous inflation and a small response to output gap. On the strong side, this study has a reasonably well defined fiscal policy and the role of money in the economy like Sri Lanka has been recognized. However, calibrated values for many structural parameters have been borrowed from studies based on advanced economies and the model is a closed economy model. The applicability of the findings to Sri Lanka is therefore questionable. Sri Lanka is an open economy with large degree of openness of above 60% over the past. Moreover, it is a generally accepted fact that a small open economy like Sri Lanka is vulnerable to shocks emanating from external sector. Thus, closed economy based study has certain limitations.

The review of the studies on empirical application of DSGE models to Sri Lanka reveals two main facts. Firstly, limitation of high frequency data that span for a long period and lack of evidences to form calibration and priors for Sri Lankan based studies. Second and the most important fact is that DSGE based studies for Sri Lanka is only at the evolutionary stage and thus there is a vast gap in the empirical literature that could be filled with more future research for Sri Lanka.

The model in the current study is more detailed than the models used by both of the past studies for Sri Lanka, though it does not fully incorporate the complexities of fiscal sector and fiscal policy for Sri Lanka. It is a medium scale DSGE model for a small open economy. The role of intermediate good imports and final good exports have been incorporated along with the non-negligible contribution of oil products in consumption and production. The production function consists of labour, investment and oil as factors of production. Nominal price rigidities have been imposed on domestic price, export price and import price and wage is also subject to nominal rigidities. Investment is subject to adjustment cost and the consumption exhibits habit persistence. Households have been disaggregated into optimising group and non-optimising group to replicate the nature of actual households in the economy. Most importantly, the role of worker remittances in smoothing consumption and mitigating current account
deficit has also been considered in this study to acknowledge the significance of worker remittances in foreign exchange earnings. The estimated model has been validated broadly based on many approaches, such as Brooks and Gelman (1998) convergence diagnostic, comparison of second moments of the models with actual moments and comparison of marginal data density of the DSGE model with that of Bayesian VAR models with different lag levels.

4. The model

The model economy consists of multi sectors characterized by a number of nominal and real rigidities. Main features of the economy are price and wage rigidities, incomplete pass through to import prices, adjustment cost in investment, habit persistence in consumption, oil in the consumption basket and as a factor of production and current account dynamics of the economy. Price rigidities, wage rigidities, investment adjustment cost and habit persistence have been included like in many other standard DSGE model that are considered essential to replicate the dynamics of the movements of actual consumption, investment, price and wages. Inclusion of detailed production sector that incorporates domestic goods, imported goods and export goods are to replicate the nature of Sri Lankan economy that imports majority of intermediate goods and mainly exports finished goods. Since oil imports to GDP ratio is around 10% and oil is being increasingly used in power generation and other production, inclusion of oil in the modelling will be helpful. Agents in the model economy consist of households that are divided into optimizing and non-optimising groups, intermediate production firms, importers, both home and foreign final good assemblers, capital leasing firm, government and monetary authority. Theoretical foundations of this model have been adapted mainly from Medina and Soto (2005, 2006 and 2007) and Smets and Wouters (2003 and 2007).

Domestic production firms produce different varieties of intermediate home goods in which they have monopoly power. Price is set in a staggered fashion characterized by Calvo type price setting. Import retailers have monopoly power over the foreign intermediate varieties they import. Their pricing behavior is also sticky similar to that of domestic intermediate firms. Changes in exchange rate are incompletely passed through to the domestic prices of imported intermediate goods. There are two different sets of final goods assemblers who receive home and foreign intermediate goods and assembles home and foreign final goods, respectively. Oil also enters into the con-
sumption basket of households in addition to the core consumption consisting of both home and foreign goods. Final home goods are consumed both by households and government, exported and used for capital accumulation while final foreign goods are consumed by households and used for capital accumulation. A representative capital leasing firm combines final home and foreign goods and assemble capital goods that are rented to domestic intermediate firms. Fiscal policy is conducted by the government and the monetary policy is conducted by the monetary authority. The economy is assumed to grow at deterministic labour productivity growth rate, $g_y$. Flow chart of the model economy is illustrated in Figure 2.

In order to guarantee that the paper is self-contained the following section illustrates the model in detail with corresponding key equations.

**Household consumption**

The model economy is inhabited by two groups of households. One group is Ricardian households, who acts rationally and takes decisions on consumption and saving and sets wage optimally. Other group is non-Ricardian households that consumes out of disposable income and takes wages set by Ricardian consumers as given. It is assumed that there will be $\lambda$ share of households that are non-Ricardian households and $(1 - \lambda)$ share of Ricardian households.

**Ricardian households**

The economy is inhabited by a continuum of optimizing households indexed by $j \in [0, 1]$. Ricardian Households gain utility from consumption ($C^R_t$) and disutility by supplying their labor ($l_t$). The expected present value of household $j$ at time $t$ is given by the following utility function.

$$U_t = E_t \{ \sum_{i=0}^{\infty} \beta^i \zeta_{C,t+i}[\log(C^R_{t+i}(j) - \hat{h}C^R_{t+i-1}) - \frac{l_{t+i}(j)^{1+\sigma_L}}{1+\sigma_L}] \}$$

where $\beta$ is the intertemporal discount factor, $C^R_t(j)$ is the consumption by Ricardian households, $l_t$ is labor. Consumption is subject to external habit formation with the habit persistence parameter $\hat{h}$. Parameter $\zeta_{C,t}$ refers to the consumption preference

\[ ^4 \text{As per the assumption of economy growing at the rate of } g_y \text{ in the steady state } \hat{h} \text{ is adjusted} \]
Figure 2: Flow chart of the model economy
shock that follows an AR (1) process. Parameter $\sigma_L$ is the inverse of labor supply elasticity that measures how much labor supply changes to a change in wage while keeping consumption unchanged. It is common in the literature to assume that $\sigma_L > 1$.

Budget constraint of Ricardian household $j$ is given by the following equation.

$$P_{C,j}C_t^R(j) + E_t\{Q_{t,t+1}D_{t+1}(j)\} + \frac{\varepsilon_l B_t^*}{(1+i_t)^{\theta(\beta_t^*)}} + M_t(j) = W_t(j)\Pi_t(j) - T_{p,t} + D_t(j) + \varepsilon_t B_{t-1}^*(j) + M_{t-1}(j) + (1-\tau) \varepsilon_t \Xi_t$$

Household has access to three types of assets: Money, $M_t(j)$, one-period non-contingent foreign bond denominated in foreign currency, $B_t^*(j)$, and one-period domestic contingent bonds, $D_{t+1}(j)$, that pays out one unit of domestic currency in a particular state. In the equation above, $Q_{t,t+1}$ is the stochastic discount factor for one period ahead nominal payoffs relevant to the domestic households. The assumption of the existence of a full set of contingent bonds ensures that consumption of all Ricardian household is the same regardless of their labour income. Nominal exchange rate (expressed in terms of domestic currency per one unit of foreign currency) is given by $\varepsilon_t$. Nominal wage set by the household is given by $W_t(j)$, profits received from domestic intermediate firms is given by $\Pi_t(j)$, worker remittances in domestic currency (further details are given below) is $\varepsilon_t \Xi_t$ and lump sum tax is given by $T_{p,t}$. Variable $i_t^*$ is the foreign interest rate.

The term $\Theta(\cdot)$ is the premium that the domestic household has to pay to borrow abroad. That is a function of aggregate net asset position of the economy, $\beta_t^*$. That is given by the net foreign asset position, $\varepsilon_t B_t^*$, relative to GDP, $P_{Y,t} Y_t$:

$$\beta_t^* = \frac{\varepsilon_t B_t^*}{P_{Y,t} Y_t}$$

Since the premium depends on the aggregate net asset position instead of individual net asset position households takes this as exogenously given when they optimize their consumption. In the steady state it is assumed that $\Theta(\cdot) = \Theta$ and $\frac{\partial}{\partial \varepsilon_t^*} \beta = 0$. When according to $h = h(1 + g_d)$, where $h$ is the habit persistent parameter in the absence of steady state growth.

$^5$In this notation $E_t(Q_{t,t+1}) = \frac{1}{1+i_t}$
a country is a net debtor $g$ correspond to the elasticity of upward-sloping supply of international funds.

Ricardian households maximize utility subject to the above budget constraint. The first order conditions yields the following Euler equation for consumption,

$$\beta E_t \left\{ (1 + i_t) \frac{P_{C,t}}{P_{C,t+1}} \frac{\zeta_{C,t+1}}{\zeta_{C,t}} \left( \frac{c_{R(t+1)}^{R} - \gamma c_{R(t)}^{R}}{c_{R(t)}^{R} - \gamma c_{R(t-1)}^{R}} \right) \right\} = 1,$$

where $P_{C,t}$ is the aggregate price index (derived later in this section) and $i$ is the domestic risk free nominal interest rate. Using this relation and the first order condition with respect to foreign bonds the following expression for uncovered interest parity (UIP) can be derived:

$$1 + i_t (1 + i_t) \beta_t = E_t P_{C,t}^{r+t} \left( \frac{p_{t+1}^{r+t}}{p_{t}^{r+t}} \frac{\zeta_{C,t+1}^{r+t}}{\zeta_{C,t}^{r+t}} \left( \frac{c_{R(t+1)}^{R} - \gamma c_{R(t)}^{R}}{c_{R(t)}^{R} - \gamma c_{R(t-1)}^{R}} \right) \right) = E_t P_{C,t}^{r+t} \left( \frac{p_{t+1}^{r+t}}{p_{t}^{r+t}} \frac{\zeta_{C,t+1}^{r+t}}{\zeta_{C,t}^{r+t}} \left( \frac{c_{R(t+1)}^{R} - \gamma c_{R(t)}^{R}}{c_{R(t)}^{R} - \gamma c_{R(t-1)}^{R}} \right) \right)$$

Using the fact that $E_t Q_{t,t+1} = E_t Q_{t,t+1}^{*}$, the above equation can be simplified as follows:

$$\frac{1 + i_t}{(1 + i_t)^{\beta}(B_t)} = E_t \left\{ \frac{p_{t+1}^{r+t}}{p_{t}^{r+t}} \frac{\zeta_{C,t+1}^{r+t}}{\zeta_{C,t}^{r+t}} \left( \frac{c_{R(t+1)}^{R} - \gamma c_{R(t)}^{R}}{c_{R(t)}^{R} - \gamma c_{R(t-1)}^{R}} \right) \right\}$$

Using the fact that $E_t Q_{t,t+1} = E_t Q_{t,t+1}^{*}$, the above equation can be simplified as follows:

$$\frac{(1+i_t)}{(1+i_t)^{\beta}(B_t)} = E_t \frac{Q_{t+1}^{*}}{c_{t}}$$

where $i_t^{*}$ is the foreign interest rate that follows an AR(1) process subject to an i.i.d shock. This shock captures the relevant foreign financial factors faced by the domestic agents, including price, risk premia and any other factors associated with the exchange rate arbitrage. This equation implies that interest rate differentials is related to both expected future exchange rate depreciation and international risk premium.

Non-Ricardian households

Assumption of the existence of a share $(\lambda)$ of credit constrained consumers serves a number of purposes. This feature replicates the real world fact of a developing country like Sri Lanka with a significant share of poor households, who consume out of their
disposable income without having access to formal credit facilities and possess no assets. They can not smooth their consumption over time regardless of fluctuations in their income. Further, existence of this class of consumers in the economy is particularly used in this model to incorporate worker remittances. Worker remittance is the primary and stable foreign exchange earner in the recent years that account for 8% of GDP and around 30% of export earnings during the sample period. Incorporating this unique feature in modeling Sri Lankan economy is therefore very relevant. It is assumed that the majority of flow of worker remittances are received by the credit constrained consumers and they utilize that to smooth their consumption.

This set of households face a static problem of maximizing period utility subject to the disposable income given by the sum of net labour income and worker remittances from abroad. The consumption of a representative non-Ricardian consumer \((j)\) is given as follows. Superscript \(nR\) denotes non-Ricardian consumers.

\[
C_{t}^{nR}(j) = \frac{W_{t}}{P_{C;t}} l_{t}(j) + \tau * \frac{\Xi_{t}}{P_{C;t}} \text{ for } j \epsilon [0, \lambda]
\]

where \(\Xi_{t}\) is the remittances received from abroad denominated in foreign currency and \(\frac{\Xi_{t}}{P_{C;t}}\) is the real remittances in domestic currency terms and \(\tau\) is the share of remittance received by non-Ricardian household\(^6\). It is reasonable to assume that this category of households do not pay lump sum tax. Remittances have been modelled mainly based on the approach applied to Philippines by Mandelman (2011), who considered full absorption of remittances by non-Ricardian households. For simplicity, it is assumed that the aggregate worker remittance is characterized by stochastic AR(1) process \(^7\) subject to an i.i.d. shock \(\varepsilon_{t}\) that depends largely on the economic conditions in the country of migrant workers.

\(^6\)Even though micro level data is hardly available on the usage of remittances by households in Sri Lanka (Maimbo and Hulugalle, 2005) several developing country based studies on remittances have concluded that remittances play an important role in improving the consumption in the recipient economies and only a marginal influence on the investment (See Barajas et al., 2009 and references therein). Also, a number of Sri Lanka related policy studies have confirmed that remittance inflows are used mainly to meet consumption expenditure and educational purposes (IMF, 2004 and FDC, 2007). Further, several recent media reports in Sri Lanka reinstate heavy usage of remittances on consumption and the need to have personal and policy initiatives to mobilize these inflows beyond consumption.

\(^7\)Another view on remittances is that remittances are associated with ‘altruistic motives’. This motive is modelled either by linking economic growth or real wage to remittances. Fall in these variables indicate economic hardship and migrant worker will remit more. However, Sri Lankan data and an existing evidence for Sri Lanka (Lueth and Ruiz-Arranz, 2007) do not support the view that remittance is counter cyclical.
\[ \Xi_t = \Xi_{t-1} \alpha \exp \varepsilon_x, \]

For simply it is assumed that non-Ricardian households take the average wage set by Ricardian household as given and supply labour at that wage.

**Aggregate consumption**

The aggregate consumption in the economy is the weighted average consumption of both Ricardian and non-Ricardian consumers, given as follows.

\[ C_t = (1 - \lambda)C_t^R + \lambda C_t^{nR} \]

Aggregate consumption basket consists of oil, \((C_O)\), and non-oil core consumption bundles, \((C_Z)\), and these are imperfect substitutes given by constant elasticity of substitution (CES) aggregator.

\[ C_t(j) = \left[ (\alpha_C)^{1/\omega_C} (C_{Z,t}(j))^{\omega_C-1} + (1 - \alpha_C)^{1/\omega_C} (C_{O,t}(j))^{\omega_C-1} \right]^{\omega_C/(\omega_C-1)} \]

here \(\alpha_C\) is the share of core consumption bundle in the aggregate consumption basket and \(\omega_C\) is the elasticity of substitution between oil and core consumption bundles. Demand functions determined by the optimal composition of consumption bundles obtained by minimizing the total expenditure is given as follows.

\[ C_{Z,t}(j) = \alpha_C \left( \frac{P_{Z,t}}{P_{C,t}} \right)^{-\omega_C} C_t(j), \quad C_{O,t}(j) = (1 - \alpha_C) \left( \frac{P_{O,t}}{P_{C,t}} \right)^{-\omega_C} C_t(j) \]

where \(P_{Z,t}\) and \(P_{O,t}\) refer to price indices of core and oil consumption bundles, respectively. In addition, aggregate price index is given by:

\[ P_{C,t} = (\alpha_C P_{Z,t}^{1-\omega_C} + (1 - \alpha_C) P_{O,t}^{1-\omega_C})^{1/(1-\omega_C)} \]

The core consumption bundle, in turn, is a composite of both final home goods and final foreign goods determined by CES aggregator,

\[ C_{Z,t}(j) = \left[ \gamma_C^{1/\eta_C} (C_{H,t}(j))^{\eta_C-1} + (1 - \gamma_C)^{1/\eta_C} (C_{F,t}(j))^{\eta_C-1} \right]^{\eta_C/(\eta_C-1)} \]
where $\gamma_C$ is the share of home consumption bundle in the core consumption basket and $\eta_C$ is the elasticity of substitution between home and foreign consumption bundles. Demand functions for home and foreign consumption bundles determined by minimizing the total expenditure is given as follows.

$$C_{H,t}(j) = \gamma_C \left( \frac{P_{H,t}}{P_{Z,t}} \right)^{-\eta_C} C_{Z,t}(j), \quad C_{F,t}(j) = (1 - \gamma_C) \left( \frac{P_{F,t}}{P_{Z,t}} \right)^{-\eta_C} C_{Z,t}(j)$$

where $P_{H,t}$ and $P_{F,t}$ refer to price indices of home and foreign consumption bundles, respectively. The price index of core consumption is given by:

$$P_{Z,t} = (\gamma_C P_{H,t}^{1-\eta_C} + (1 - \gamma_C) P_{F,t}^{1-\eta_C})^{\frac{1}{1-\eta_C}}$$

Wage setting

Each household $j$ supplies differentiated labour supply, in which he has monopoly power. Perfectly competitive labor service assemblers hire labor from each household and combine to aggregate labor supply as follows:

$$l_t = \left( \frac{1}{0} l_t(j) \frac{\varepsilon_L}{\varepsilon_L - 1} dj \right)^{\frac{\varepsilon_L}{\varepsilon_L - 1}}$$

Wage setting in this set up is subject to Calvo (1983) type nominal rigidity. In each period, households face a probability of $1 - \phi_L$ to re-optimize their nominal wage. In which, $\phi_L$ is a measure of degree of nominal wage rigidity. The larger this parameter the longer it takes to adjust wages, i.e. wages are more sticky. This labor is used by domestic intermediate firms in their production function as a factor of production along with oil and capital. Parameter $\varepsilon_L$ refers to elasticity of substitution among labor varieties.

Demand for each type of labor services is derived by minimizing its cost, which is given as follows:

$$l_t(j) = \left( \frac{W_t(j)}{W_t} \right)^{-\varepsilon_L} l_t$$
It is assumed that those who do not re-optimize their wages during periods between $t$ and $t+i$ set their wage at time $t+i$ based on implicit inflation target of the monetary authority. The rule, $\Gamma^i_{W,t}$, is as follows:

$$\Gamma^i_{W,t} = (1 + g_y)(1 + \pi_{t+i})\Gamma^{i-1}_{W,t} \text{ and } \Gamma^0_{W,t} = 1.$$ 

Households who cannot re-optimize update their wage based inflation target subject to the long-term trend in labor productivity, $g_y$. Including this term in the rule prevents an increasing dispersion in the real wages across households along the steady-state balanced growth path.

It is assumed that once household sets his wage he must supply quantity of labor demanded at that wage. The re-optimizing household $j$ at time $t$ must solve the following problem:

$$\max_{W_t(j)} E_t \left\{ \sum_{i=0}^{\infty} \phi^i L^i \Lambda_{t,t+i} \left[ \frac{\Gamma^{i}_{W,i} W_i(j)}{P_{C,t+i}} I_{t+i}(j) - \frac{I_{t+i}(j)^{1+\sigma_i}}{1+\sigma_i} \left( C_{t+i} - h C_{t+i-1} \right) \right] \right\}$$

where $\Lambda_{t,t+i}$ is the relevant discount factor between periods $t$ and $t+i$.

**Investment**

There is a representative capital producer that decides on how much capital to accumulate in each period by combining both home and foreign goods using a CES technology as follows.

$$I_t = \left[ \frac{1}{\gamma} I_{H,t}^{1-\frac{1}{\eta_H}} + (1 - \gamma) \frac{1}{\eta_F} I_{F,t}^{1-\frac{1}{\eta_F}} \right]^{\frac{1}{\eta}}$$

where $\eta$ is the elasticity of substitution between home and foreign goods in investment and $\gamma$ is the share of home goods in investment. The demand functions for home and foreign goods for investment is derived by minimizing the cost of investment and are given by:

$$I_{H,t} = \gamma I \left( \frac{P_{H,t}}{P_{T,t}} \right)^{-\eta_H} I_t, \quad I_{F,t} = (1 - \gamma) I \left( \frac{P_{F,t}}{P_{T,t}} \right)^{-\eta_F} I_t,$$

---

\(^8\)Since utility exhibits habit formation in consumption the relevant discount factor is given by $\Lambda_{t,t+i} = \beta^t \left( \frac{C_i(j) - h C_{t+i-1}}{C_{t+i}(j) - h C_{t+i-1}} \right)$.
where $I_t$ is the total investment and $P_{I,t}$ is the investment price index that is given by:

$$P_{I,t} = (\gamma_t P_{H,t}^{1-\eta_t} + (1 - \gamma_t) P_{F,t}^{1-\eta_t})^{1-\eta_t}$$

This model assumes that adjusting investment each period is costly. This investment adjustment cost is introduced to model inertia in investment like in many other studies. The representative capital producer should solve the following problem subject to the law of motion of the capital stock.

$$\max_{K_{t+1},I_{t+1}} = E_t \left\{ \sum_{t=0}^{\infty} \frac{\Lambda_{t,t+1} Z_{t+1} K_{t+1} - P_{I,t+1} I_{t+1}}{P_{C,t+1}} \right\},$$

subject to the law of motion of capital stock given by:

$$K_{t+1} = (1 - \delta) K_t + \varsigma_{I,t} S \left( \frac{I_t}{I_{t-1}} \right) I_t,$$

where $Z_t$ is the rental price of capital and $\delta$ is the rate of depreciation of capital and $\left( \frac{I_t}{I_{t-1}} \right) = (1 + g_y)$. Investment adjustment cost is given by the function $S(\cdot)$, that satisfies: $S(1 + g_y) = 1, S'(1 + g_y) = 0,$ and $S''(1 + g_y) = -\mu_y < 0$. This cost disappears in the long-run and investment will be fully adjustable. A stochastic shock to the investment is introduced by the term $\varsigma_{I,t}$, that changes the rate of transformation of investment into capital. The higher the value of this term the higher will be the level of capital for a given amount of investment.

The first order conditions for the above maximization problem is given by:

$$\frac{P_{I,t}}{P_{C,t}} = \frac{Q_{t}}{P_{C,t}} \left[ S \left( \frac{I_t}{I_{t-1}} \right) + S' \left( \frac{I_t}{I_{t-1}} \right) \frac{I_{t-1}}{I_t} \right] \varsigma_{I,t} - E_t \left\{ \Lambda_{t,t+1} \frac{Q_{t+1}}{P_{C,t+1}} \left[ S' \left( \frac{I_{t+1}}{I_t} \right) \left( \frac{I_{t+1}}{I_t} \right)^2 \right] \varsigma_{I,t+1} \right\},$$

$$\frac{Q_{t}}{P_{C,t}} = E_t \left\{ \Lambda_{t,t+i+1} \left( \frac{Z_{t+1}}{P_{C,t+1}} + \frac{Q_{t+1}}{P_{C,t+1}} (1 - \delta) \right) \right\},$$

These two equations determine the evolution of the shadow price of capital, $Q_t$, and real investment expenditure.

**Domestic production**

Domestic production sector consists of two types of firms: domestic final goods producers and domestic intermediate goods producers.
Domestic intermediate goods producers

There is a set of firms that produce varieties of intermediate goods with monopoly power. Factors of productions includes labor, capital and oil. The production technology of a representative firm \( (z_H) \) is given by the following CES aggregator:

\[
Y_{H,t}(z_H) = A_{H,t} \left[ \alpha_H \left( V_{H,t}(z_H) \right)^{\frac{\omega_H - 1}{\omega_H}} + (1 - \alpha_H) \left( O_{H,t}(z_H) \right)^{\frac{\omega_H - 1}{\omega_H}} \right],
\]

where \( A_{H,t} \) is the total factor productivity shock that evolve according to AR(1) process as given below.

\[
A_{H,t} = A_{H,t-1}^{\rho_{aH}} \exp \xi_{aH,t}
\]

The variable \( O_{H,t} \) is the amount of oil input in the production, while \( \alpha_H \) represents weight of the composite of labor and capital in the production and \( \omega_H \) is the elasticity of substitution between oil and other production factors. Composite of labor and capital of a representative firm \( (z_H) \), \( V_{H,t}(z_H) \), is given by the following Cobb-Douglas technology:

\[
V_{H,t}(z_H) = \left( 1 + g_y \right) l_t(z_H)^{\eta_I} \left( K_t(z_H) \right)^{1-\eta_I},
\]

where \( \eta_I \) is the share of labor utilized in the production.

It is assumed that prices of home goods both for domestic and foreign markets are sticky and follow Calvo type pricing. Signals for price adjustment for the domestic market arrives with probability \( 1 - \phi_{H_D} \) while that for the foreign market arrives with probability \( 1 - \phi_{H_F} \) each period. Further it is assumed that these probabilities are independent of the time lapse from the last price change and they are common to all the firms. Using these probabilities, average price duration for domestic market and foreign market can be calculated by \( (1 / 1 - \phi_{H_D}) \) and \( (1 / 1 - \phi_{H_F}) \), respectively. Those firms that do not re-optimize their price in the current period follows a passive rule as discussed in wage setting arrangement. A representative domestic firm \( Z_H \) and re-optimizes its prices for domestic, \( P_{H,t}(Z_H) \), and foreign markets, \( P_{H,t}^*(Z_H) \), by solving the following optimization problems.

\[
\max_{P_{H,t}(Z_H)} = E_t \left\{ \sum_{i=0}^{\infty} \phi_{H_D}^{i} A_{t+i} \left[ \frac{\Gamma_{H,t}P_{H,t}(Z_H) - MC_{t+i}}{P_{C,t+i}} Y_{H,t+i}(Z_H) \right] \right\},
\]
\[
\max_{P_{H,t}(Z_H)} \{ E_t \left\{ \sum_{i=0}^{\infty} \phi^i_{H,t} \Lambda_{t+1} \left[ \frac{\Gamma_{H,t} P_{H,t}(Z_H) - M \gamma_{H,t}^{1+i}}{P_{C,t+1}} Y^*_{H,t+i}(Z_H) \right] \right\} \}
\]

where \( M \gamma_{H,t} \) is the marginal cost of producing the variety and it evolves as follows:

\[
M \gamma_{H,t} = \frac{W_{H,t}(Z_H) + Z_{t+1} K_t(Z_H) + P_{O,t} O_{H,t}(Z_H) + P_{O,t} O_{H,t}(Z_H)}{Y^*_{H,t+i}(Z_H)}
\]

Domestic final goods retailers

There is a set of domestic final goods assemblers that use domestic intermediate varieties to assemble home goods using CES technology. Demand for these home goods come from both domestic consumers including the government and consumers abroad. The demand functions are given as follows.

\[
Y^*_{H,t}(Z_H) = \left( \frac{P_{H,t}(Z_H)}{P_{H,t}} \right)^{-\varepsilon_H} Y_{H,t}, \quad Y^*_{H,t}(Z_H) = \left( \frac{P_{H,t}(Z_H)}{P_{H,t}} \right)^{-\varepsilon_H} Y_{H,t},
\]

where \( P_{H,t}(Z_H) \) is the price of variety \( Z_H \) when used to assemble home goods sold in the domestic market and \( P_{H,t}(Z_H) \) is the foreign currency price of this variety when used to assemble home goods sold abroad. Aggregate price indices of home goods sold domestically and abroad are given by \( P_{H,t} \) and \( P_{H,t} \) respectively.

Imported goods

Intermediate foreign goods importers

There is a set of monopolistically competitive importing firms that import intermediate varieties abroad and resell domestically to final foreign goods retailers. Domestic currency price stickiness is assumed in imported intermediate foreign goods to allow for incomplete exchange rate pass-through into import prices in the short-term. Import firms adjust their domestic prices only infrequently when they receive price signal. Similar to the set-up assumed for domestic intermediate goods firms, a representative import firm \( Z_F \) that receives signal to re-optimize its price sets its price in the following manner by maximizing present value of expected profits.

\[
\max_{P_{F,t}(Z_F)} \{ E_t \left\{ \sum_{i=0}^{\infty} \phi^i_{F,t} \Lambda_{t+1} \left[ \frac{\Gamma_{F,t} P_{F,t}(Z_F) - M \gamma_{F,t}^{1+i}}{P_{C,t+1}} Y^*_{F,t+i}(Z_F) \right] \right\} \}
\]

Updating rule for the non-optimising firms are same as the rule adopted for wage
setting and home goods price setting. For simplicity it is assumed that the foreign price of representative firm $Z_F$ is the same for all importing firms, $P_{F,t}^e(Z_F) = P_{F,t}^*$. 

**Final foreign goods retailers**

Similar to domestic final goods assemblers there is a set of assemblers to assemble final foreign goods using imported varieties based on CES technology. These final foreign goods are partly consumed domestically and partly used to assemble new capital goods. The CES aggregator is given as follows:

$$Y_{F,t}(Z_F) = \left( \int_0^1 Y_{F,t}^e(Z_F)^{\varepsilon_F - 1} dZ_F \right)^{\frac{1}{\varepsilon_F - 1}},$$

where $\varepsilon_F$ is the elasticity of substitution among intermediate import good varieties. Under the assumption of perfectly competitive retail market for final foreign goods, demand for imported intermediate varieties is derived by profit maximization of these retailers. Demand for imported intermediate variety $Z_F$ is given as follows:

$$Y_{F,t}(Z_F) = \left( \frac{P_{F,t}^e(Z_F)}{P_{F,t}} \right)^{-\varepsilon_F} Y_{F,t},$$

where $P_{F,t}(Z_F)$ is the domestic currency price of imported variety $Z_F$ and $P_{F,t}$ is the aggregate price of imported goods in the market.

**Fiscal policy**

The government and fiscal policy are recent additions to the standard New Keynesian DSGE model. Detailed modeling of fiscal policy in itself is a separate area of research. Generally followed simplified approach to the fiscal policy is to assume that the government always runs a balanced budget and the government expenditure is given by an autoregressive process. According to this approach, the focus will be on the effect of aggregate level government spending rather than the effect of financing government budget balances (Gali and Monacelli, 2008). It is assumed that the government budget is balanced each period and expenditure is fully financed by tax.\(^9\) It is assumed in this model that government consumption is home-biased, thus govern-

\(^9\)Fiscal policy has been modelled exogenously in this study. Even though fully balancing government budget appears as an unrealistic assumption, this has been a common practice in the literature when the study does not fully focus on the fiscal policy, yet government sector is included in order to assess the effect of government expenditure shock on the economy. Detailed modelling of fiscal policy has been left for future research.
ment consumes only home goods. This home-biased assumption is very common in the DSGE literature and fairly represent government’s preference towards home goods. Government expenditure is given AR(1) process as follows:

\[ G_t = G_{t-1}^{exp} \exp \xi_{G,t} \]

Sri Lankan fiscal policy is some what complex to include in this paper as the focus is on the business cycles rather than the fiscal policy and its implications\(^{10}\). Therefore, detailed modeling of Sri Lankan fiscal policy is left for future research agenda, while including the government expenditure and its impacts in this model.

Sri Lanka experienced prolonged civil conflict for decades and in early 2009 the civil war was brought to an end. It is natural to expect that the saving on national defence expenditures to have a positive impact on the government expenditure, since this time period coincides with the sharp fall in GDP growth in 2008-2009. However, the expected reduction in defence related procurement expenditure was not adequately strong to bring a notable reduction in government expenditure. This is because the government had to continuously spend on resettlement, rehabilitation and reconstruction activities in the war-affected regions even after the end of war. The dip in the GDP during 2008-2009 has been officially attributed to the financial crisis related external forces and fall in agricultural output caused by adverse weather. These facts have been widely acknowledged in the Annual Reports of Central Bank of Sri Lanka in these periods. Thus, the author does not believe that this episode needs a special attention in the DSGE modelling. Further, a shock to the government expenditure is already incorporated in this model, though the actual data on government expenditure was not observed due to the difficulties in obtaining high frequency fiscal related data in Sri Lanka.

**Monetary policy**

Monetary policy is conducted by a monetary authority in this model characterized by a policy reaction function. The monetary policy rule is a simple Taylor rule augmented with deviation in nominal exchange rate\(^{11}\). The monetary policy rule is given as follows.

---

\(^{10}\)Sri Lanka has been experiencing high fiscal deficit and chronic public debt that is high in the region. In order to discipline fiscal management a rule based policy was legally enacted in 2003, with certain targets for deficit and public debt. However, these targets have not been met yet.

\(^{11}\)Different other reaction functions including forward looking version of the current rule were tested. However, the current rule is well supported by the data than any other rule.
\[
\frac{1 + i_t}{1 + i} = \left(1 + \frac{\pi_{t+1}}{1 + \pi}ight)^{\varphi_i} \left(1 + \frac{\pi_t}{1 + \pi}ight)^{(1 - \varphi_i)\varphi_y} \left(\frac{\pi_t}{\pi_{t-1}}\right)^{(1 - \varphi_i)\varphi_y} \exp \zeta_{m,t}
\]

where \(\varphi_i\) measures degree of interest rate smoothing by the monetary authority and \(\varphi_y\) measures relative importance given to inflation deviation from the implicit target, while \(\varphi_y\) measures weights on output deviation from the potential output. Parameter \(\varphi_e\) measures the relative weight on rate of depreciation in nominal exchange rate. In addition to inflation and output deviations, role of deviations in exchange rate also has been highly emphasized in monetary policy rules for less-advanced economies in the literature. This is because in most of these countries direct controls on exchange rate in the form of managed floating or central bank interventions in the foreign exchange markets are still in operation. However, this kind of direct controls are over and above the usual indirect controls through CPI. Therefore the assigned weights to exchange rate deviation are relatively lower than that of inflation and output deviations (see Mohanty and Klau (2004), Anand et al. (2010), Batini (2009)). Variable \(\zeta_{m,t}\) is defined as monetary policy shock that captures unanticipated changes in interest rate.

**Foreign sector**

Real exchange rate linking relative prices of foreign, \(P_t^*\), and domestic economy, \(P_{C,t}\), is given below:

\[
RER_t = \frac{\varepsilon_t P_t^*}{P_{C,t}}
\]

It is assumed for simplicity that pass-through of international oil price to domestic oil price is complete and the law of one price holds for oil prices. The domestic currency price of oil is therefore given as follows.

\[
P_{O,t} = \varepsilon_t P_{O,t}^*
\]

The relationship between foreign price index, \(P_t^*\), and price of imported goods, \(P_{F,t}^*\), is not one to one, but subject to a shock to the price of imports given by \(\zeta_{F,t}^*\).

\[
P_{F,t}^* = P_t^* \zeta_{F,t}^*
\]

This shock includes changes in relative productivity across sectors in the foreign economy.

Export demand for *home* good is given as follows:
\[ Y_{H,t}^* = \zeta^* \left( \frac{P_{H,t}}{P_t} \right)^{-\eta^*} Y_t^* \]

where \( \zeta^* \) is the share of home goods in the consumption basket of foreign agents and \( \eta^* \) price elasticity of foreign demand.

Foreign variables, such as foreign inflation, foreign output and foreign interest rate, are modeled as AR(1) processes for simplicity.

**General equilibrium**

General equilibrium is achieved when all the markets are in equilibrium. General equilibrium condition in the goods market is as follows:

\[ Y_{H,t}(Z_H) = \left( \frac{P_{H,t}(Z_H)}{P_{H,t}} \right)^{-\epsilon_H} Y_{H,t} + \left( \frac{P_{H,t}(Z_H)}{P_{H,t}} \right)^{-\epsilon_H} Y_{H,t}^* \]

where \( Y_{H,t} = C_{H,t} + I_{H,t} + G_{H,t} \).

Once prices and wages are set in the domestic market the quantity demanded of goods and labour have to be supplied at those prices. Equilibrium in the labor market implies that the labor demanded by intermediate varieties producers should be equal to labor supply.

On the external side, current account is assumed to be equal to the capital account since foreign reserves accumulation by the central bank is not included in the model. Evolution of net foreign asset accumulation considering all market equilibrium conditions and budget constraints of households and government is as follows:

\[ \frac{b_t}{(1+i_t)(\frac{b_{t-1}}{P_{Y,t}Y_t})} = \frac{\epsilon_{t-1} Y_{t-1}}{P_{Y,t}Y_t} + \frac{P_{X,t}X_t}{P_{Y,t}Y_t} - \frac{P_{M,t}M_t}{P_{Y,t}Y_t} + \frac{\epsilon_t \xi_t}{P_{Y,t}Y_t} \]

where \( P_{Y,t}Y_t \) is the nominal GDP defined as:

\[ P_{Y,t}Y_t = P_{C,t}C_t + P_{H,t}G_t + P_{I,t}I_t + P_{X,t}X_t - P_{M,t}M_t, \]

of which

\[ P_{X,t}X_t = \epsilon_t P_{H,t}^* Y_{H,t}^*, \]

\[ P_{M,t}M_t = \epsilon_t (P_{F,t}^* Y_{F,t} + P_{O,t}^* (C_{O,t} + O_{H,t})) \]
where \( C_{O,t} \) is the domestic consumption of oil and \( O_{H,t} \) is the oil usage in production. Import includes both foreign goods imports and oil imports.

Worker remittances in domestic currency, \( \varepsilon_{l,t} \), also increases net foreign asset accumulation.

Aggregate resource constraint for \( Y_{F,t} \) is

\[
Y_{F,t} = C_{F,t} + I_{F,t}
\]

assuming that the government consumes only home goods.

5. Model estimation

Bayesian approach

Bayesian approach to estimate DSGE model has become increasingly popular in DSGE based research because of its certain superiority over the other competing approaches. Allowing inclusion of priors helps to avoid posterior distribution moving to and peaking at unacceptable region. Bayesian estimation is broad based that fits the entirely solved DSGE model as opposed to particular equilibrium relationship based on Generalized Methods of Moments (GMM) estimation. Further, it is a full information approach and estimation is based on likelihood generated by the solution of the DSGE model.

Procedures of Bayesian estimation technique used in the estimation of this study can be simplified to the following 4 major steps.

1. Set the priors for parameters to be estimated.
2. Detrend and log-linearize model equations and solve the model by obtaining state space representation.
3. Use Metropolis-Hastings (MH) algorithm to take draws to simulate the posterior distribution to be used for inference. Convergence of MH iterations is checked.
4. Compute marginal likelihood of the model.
Description of priors for Step 1 is provided under subsection priors for estimated parameters. Detailed explanation for Steps 2, 3 and 4 are given below.

In order to make model variables stationary, prior to log-linearization, detrending of the model is performed by dividing real variables by deterministic trend in technology and dividing nominal variables by consumption price index. Detrended model is then log-linearized around the steady state. Log-linearised version of the full DSGE model that is given in Appendix A can be represented in a linear rational expectation system as follows as stated in Lubik and Schorfheide (2005).

\[ \Gamma_0(\theta)s_t = \Gamma_1(\theta)s_{t-1} + \Gamma_\epsilon(\theta)\epsilon_t + \Gamma_\eta(\theta)\eta_t \]

where \( s_t \) is the vector of model variables such as \( \hat{c}_t, \hat{k}_t, \hat{l}_t \) that are expressed as log-deviations from steady states and \( \epsilon_t \) is the vector of innovations to exogenous processes. Vector \( \eta_t \) contains rational expectations forecast errors \((\eta_t = s_t - E_{t-1}(s_t))\). Matrices \( \Gamma_i \) consists of non-linear functions of structural parameters contained in vector \( \theta \). The above system can be solved up to first order and expressed in state space form as follows:

\[ s_t = \Omega_s(\theta)s_{t-1} + \Omega_\epsilon(\theta)\epsilon \]

here \( \Omega_s \) and \( \Omega_\epsilon \) are functions of structural parameters.

Let \( y_t \) is the vector of observable variables. Measurement equation that links model variables to the observable variables is given as:

\[ y_t = Bs_t \]

Matrix \( B \) selects elements from \( s_t \) and does not depend on \( \theta \).

In this model \( y_t \) consists of following 9 observable variables. Measurement equation is expressed as follows.
\[
\begin{bmatrix}
\Delta \log (Y_t) \\
Int_t \\
\pi_t \\
\Delta \log (Exr_t) \\
\Delta \log (Con_t) \\
\Delta \log (Inv_t) \\
CA_t/Y_t \\
Oilp_t \\
rem_t
\end{bmatrix}
= \begin{bmatrix}
y_t - y_{t-1} + g_y \\
\dot{y}_t \\
\pi_t - \bar{\pi} \\
e_t - e_{t-1} \\
c_t - c_{t-1} + g_y \\
inv_t - inv_{t-1} + g_y \\
ca_y_t \\
pr^*_{o,t} - pr^*_o,t-1 \\
\Xi_t
\end{bmatrix}
\]

All the trending variables are observed as first differences in logs. Growth rates of output, \(\Delta \log (Y_t)\), consumption, \(\Delta \log (Con_t)\), investment, \(\Delta \log (Inv_t)\), are linked to the growth rate of model variables with the deterministic trend in labor productivity. Exchange rate is the rate of depreciation, \(\Delta \log (Exr_t)\), demeaned. International oil price, \(Oilp_t\), has shown an upward trend, so that observed as oil price inflation, demeaned. Quarterly inflation, \(\pi_t\), is observed as a deviation from implicit target for inflation, that is assumed to be equal to 5% (annually)\(^{12}\). Interest rate, \(Int_t\), is the nominal quarterly interest rate, demeaned. Current account to GDP, \(CA_t/Y_t\), is observed as a ratio, demeaned. Worker remittances, \(rem_t\), is observed as remittances in US dollar (detrended). Though hours worked or employment data is generally observed as important data set to study business cycles, it could not be included in this study\(^{13}\). Detailed description of data and transformation is given in Appendix A.

Let \(Y^T \equiv \{y_1, \ldots, y_T\}\) that contains observations until period \(T\). State-space representation of the model can be used to compute conditional likelihood function, using Kalman filter under the assumption of normally distributed white noise exogenous shocks. The Bayesian estimation combines the prior density, \(p(\theta)\), and likelihood function, \(p(Y^T | \theta)\), to find the posterior density, \(p(\theta | Y^T)\), using Bayes’ theorem. Posterior density is the density of parameters with the knowledge of data. Posterior density is given by:

\(^{12}\)Perera and Jayawickrema (2014) suggest that the implicit inflation target is 5% annually based on several central bank policy documents and reports.

\(^{13}\)Consistent labor force data was not available throughout the sample period due to labor force surveys conducted excluding North and East (N&E) provinces in early years of the sample because of the civil war.
Next, mode of the posterior density is approximated using a numerical algorithm. This mode is then used as the starting value of Metropolis-Hastings algorithm (MH) to simulate posterior distribution. Using this algorithm posterior mean and confidence intervals of the parameters are derived from generated draws.

The MH algorithm is briefly explained as follows (based on Griffoli, 2013).

1. Select a starting value $\theta^0$, i.e. posterior mode.

2. Draw $\theta^*$ from joint distribution $j(\theta^*/\theta^{t-1}) \sim N(\theta^{t-1}, \Sigma_m)$, where $\theta^{t-1}$ is the mean and $\Sigma_m$ is the inverse of the Hessian computed at the posterior mode.

3. Acceptance ratio is computed as follows:

\[
r = \frac{p(\theta^* | Y^T)}{p(\theta^{t-1} | Y^T)} = \frac{K(\theta^* | Y^T)}{K(\theta^{t-1} | Y^T)}
\]

4. Acceptance or rejection rule for the draw $\theta^*$ is given below:

\[
\theta^t = \begin{cases} 
\theta^* & \text{with probability } \min(r, 1) \\
\theta^{t-1} & \text{otherwise}
\end{cases}
\]

when a candidate draw is accepted the draw is kept and the mean of the posterior is updated. Otherwise, previous draw is kept. This acceptance rule is to ensure that the draws are taken from entire domain of the posterior distribution and the draws are not too quickly accepted or rejected. The proportion of accepted draws over the total number of draws, known as Acceptance rate, largely depends on the scale factor. If the scale factor is very small the acceptance rate will be high and vice versa. The acceptance rate is expected to be within 20%-40% and the scale factor is adjusted in the estimation to bring the acceptance rate within this range.

5. Steps 2-4 are repeated large number of times until convergence.

Calibrated parameters

This model has a large number of parameters. It is hard to find appropriate priors for these parameters and estimate them based on a short sample of data. In order to deal
with this problem, a set of parameters have been calibrated and the rest are estimated.

Calibrated parameters have been chosen either based on Sri Lankan data or in the absence of availability the parameter values have been chosen from other studies based on other developing and emerging economies. Intertemporal discount factor $\beta$ has been calculated based on average quarterly real interest rate in Sri Lanka during the sample period. Since Sri Lanka is relatively a high inflation country the average quarterly real interest rate is relatively lower at 0.4% despite of high nominal interest rate. The resulting estimate of beta is 0.995. Steady state labour productivity growth $g_y$, is calculated to be 4.1% (annually). The labour productivity is calculated by dividing real GDP by employed population to be consistent with the practice adopted at Central bank of Sri Lanka. This rate is also consistent with 5.6% annual growth in real GDP and 1.5% annual growth in employed population during the sample period. Steady-state inflation target $\pi^*$ is set to be 5% on annual basis. Assuming a lower rate of 5% as inflation target as opposed to the average inflation of 9% has a valid reason. The sample covers the past years with unusually high inflation, especially in 2008. However, the inflation has moderated in the recent years. It is not rational to expect that the monetary authority will wait to respond to inflation until it goes above the historically high average inflation. Moreover, in the absence of explicit inflation target in Sri Lanka, this rate is also an implicit target assumed in various official documents of the Central Bank of Sri Lanka in the recent times. Another study on the monetary policy rule for Sri Lanka also assumed this rate as the inflation target (Perera and Jayawickrema, 2013).

Share of oil in consumption basket is assumed to be 6%, to be largely consistent with relative direct weight given on oil and energy related products in Colombo Consumer Price Index (2006/2007=100), which is the official consumer price index in Sri Lanka. Share of oil as a factor of production in the production function is hard to calculate since micro level data on production function is not available. Average share of oil usage in industry and power generation during the sample period is around 40% of the total oil imports. Given the fact that the average oil imports as a percentage of GDP in the sample period is around 10%, that gives an approximation of 4% of oil usage for production. This also confirms the remaining share of 6% for consumption. According to this the share of core consumption in the consumption basket, $\alpha_C$, is 94%. Share of home goods in core consumption basket, $\gamma_C$, is assumed to be 65%, consistent with average expenditure on imported goods of 35% according to consumer price index. In
the absence of accurate data on the share of home goods in investment basket, $\gamma_I$, it is assumed that share of foreign investment made through Board of Investments (BOI) as a share of total investment is the proxy for share of foreign goods in investment basket that is 58% in the sample period. Thus, share of home goods in the investment basket will be 42%. After accounting for the use of oil as factor of production, the share of labor and capital in production of home good, $\alpha_H$, is assumed to be 96% . Referring to Duma (2007), who suggests that share of capital in production in most of the Asian countries is around 35%, share of labor in value added production of home goods, $\eta_H$, is set as 65%.

The share of credit-constrained household, $\lambda$, is hard to assume. There is no exact measurement of this parameter for Sri Lanka. It can be conveniently assumed that credit-constrained household will not have access to formal credit channel and therefore should depend on informal sources. A widely quoted micro-level based survey for Sri Lanka by Tilakaratna (2005) revealed that 32% of the household in the total sample borrowed from informal sources. Also, first 3 deciles of households receive very low income in the income distribution. Taking these facts this study assumes that 30% ($\lambda = 0.3$) of household in the economy are non-Ricardian. More than 70% of Sri Lankan migrant workers are unskilled and represent poor and middle income segment of the population. IPS (2013) confirmed that majority of remittance receiving households are in the poorest segment. Considering these findings, it is assumed that non-Ricardian households receive 60% of the worker remittances ($\tau = 0.6$).

Further, relative shares of some aggregates on GDP were calculated based on their annual averages in the sample period. Accordingly, private consumption to GDP, $\frac{P_{OC}}{P_{GY}}$, is calculated as 70% while import to GDP ratio, $\frac{P_{IM}}{P_{GY}}$, is 40% and net export to GDP ratio, $\frac{P_{X}-P_{M}}{P_{GY}}$, is -9%. Investment to GDP ratio, $\frac{P_{I}}{P_{GY}}$, calculated to be 22%. Current account balance as a percentage of GDP, $\frac{CA}{P_{GY}}$ is -3.9%. Share of oil on total imports, $\frac{P_{OM}}{P_{YM}}$, is calculated to be 15% and worker remittances to GDP ratio, $\frac{P_{R}}{P_{GY}}$, is 8%. Depreciation is assumed to be 12% annually to be consistent with other developing country based studies. Estimates of elasticities of labor varieties and consumption goods are rarely available through micro-based studies in emerging and developing economies. Elasticity of substitution among labor varieties, $\epsilon_L$, elasticity of substitution among home goods varieties, $\epsilon_H$, elasticity of substitution among imported foreign goods varieties, $\epsilon_F$, are assumed to be 6 as per Choudhri and Malik (2012) . List of calibrated parameters are listed in Appendix Table A.1.
Priors for estimated parameters

Inverse of labour supply elasticity, $\sigma_L$, estimate was not available for Sri Lanka. In order to be consistent with the value assigned in many other studies (e.g. Smets and Wouters, 2003) prior mean for inverse of labour supply elasticity, $\sigma_L$, is assumed to be 2, with gamma distribution. Existing literature on emerging market has documented a lower habit persistence in consumption, $h$, in emerging and developing economies than that of advanced economies (see Haider and Khan (2008), Anand et al. (2010)). According to this evidence a lower value of 0.5 has been assigned to this parameter. It is common in DSGE literature to assign price stickiness parameter a value of 0.75, assuming an average price adjustment period of 4 quarters. The same Calvo probability of 0.75 has been assigned to domestic prices of home goods, $\phi_{HD}$, foreign prices of home goods, $\phi_{HF}$, domestic prices of foreign goods, $\phi_F$, and nominal wages, $\phi_L$. Calvo probabilities follow beta distribution. Intratemporal elasticity in consumption, $\eta_C$, and intratemporal elasticity in investment, $\eta_I$, are assumed to be 1.0 with a large standard deviation to account for uncertainty of the parameter values. Elasticity of substitution of oil in the consumption basket, $\omega_C$, and in investment, $\omega_I$, also have been assigned 0.3 again with a broader standard deviation. This lower value of substitution is to insist imperfect substitutability of oil both in consumption and production. Coefficient for investment inertia is assumed to be 2.0 as given in Medina and Soto (2007) and Gabriel et al. (2010). All elasticities of substitution and intratemporal elasticities are assumed to follow inverse gamma distribution.

Monetary policy parameters are largely consistent with values assigned in the literature. Given this, the prior mean of interest rate smoothing coefficient, $\varphi_i$, is set to be 0.75. Parameter value for reaction to inflation deviation from target, $\varphi_p$, is assumed to be 1.5, and reaction to output gap, $\varphi_y$, is assumed to be 0.5, while $\varphi_e$ is set as 0.2. Reaction coefficients of inflation, output growth and exchange rate have been limited to positive values, so that gamma distribution is assumed. Persistence parameters are assumed to be 0.7 for all the shock and the standard deviation of all the shocks is assumed to be 1%.
6. Estimation outcomes

Parameter estimation

Posterior mean and confidence interval of the structural parameters along with corresponding mean, probability distribution and standard deviation of prior are reported in Table 2. One million draws have been drawn and 50% of the draws were discarded to eliminate the influence of priors on the posterior estimation and remaining 500,000 draws have been used for inferences. Complying with acceptance rate between 20% and 40% as recommended in the literature (see Griffoli, 2013) the model has reported an acceptance rate of around 27% for both chains consisting of 250,000 draws each. Dynare version 4.4.3 has been used for the estimation.

Posterior mean of inverse of labor supply elasticity is estimated to be 2.6 indicating labour supply elasticity of 0.38. This indicates that 0.38% increase in labor supply for 1% increase in real wage and thus highlighting inelastic labor supply. Consumption reveals high degree of habit persistent as opposed to a lower value assigned in the prior. Calvo probabilities of labor, $\phi_L$, domestic prices of foreign goods, $\phi_F$, and foreign price of home goods, $\phi_{HF}$, are high and providing evidence that nominal wages are negotiated and re-optimized in more than one year and import prices and export prices are sticky and takes more than a year to adjust to a new level. However, domestic price of home goods, $\phi_{HD}$, are less sticky and adjusted within 2 quarters. Elasticity of substitution of oil is very low both in consumption and production indicating that oil can be hardly replaced with other goods in consumption and in production.

Regarding monetary policy related parameters, interest rate exhibit high degree of smoothing and confirms that monetary authority adjust nominal interest rate only gradually. Reaction of interest rate to changes in inflation is 1.3 and that of output is only 0.18. Lower reaction to output gap is not exceptional because some studies assign 0.125 as prior mean for output gap reaction after adjusting the weight of 0.5 to suit quarterly measured output gap. Surprisingly, the reaction coefficient to the depreciation of exchange rate is higher than that of output growth and inflation. This result could be partly validated since Central Bank of Sri Lanka intervene in the foreign exchange market at volatile times even though exchange rate regime is referred to as 'freely floating'. Lubik and Schorfheide (2007) argued that openness changes the structure of the economy and its reaction to monetary policy. Further, domestic business fluctuations likely to have considerable relative price component and the central
banks in the open economies might be interested in explicitly reacting to and smoothing exchange rate fluctuation. Sri Lanka being a small open economy with over 60% degree of openness higher reaction to exchange rate fluctuations is natural. Also, key objective of the Central Bank of Sri Lanka requires maintaining stability of internal and external value (exchange rate) of Sri Lankan rupee. This leads to a conclusion that monetary policy in Sri Lanka places high importance to the exchange rate movements.

Table 2: Estimation outcome of structural parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior</th>
<th>Posterior</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inverse of elasticity of labor supply</td>
<td>( \sigma_L )</td>
<td>2.0 1.0 Gamma</td>
</tr>
<tr>
<td>Habit persistent</td>
<td>( \hat{h} )</td>
<td>0.5 0.2 Beta</td>
</tr>
<tr>
<td>Intratemporal elasticity in consumption</td>
<td>( \eta_C )</td>
<td>1.0 0.25 Inv Gamma</td>
</tr>
<tr>
<td>Intratemporal elasticity in investment</td>
<td>( \eta_I )</td>
<td>1.0 0.25 Inv Gamma</td>
</tr>
<tr>
<td>Investment inertia</td>
<td>( \mu_S )</td>
<td>2.0 0.25 Inv Gamma</td>
</tr>
<tr>
<td>Calvo probability in wage</td>
<td>( \phi_L )</td>
<td>0.75 0.1 Beta</td>
</tr>
<tr>
<td>Calvo probability in domestic price of home good</td>
<td>( \phi_{HD} )</td>
<td>0.75 0.1 Beta</td>
</tr>
<tr>
<td>Calvo probability in foreign price of home good</td>
<td>( \phi_{HF} )</td>
<td>0.75 0.1 Beta</td>
</tr>
<tr>
<td>Calvo probability in domestic price of foreign good</td>
<td>( \phi_F )</td>
<td>0.75 0.1 Beta</td>
</tr>
<tr>
<td>Elasticity of substitution of oil in consumption</td>
<td>( \omega_C )</td>
<td>0.3 0.25 Inv Gamma</td>
</tr>
<tr>
<td>Elasticity of substitution of oil in production</td>
<td>( \omega_H )</td>
<td>0.3 0.25 Inv Gamma</td>
</tr>
<tr>
<td>Smoothing coefficient of interest rate</td>
<td>( \varphi_i )</td>
<td>0.75 0.1 Beta</td>
</tr>
<tr>
<td>Reaction to inflation deviation</td>
<td>( \varphi_{\pi} )</td>
<td>1.5 0.15 Gamma</td>
</tr>
<tr>
<td>Reaction to output gap</td>
<td>( \varphi_y )</td>
<td>0.5 0.15 Gamma</td>
</tr>
<tr>
<td>Reaction to EXR deviation</td>
<td>( \varphi_{e} )</td>
<td>0.2 0.15 Gamma</td>
</tr>
<tr>
<td>Intratemporal elasticity in foreign demand</td>
<td>( \eta^s )</td>
<td>1 0.25 Inv Gamma</td>
</tr>
<tr>
<td>Elasticity of external premium</td>
<td>( \theta )</td>
<td>0.01 0.25 Inv Gamma</td>
</tr>
</tbody>
</table>
Prior information and posterior estimates of the shocks are reported in Appendix Table A.2. Regarding the persistence of shocks, monetary policy, import price and remittance shocks are less persistent while oil price, productivity and foreign interest shocks are very persistent. On the other hand the volatility estimates of shocks reveal that shock to the consumption preference, oil price and import price are highly volatile.

Identification analysis performed along with the estimation satisfies that all the parameters of the model are identified with the observed variables, though strength of identification is relatively weaker for some of the parameter. Plots of prior distribution and posterior distribution with posterior mode are shown in Appendix Figures A.1 and A.2. Gray and black lines indicate prior and posterior distributions, respectively, while dotted line connects the posterior mode. These plots confirm that prior and posterior distribution are considerably different and the observed data has helped to update the knowledge about priors for many parameters except for a few. Those parameters, such as $\phi_L, \omega_H$ and $\eta^*$, are said to be only weakly identified.

Convergence diagnostic could also be used to validate the model. Convergence diagnostics test based on Brooks and Gelman (1998) have been carried out for both univariate and multivariate statistics. Both of these statistics have converged at one million draws. To be concise, only the convergence of multivariate statistic has been reported in Appendix Figures A.3.
Model validation

Robustness check

It is obvious from the discussion in Section 3.2 and the analysis carried out in the next chapter that GDP growth reported a large shift in 2001. Further, this is when exchange rate regime in Sri Lanka was changed from managed float to free float. Given that the model has reported high monetary policy response to exchange rate movement, it would be interesting to find out the model predictions when the sample covers a period after 2001. The outcome of the estimation using a sample with 2002-2014 has been reported in Appendix Table A.3.

As shown in the table, the results are fairly robust. However, it is worth discussing about two important differences. Firstly, the reported habit persistence is lower with this shorter sample. It is acceptable, since consumption was highly volatile in this period compared to the overall sample as shown in Figure 1. Next, monetary policy response to exchange rate movements have become lower when the sample covers only the periods of freely floating exchange rate. During this period, responses to inflation is higher than that of exchange rate movements.

Model fit

Model estimated based on Bayesian approach are validated by comparing the marginal likelihood of the estimated DSGE model with another version of the same model or Vector Autoregression (VAR) models estimated with the same data set. If the marginal log data density of the estimated DSGE model is greater than that of other competing models then DSGE model is considered as better model than other model. The DSGE model in this paper is validated against Bayesian VAR (BVAR) models estimated using Minnesota prior with the same data up to lag level 6. The marginal likelihoods of the BVAR models and DSGE model have been calculated by the modified harmonic mean algorithm proposed by Geweke(1998). Table 3 reports the outcome of this validation. According to the table, DSGE model outperforms all the versions of BVAR model as it has the highest marginal likelihood of the data.
Table 3: Marginal likelihood- DSGE vs VAR

<table>
<thead>
<tr>
<th>Model</th>
<th>Marginal Log Data Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>DSGE</td>
<td>1387.6</td>
</tr>
<tr>
<td>BVAR(1)</td>
<td>1122.4</td>
</tr>
<tr>
<td>BVAR(2)</td>
<td>1293.6</td>
</tr>
<tr>
<td>BVAR(3)</td>
<td>1276.6</td>
</tr>
<tr>
<td>BVAR(4)</td>
<td>1311.4</td>
</tr>
<tr>
<td>BVAR(5)</td>
<td>1287.8</td>
</tr>
<tr>
<td>BVAR(6)</td>
<td>1289.7</td>
</tr>
</tbody>
</table>

Another approach to validate the estimated model is by comparing second moments of the actual data and the moments generated by the model to see whether the model is able to reproduce the actual moments fairly well. Table 4 reports the second moments of the original data with the theoretical moments generated by the model using posterior distribution. Accordingly, the model has performed a fairly good job in reproducing standard deviation of most of the variables though the model slightly over estimates the volatility of investment. Also, the model fairly replicates cross correlation between other observables and output growth. The sign of cross correlation in actual data has been correctly reproduced by the model. The size of the cross correlation is also fairly consistent with the correlation found in actual data.

Table 4: Comparison of moments- Actual vs Model

<table>
<thead>
<tr>
<th></th>
<th>Volatility-Standard deviation(%)</th>
<th>Correlation with output growth (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Actual</td>
<td>Model</td>
</tr>
<tr>
<td>Output growth</td>
<td>0.9</td>
<td>1.3</td>
</tr>
<tr>
<td>Inflation</td>
<td>2.0</td>
<td>2.2</td>
</tr>
<tr>
<td>Interest rate</td>
<td>0.9</td>
<td>1.1</td>
</tr>
<tr>
<td>Exchange rate</td>
<td>2.4</td>
<td>3.2</td>
</tr>
<tr>
<td>Consumption growth</td>
<td>1.5</td>
<td>1.6</td>
</tr>
<tr>
<td>Investment growth</td>
<td>4.1</td>
<td>5.7</td>
</tr>
</tbody>
</table>
This model employs a large number of frictions and that makes the model more complex. Many frictions, such as habit persistence, price rigidities and wage rigidities are imposed in order to comply with popular DSGE models to capture dynamics of the actual data. Yet, the relevance of these frictions to Sri Lanka might be questionable. These frictions are important in the modeling only if the inclusion helps to improve marginal data density and to capture actual moments of the observed data. For this purpose the model is re-estimated after these frictions are cut-down completely one at a time. The moments and marginal log data density of these model variations have been reported in Table 5 along with the actual moments and that of the base model originally considered in this study. All these model variations have generated lower marginal data density than the base model. The model versions excluding price rigidity in home goods (\(\phi_{H_D} = 0\)) and excluding wage rigidities (\(\phi_{L} = 0\)) have reported marginal likelihood that are very close to the base model. Still, the second moments of these versions are not superior to the moments generated by the base model. This analysis therefore supports the fact that the complexities imposed in this model incorporating many frictions are really essential in replicating second moments of the actual data.

Table 5: Empirical importance of the frictions

<table>
<thead>
<tr>
<th></th>
<th>Actual</th>
<th>Base Model</th>
<th>(h = 0)</th>
<th>(\phi_{H_D} = 0)</th>
<th>(\phi_{H_F} = 0)</th>
<th>(\phi_{F} = 0)</th>
<th>(\phi_{L} = 0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marginal Log Data Density</td>
<td>1387.6</td>
<td>1321.6</td>
<td>1376.4</td>
<td>1339.9</td>
<td>1285.8</td>
<td>1365.7</td>
<td></td>
</tr>
<tr>
<td>Volatility</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output growth</td>
<td>0.9</td>
<td>1.3</td>
<td>1.7</td>
<td>1.3</td>
<td>1.4</td>
<td>2.2</td>
<td>1.1</td>
</tr>
<tr>
<td>Inflation</td>
<td>2.0</td>
<td>2.2</td>
<td>2.1</td>
<td>2.7</td>
<td>4.8</td>
<td>3.3</td>
<td>3.1</td>
</tr>
<tr>
<td>Interest rate</td>
<td>0.9</td>
<td>1.1</td>
<td>1.1</td>
<td>2.3</td>
<td>5.4</td>
<td>3.2</td>
<td>2.4</td>
</tr>
<tr>
<td>Exchange rate</td>
<td>2.4</td>
<td>3.2</td>
<td>3.3</td>
<td>3.4</td>
<td>5.8</td>
<td>4.2</td>
<td>3.7</td>
</tr>
<tr>
<td>Consumption growth</td>
<td>1.5</td>
<td>1.6</td>
<td>3.2</td>
<td>1.8</td>
<td>1.8</td>
<td>1.7</td>
<td>1.5</td>
</tr>
<tr>
<td>Investment growth</td>
<td>4.1</td>
<td>5.7</td>
<td>5.0</td>
<td>5.9</td>
<td>5.5</td>
<td>5.0</td>
<td>4.6</td>
</tr>
<tr>
<td>Correlation with output growth (%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inflation</td>
<td>-14.6</td>
<td>-20.8</td>
<td>-29.6</td>
<td>-32.4</td>
<td>-23.0</td>
<td>-15.5</td>
<td>-11.3</td>
</tr>
<tr>
<td>Interest rate</td>
<td>-40.3</td>
<td>-31.6</td>
<td>-24.0</td>
<td>-23.5</td>
<td>-11.7</td>
<td>-13.3</td>
<td>-11.0</td>
</tr>
<tr>
<td>Exchange rate</td>
<td>-35.0</td>
<td>-26.7</td>
<td>-19.3</td>
<td>-11.7</td>
<td>-11.8</td>
<td>-24</td>
<td>-10.0</td>
</tr>
<tr>
<td>Consumption growth</td>
<td>61.1</td>
<td>52.1</td>
<td>80.5</td>
<td>36.52</td>
<td>19.6</td>
<td>39.6</td>
<td>47.5</td>
</tr>
<tr>
<td>Investment growth</td>
<td>71.6</td>
<td>62.9</td>
<td>42.6</td>
<td>60.9</td>
<td>39.5</td>
<td>35.6</td>
<td>47.4</td>
</tr>
</tbody>
</table>
7. Application of the estimated model

The estimated model is used to study business cycles and identify the drivers of the cycles. This section includes historical decomposition analysis, forecast error variance decomposition and impulse responses.

Historical decomposition

Historical decomposition is to study the influence of the structural shocks on the explanation of historical movements of the observed data. Different shocks influence variables at different pace and direction and the sum of the impacts of all shocks should approximately correspond to the value of observed data at a time period. All 11 shocks are grouped into 5 types of shocks: domestic demand shocks, domestic supply shocks, monetary policy shocks, international oil price shock and other external shocks. Since this study has a special emphasis on the effect of international oil price on the economy it has been reported separately from other external shocks. Domestic demand shocks include consumption preference shock and government expenditure shock, while domestic supply shocks cover transitory productivity and shock to the investment adjustment cost. Monetary policy shock is a shock to the nominal interest rate. Other external
shocks include foreign inflation shock, foreign interest rate shock, foreign output shock, import price shock and a shock to worker remittances. Figures 3, 4 and 5 portray the historical movements of GDP growth, inflation and interest rate, respectively, along with the contributions of 5 classes of shocks.

According to Figure 3, negative GDP growth rates have been reported mainly at 3 different time periods, such as 1998-1999, 2001 and 2008-2009. The first and second recessions were mainly driven by domestic supply shocks, with non-negligible contribution from other external shocks. The recession during recent financial crisis is largely explained by the demand shocks as expected. Domestic supply shock also reports a negative movement during this time period. These findings are in-line with the claims of the Central Bank of Sri Lanka in its annual reports of the respective periods. The role of international oil price in explaining historical movements of output growth is not very prominent.

Figure 4: Historical decomposition of inflation

Figure 4 confirms that inflation has been highly volatile during many periods in the past, though it has moderated in the recent times. Inflation has been cyclical around the three recession periods discussed above. Unlike GDP growth, international
oil price had played a major role in explaining the historical movements of inflation in the past. Hyper inflation recorded during latter part of 2007 to early 2008 have been explained mainly by oil price, domestic supply shocks and other external shocks. The same interpretation is applicable to other high fluctuations in inflation recorded in the past. Sharp deceleration in inflation by the end of 2008 is fully explained by the unprecedented fall in international oil price. Monetary policy also has helped to explain the inflation when it was deviating from the steady states only marginally in the normal times. Moderation in inflation during the recent times after 2011 have been influenced by domestic demand shocks and oil price.

Interest rate movements around the steady state are persistent over 2 to 3 years unlike the historical movements of GDP growth and inflation (Figure 5). This confirms that policy rate are adjusted only gradually. Other external shocks are behind all large fluctuations around the mean, except the peak in interest rate during 2007 and 2008. That peak is influenced greatly by the domestic supply shock, while oil price and other external shocks have also contributed to that.

Figure 5: Historical decomposition of interest rate
Variance decomposition

Contribution of structural shocks to the conditional forecast error variance of the key observable variables at immediate period (quarter 1) and business cycle frequency (quarter 12) and long-term (quarter 40) are reported in Table 6 and discussed briefly in this subsection. As reported in the table domestic supply shocks and other external shocks explain major part of forecast error of output, while domestic supply shocks and oil price shock explain most part of the forecast error of inflation. Interest rate forecast error variance is explained by mainly by other external shocks in the immediate quarter. There is no notable differences in the outcome based on the timing of the decomposition except for the interest rate. Even though other external shocks explain most of the forecast error variance on interest rate in the immediate quarter that gradually declines in the medium term and long-term. Domestic demand shocks explain most part of the forecast error in the long-term.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Period</th>
<th>Supply</th>
<th>Demand</th>
<th>Monetary Policy</th>
<th>External Oil</th>
<th>Other External</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output growth</td>
<td>1</td>
<td>42%</td>
<td>12%</td>
<td>4%</td>
<td>2%</td>
<td>40%</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>43%</td>
<td>13%</td>
<td>4%</td>
<td>3%</td>
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</table>
Impulse responses

Responses of key macroeconomic variables to transitory productivity shock, monetary policy shock, oil price shock and shock to the remittances are discussed in detail in this part.

Contractionary monetary policy shock measured by an unexpected increase nominal interest rate leads to contraction in both consumption and investment as illustrated in Figure 6. Responses of both of these variables are hump shaped with the peak reductions reported around 3 to 4 quarters after the shock. These delayed responses can be explained by the habit persistence in consumption and the existence of investment adjustment cost. The combined effect of the contraction in consumption and investment, in other words aggregate demand, results in hump shaped decline in output. The increase in nominal interest rate helps to appreciate real exchange rate through an appreciation of nominal exchange rate\(^\text{14}\). The exchange rate appreciation also explains the contraction in output. Real appreciation makes foreign good cheaper than the home good and also exports of home good will be less competitive in the global market. As a result, domestic production will be reduced and the employment will fall. Further, inflation falls due to the appreciation of exchange rate. The effect of monetary policy on inflation is not persistence and inflation reaches the steady state level within 2 years. Given lower inflation and nominal rigidities in nominal wages this leads to an increase in real wages.

A positive transitory productivity shock reported positive responses on output, consumption and investment as shown in Figure 7. The increased labor productivity reduces marginal cost and as a result inflation declines as per the Phillips curve. Inflation response does not stay for longer and dies out by the end of one year. Nominal interest rate declines since monetary authority adopts an accommodative monetary policy and this policy stance continues relatively longer. Real exchange depreciates due to the combined effect of nominal exchange rate depreciation explained through uncovered interest parity and reduction in domestic prices. Real wage increases because of relatively unchanged nominal wages explained by nominal rigidities and fall in domestic prices. Expansion in aggregate demand and output should be expected to increase

\(^\text{14}\)Both nominal and real exchange rate have been modelled in such a way that a decline refers to an appreciation.
labour demand and employment. However, reported responses of both employment is on the opposite direction. This contradicting evidence is not new in the literature. Gali (1999), Francis and Ramey (2005) and Smets and Wouters (2007) and many other studies cited therein have reported similar findings for the USA and some other industrial countries. Francis and Ramey (2005) confirmed that the model with habit persistence and investment adjustment cost was able explain this phenomenon well. Existence of habit persistence and investment adjustment cost result in sluggish responses of consumption and investment in the short-term and household spend the new wealth on leisure. Smets and Wouters (2007) explained this response in sticky price model through the existence of nominal and real rigidities and comply with the findings of Francis and Ramey (2005). They also claimed that the productivity increase reduces fixed cost per unit so that less labour is required to produce same level of output. Medina and Soto (2007) have noted that when increase in aggregate demand and accommodative monetary policy are not strong enough employment will not increase. All these explanations are valid in Sri Lankan case as well.

Figure 6: Responses to monetary policy shock
Figure 7: Responses to transitory productivity shock

Figure 8 reports the responses to international oil price. An increase in international oil price has immediate impact on consumption by reducing it. Since oil is included in the consumption basket reduction in oil consumption reduces consumption. Oil can not be substituted with either home or foreign goods, since estimated elasticity of substitution of oil in consumption is only 0.18. Further, negative income effect reduces consumption of other goods as well. To overcome this the labour supply increases initially as shown by the response of employment. Oil is included as a factor of production also. Again elasticity of substitution of oil in production is only 0.22. Thus, oil price increase results in increasing marginal cost of domestic home goods production, so that inflation increases. Short-lived response of inflation is puzzling, though a part of this could be explained by the immediate monetary tightening by the central bank. Output contraction explains the reduction in investment. This output contraction reduces demand for labour. High inflation will reduce real wage. Monetary authority attempts to arrest the surge in inflation by increasing nominal interest rate. This explains the appreciation of real exchange rate reported in the figure.
Figure 8: Responses to international oil price shock

Responses of key macroeconomic variables to other shocks are reported in Appendix Figures A.4 to A.11.

The impact of remittances

Remittance inflows from migrant workers have long been an important source of foreign exchange earning and it has become the primary net exchange earner in the recent past. Remittances as a share of GDP has been 8% during the sample period. Consistent and significant share of migrant worker remittances serves as a cushion against widening trade deficits and mitigates current account deficits. As noted earlier in this study Sri Lanka receives the highest per capita remittance inflows in the South Asian region. The role played by the remittance inflows in developing economies, including Sri Lanka, in poverty alleviation and consumption smoothing is well accepted in the literature.

Figure 9 shows the historical movements of GDP, private consumption and remittances inflows, measured by year-on-year growth. This graph endorses the fact that remittances are procyclical in Sri Lankan context. The peak growth of remittances was recorded in 2005-2006 that was associated with post-tsunami recovery.
Figure 9: Movements of selected variables- year-on-year
Impulse responses of an unexpected increase in worker remittances is shown in Figure 10. An increase in remittances improves current account balances and results in appreciation of nominal and real exchange rates. These remittances immediately boost consumption of non-Ricardian households, while the consumption increase of Ricardian households are only gradual. Increase in consumption helps to expand aggregate demand and output. Monetary authority adopts a loose monetary policy to respond to this.

Second moments related to remittances are reported in Table 7. According to the table remittances are volatile compared to the GDP. However, there is evidence that...
remittance inflows exhibit less volatility compared to other volatile foreign exchange inflows such as official development aid, foreign direct investment and portfolio investment (Lueth and Ruiz-Arranz, 2007). The model predicts the volatility of the remittances and cross correlation with consumption quite accurately. Consumption growth is highly positively correlated with remittances and remittance inflows improve current account to GDP ratio as observed in the data and predicted by the model.

8. Conclusion and way forward

Conclusion

DSGE models that are currently being treated as one of the key macroeconomic modeling tools by advanced economies, are gradually being employed for emerging and developing economies. A few central banks in these economies have already developed their own DSGE models and use them for internal policy analysis. This study estimates a medium-scale small open economy DSGE model for Sri Lanka. Bayesian estimation technique has been employed in estimating a set of parameters, while some of the parameters and steady state relationships are calibrated. Several nominal and real rigidities of the standard DSGE models and some specific features of Sri Lankan economy are added in the modeling. The estimated model is then used to find out main driving forces of Sri Lankan business cycle fluctuations.

Findings related to the estimation of the model reveal the existence of habit persistence in consumption, presence of investment adjustment cost and nominal wage and price rigidities. Further, the estimated model is fairly successful in replicating the second moments of actual data. Moreover, the estimated DSGE model outperforms BVAR models in terms of higher marginal data density.

Preliminary analysis on the stylized facts of business cycles in Sri Lanka exhibits that output is fairly stabilized during the study period and trade balance, investment and consumption volatilities are much higher than the volatility of output. The application of the estimated model concludes that domestic supply shocks and external shocks, including the oil price shock explain major part of the historical movements of output growth and inflation, while other external shocks drives interest rate variations in Sri Lanka. Also, the role of international oil price is prominent in explaining Sri Lankan inflation. These findings comply with the common belief about the Sri Lankan economy.

\footnote{For example, Chile (MAS model), Thailand (BOTMM model), Indonesia (BISMA model).}
Impulse responses of variables to the key structural shocks are broadly economically interpretable and gives some insight about Sri Lankan economy.

**Future work**

This study is one of the first attempts to model Sri Lankan economy in a DSGE set up. So that not all the frictions that are applicable to less developed economies are incorporated in this study. If all those frictions and imperfections in the markets are included in a single study the model will not be tractable. A step-by step approach of adding more complexities over the time has been followed by some other studies on other economies in the region. For instance, Gabriel et al. (2010) developed a closed economy DSGE model with financial frictions and informal sector for India in order to simplify the analysis even though India is an open economy. Similarly, several parallel studies have been carried out for India and Pakistan by a number of researchers adding idiosyncratic features of their economies to the standard DSGE model\(^\text{16}\). Therefore the model applied in this study should not be treated as a complete model to fully represent real world economic characteristics of Sri Lanka and it should not be expected to answer all policy related questions. Rather, this study could be treated as the first step towards DSGE model based research agenda for Sri Lanka and introducing DSGE modeling in policy framework to Sri Lanka in the future. Several extensions to this version are worth considering for the future work. Firstly, incorporating a customized fiscal policy rule for Sri Lanka that not only includes the impacts of government expenditure but the sources of financing the chronic fiscal deficit is the essential extension. Secondly, Sri Lankan financial sector is dominated by banking sector. Incorporating banking sector and financial linkages with financial market frictions will enrich the model. Further, current monetary policy regime in Sri Lanka is monetary aggregate targeting framework. Thus, complete ignorance of the role of the money in the modeling is not fully justifiable unlike in advanced economy based models. Another possible way of extension includes extending the application of the already estimated model to answer several other policy related questions.

Appendix A: Model description and data

Steady state relationships

Following relative prices are normalized to 1 at the steady state:

\[ pr_I = pr_{HD} = pr_F = wr = 1 \]

Real price of investment and Tobin’s Q:

\[ pr_I = Qr \]

where \( Qr = \frac{Q}{P_C} \)

Rental rate of capital

\[ Zr = ((1 + r) - (1 - \delta))pr_I \]

where \( Zr = \frac{Z}{P_C} \)

Capital -Labour ratio is given as:

\[ \frac{K}{L} = (\frac{Wr}{Zr})^{\eta_H} \frac{(1-n_H)}{n_H} \]

where \( Wr = \frac{W}{P_C} \)

Production- Labour ratio is given by:

\[ \frac{Y_L}{L} = (Wr + Zr \frac{K}{L})^{\eta_H} \frac{\epsilon_H}{(\epsilon_H-1)} \]

Net foreign asset position:

\[ B^* \left( \frac{1}{(1+r)} \text{OB} - \frac{1}{(1+\pi^*)(1+n)(1+g_y)} \right) = \frac{P_X X}{P_Y Y} - \frac{P_M M}{P_Y Y} + \frac{REM}{P_Y Y} \]

Log-linearized model

Consumption and Labour

- Consumption of Ricardian household

\[ c^R_t = -\frac{1}{1+h}E_t[\hat{i}_t - \pi_{t+1}] + \frac{1}{1+h}E_t[\hat{c}^R_{t+1}] + \frac{h}{1+h}c^R_{t-1} + \frac{1}{1+h}[s_{C,t} - \ldots] \]
\[ E_t[\hat{C}_{t+1}] \]

- Consumption of non-Ricardian household

\[ c^R_t = \frac{W}{P^c} (w^R_t + \hat{l}_t) + \tau (\hat{r}r_t + \Xi_t) \]

- Overall consumption

\[ \hat{c}_t = (1 - \lambda)\hat{c}_t^R + \lambda \hat{c}^R_t \]

- Uncovered interest parity condition

\[ \hat{i}_t = \hat{i}_t^* + \beta \hat{b}_t + E_t[\Delta e_{t+1}] \]

- Labour supply

\[ [\kappa_L + (1 + \beta)\hat{w}_t] = \kappa_L \left( \sigma_L \hat{L}_t + \frac{1}{1-h} \hat{c}_t - \frac{h}{1-h} \hat{c}_{t-1} \right) + \hat{w}_t - \beta E_t[\hat{w}_{t+1}] - (1 + \beta)\hat{\pi}_{C,t} + \beta E_t[\hat{\pi}_{C,t+1}] \]

where \[ \kappa_L = \frac{(1 - \beta \phi_L)(1 - \phi_L)}{\phi_L (1 + \sigma_L \epsilon_L)} \]

- Consumption goods bundles

\[ \hat{c}_{Z,t} = \hat{c}_t - \omega_C \hat{r}_{Z,t} \]
\[ \hat{c}_{O,t} = \hat{c}_t - \omega_C \hat{r}_{O,t} \]
\[ \hat{c}_{H,t} = \hat{c}_{Z,t} - \eta_C \hat{r}_{H,t} \]
\[ \hat{c}_{F,t} = \hat{c}_{Z,t} - \eta_C \hat{r}_{F,t} \]
\[ \alpha_C \hat{r}_{Z,t} + (1 - \alpha_C) \hat{r}_{O,t} = 0 \]
\[ \gamma_C \hat{r}_{H,t} + (1 - \gamma_C) \hat{r}_{F,t} = \hat{r}_{Z,t} \]

Relative prices

\[ \hat{\pi}_{Z,t} = \hat{p}_{Z,t} - \hat{p}_{Z,t-1} + \hat{\pi}_{C,t} \]
\[ \hat{\pi}_{H,t} = \hat{p}_{H,t} - \hat{p}_{H,t-1} + \hat{\pi}_{C,t} \]
\[
\hat{\pi}_{H,F,t} = \hat{pr}_{H,F,t} - \hat{pr}_{H,F,t-1} + \hat{\pi}_t
\]
\[
\hat{\pi}_{F,t} = \hat{pr}_{F,t} - \hat{pr}_{F,t-1} + \hat{\pi}_{C,t}
\]
\[
\Delta \hat{e}_t = \hat{r}_{e,t} - \hat{r}_{e,t-1} + \hat{\pi}_{C,t} - \hat{\pi}_t
\]

**Investment**

- **Capital Accumulation**
  \[
  \hat{k}_{t+1} = \frac{1-\delta}{(1+n)(1+g_y)} \hat{k}_t + (1 - \frac{1-\delta}{(1+n)(1+g_y)})(\hat{inv}_t + \hat{\zeta}_{I,t})
  \]

- **Investment good bundle**
  \[
  \hat{inv}_{H,t} = \hat{inv}_t - \eta_H(\hat{pr}_{H,D,t} - \hat{pr}_{I,t})
  \]
  \[
  \hat{inv}_{F,t} = \hat{inv}_t - \eta_F(\hat{pr}_{F,t} - \hat{pr}_{I,t})
  \]
  \[
  \hat{pr}_{I,t} = \gamma_I \hat{pr}_{H,D,t} + (1 - \gamma_I) \hat{pr}_{F,t}
  \]

- **Supply and demand for investment goods (detrended and log-linearized)**
  \[
  \hat{pr}_{I,t} = \frac{\varphi_r}{\varphi_{pr}} \hat{q}_r + (\hat{\zeta}_{I,t}) - \frac{\varphi_r}{\varphi_{pr}} (1 + \frac{1}{1+r})(1+g_y)^2 \hat{inv}_t + \frac{\varphi_r}{\varphi_{pr}} \mu_S (1+g_y)^2 \hat{inv}_{I,t-1} + \frac{\varphi_r}{\varphi_{pr}} \mu_S (1+g_y) \hat{q}_r \hat{inv}_t
  \]
  \[
  \hat{q}_r = E_t[\hat{pi}_{C,t+1} - \hat{\iota}_t] + \frac{1}{1+r} \varphi_r E_t[\hat{pr}_{H,D,t}] + \frac{1}{1+r} \varphi_r E_t[\hat{pr}_{F,t}]
  \]

**Cost minimization and Inflation dynamics**

- **First order conditions for cost minimization and marginal cost**
  \[
  \frac{1}{\varphi_H} \hat{k}_t = \frac{1}{\varphi_H} \hat{l}_t = \frac{1}{\varphi_H} \hat{\zeta}_{T,t} - \hat{l}_t = \varphi_r \hat{q}_r
  \]
  \[
  \frac{1}{\varphi_H} \hat{q}_r = \frac{1}{\varphi_H} \hat{\iota}_t + \frac{1}{\varphi_r} \varphi_r E_t[\hat{pr}_{H_D,t}] + \frac{1}{\varphi_r} E_t[\hat{pr}_{H,F,t}] - \frac{1}{\varphi_H} \hat{\zeta}_{T,t} - \hat{l}_t
  \]
  \[
  \hat{w}_r = 0
  \]
  \[
  \hat{mcr}_{H,t} = \frac{Z_{rh}}{MC_{rH,Y_H}} (\hat{q}_r + \hat{k}_t) + \frac{Wr}{MC_{rH,Y_H}} (\hat{wr}_t + \hat{l}_t) + \frac{P{0}_{rH}}{MC_{rH,Y_H}} (\hat{pr}_{O,t} + \hat{o}_{H,t}) - \hat{y}_{H,t}
  \]

- **Phillips curve for home goods consumed domestically (detrended and log-linearized)**
  \[
  \hat{\pi}_{H,D,t} = \beta E_t[\hat{\pi}_{H,D,t+1}] + \kappa_{H,D}[\hat{mcr}_{H,t} - \hat{pr}_{H,D,t}]
  \]
where $\kappa_{HD} = \frac{(1-\beta \phi_{HD})(1-\phi_{HD})}{\phi_{HD}}$

- Phillips curve for exported home goods (detrended and log-linearized)

$$\pi_{H,t} = \beta E_t[\pi_{H,t+1}] + \kappa_{H_F}[\Delta r_H - \Delta \pi_{H,F,t}]$$

where $\kappa_{H_F} = \frac{(1-\beta \phi_{H_F})(1-\phi_{H_F})}{\phi_{H_F}}$

- Phillips curve for imported goods (detrended and log-linearized)

$$\pi_{F,t} = \beta E_t[\pi_{F,t+1}] + \kappa_{H_F}[\Delta \pi_{H,F,t} + \Delta \pi_{F,O,t}]$$

where $\kappa_{F} = \frac{(1-\beta \phi_{F})(1-\phi_{F})}{\phi_{F}}$

**Monetary Policy and Interest rate**

- Monetary policy feedback rule:

$$i_t = \varphi_i i_{t-1} + (1 - \varphi_i) \varphi \pi_{C,t} + (1 - \varphi_i) \varphi \pi_{F,t} + (1 - \varphi_i) \varphi \pi_{O,t} + (1 - \varphi_i) \varphi \pi_{M,t} + \phi_{t} + \phi_{t} \pi_{M,t} + \phi_{t} \pi_{O,t}$$

- Real interest rate (ex-post)

$$r_t = i_t - \pi_t$$

**Foreign Sector**

- Foreign demand for home goods

$$y_{H,t} = y_{t} - \pi_{H,F,t}$$

- Domestic price for the oil

$$\pi_{O,t} = \Delta r_H + \Delta \pi_{O,F,t}$$

- Balance of payments

$$\Delta \pi_{M,Y} - \Delta \pi_{X,Y} - \Delta \pi_{Y,Y}$$

- Real export, real imports and their deflators

$$x_t = \pi_{H,t}$$
\[ \hat{p}_{X,t} = \hat{p}_{H,t} + \hat{r} e_{t} \]

\[ \hat{m}_t = (1 - \gamma_C) \frac{P_L}{P_M} \hat{c}_{F,t} + (1 - \gamma_I) \frac{P_L}{P_M} \hat{i}_{F,t} + \frac{P_O(C_O+O_H)}{P_M} \left( \frac{C_O}{C_O+O_H} \hat{c}_{O,t} + \frac{O_H}{C_O+O_H} \hat{o}_{H,t} \right) \]

\[ \hat{p}_{M,t} = \hat{r} e_{t} + \left( 1 - \frac{P_O(C_O+O_H)}{P_M} \right) \hat{s}_{F,t} + \frac{P_O(C_O+O_H)}{P_M} \hat{p}_{O,t} \]

### Aggregate Equilibrium

- **Total demand for home goods**
  \[ \frac{P_H}{P_Y} y_{H,t} = \gamma_C \frac{P_L}{P_Y} \hat{c}_{H,t} + \frac{P_L}{P_Y} (g_t - \hat{p}_{H,D,t}) + \gamma_I \frac{P_I}{P_Y} \hat{i}_{H,t} + \frac{P_H}{P_Y} \hat{Y}_{H,t} \]

- **Total supply of home goods**
  \[ \hat{y}_{H,t} = \hat{a}_{H,t} + \alpha_H^{1/\omega_H} (A_H \frac{\hat{O}_H}{Y_H})^{\omega_H-1} \hat{o}_{H,t} + (1 - \alpha_H) \frac{1}{\omega_H} (A_H \frac{\hat{Y}_H}{Y_H})^{\omega_H-1} \hat{Y}_{H,t} + (1 - \alpha_H) \frac{1}{\omega_H} (A_H \frac{\hat{V}_H}{Y_H})^{\omega_H-1} (1 - \eta_H) (k_{t+1}) \]

- **Real GDP**
  \[ \frac{\hat{y}_t}{P_Y} = \frac{P_L}{P_Y} \hat{c}_t + \frac{P_L}{P_Y} (g_t - \hat{p}_{H,D,t}) + \frac{P_I}{P_Y} \hat{i}_{t} + \frac{P_X}{P_Y} \hat{x}_t - \frac{P_M}{P_Y} \hat{m}_t \]

### Exogenous processes

\[ \hat{\xi}_t = \rho_\xi \hat{\xi}_{t-1} + \epsilon_{\xi,t} \quad \epsilon_{\xi,t} \sim N(0, \sigma_\xi^2) \]

where \( \xi = a_H, \varsigma_m, \varsigma_C, \varsigma_I, \varsigma_F, g, y^*, i^*, \pi^*, p^*, O, \Xi \)

### Data description and transformation

Quarterly data for a sample period of 1996:Q2 to 2014:Q4 has been chosen for the estimation. The selection of sample is limited by the availability of quarterly GDP data series in Sri Lanka. Data includes 9 observable macroeconomic variables. Real GDP growth, CPI based inflation, 91-day Treasury bill rate, nominal US dollar exchange rate, real consumption growth and real investment growth are included since these are the key variables of the economy. In addition to these, international oil price, worker remittances and current account to GDP ratio are also included as the observables since these are related to the key external shocks of this study.  

\(^{17}\)Short samples are common for emerging and developing countries due to the limitations in obtaining consistent high frequency data.
Real GDP is seasonally adjusted using census x12 and expressed as first differences in logs. Inflation is measured by Colombo Consumer Price Index (CCPI, 2006-2007=100) and it is seasonally adjusted using census x12. Inflation is expressed as the deviation of quarterly inflation from the implicit target of 5% (annual), instead of demeaned series. The reason is that Sri Lanka experienced higher inflation during the past though it is moderated in the recent years. Hence, the average inflation during the sample period is high, around 9% annually. The Central Bank of Sri Lanka has been stressing in its policy documents as it aims at maintaining inflation around the mid-single rate. Therefore, observing inflation as demeaned series and expecting monetary authority to respond to inflation only when it is above the mean is not very relevant. Nominal exchange rate is expressed in terms of domestic currency per one unit of foreign currency (US dollar) \(^{18}\) and has been observed and expressed as log difference, demeaned. Therefore, as per the model definition an increase in exchange rate should reflect a depreciation of domestic currency. Treasury bill yield with 91-day maturity is the nominal interest rate that is observed as a demeaned series. Policy interest rate was not observed since it was not stationary.

Consumption and investment data are not available on quarterly basis in Sri Lanka. However, these two are important variables and generally observed in the literature. Also, the model has a few parameters related to these variables to be identified and two related shocks. Therefore, it is more appropriate to generate quarterly series by disaggregating annual data. Chow-Lin (1971) procedure has been chosen to interpolate annual real consumption and real investment. Quarterly real GDP has been used as indicative variable in deriving the interpolated series for consumption and investment. Rashid and Jehan (2013) have used Chow-Lin (1971) procedure to disaggregate annual investment and used CPI and industrial production index as indicator variables in a study for Pakistan. The interpolated series were then seasonally adjusted using census x12. Quarterly consumption and investment exhibit similar correlation with quarterly GDP data as observed in annual data. Both real consumption and real investment have been observed as first differences in logs.

International oil price is the actual price, denominated in dollars, paid by the Ceylon Petroleum Corporation when purchasing the oil in the international market. The nominal oil price in dollars is deflated based on CPI index of USA. The data is observed

\(^{18}\)Neither nominal effective exchange rate (NEER) nor real effective exchange rate (REER) were not observed since these data were not available throughout the sample period.
as real oil price in log difference. Current account to GDP has been observed as a ratio, demeaned. Worker remittance, denominated in US dollar and deflated to real remittances, is observed as HP detrended series. GDP, consumption, investment, CPI were obtained from Department of Census and Statistics of Sri Lanka and remaining data were from Central Bank of Sri Lanka.

Appendix B: Model estimation and outcome

Table A.1: Calibrated parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Subjective discount factor</td>
<td>0.995</td>
</tr>
<tr>
<td>$g_y$</td>
<td>Steady state productivity growth</td>
<td>4.1% (annual)</td>
</tr>
<tr>
<td>$\bar{\pi}$</td>
<td>Steady state inflation target</td>
<td>5% (annual)</td>
</tr>
<tr>
<td>$I/Y$</td>
<td>Investment/ GDP ratio</td>
<td>22%</td>
</tr>
<tr>
<td>$C/Y$</td>
<td>Consumption/ GDP ratio</td>
<td>70%</td>
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<tr>
<td>$M/Y$</td>
<td>Import/ GDP ratio</td>
<td>40%</td>
</tr>
<tr>
<td>$NX/Y$</td>
<td>Net export/ GDP ratio</td>
<td>-9%</td>
</tr>
<tr>
<td>$M_O/M$</td>
<td>Oil import/ Import ratio</td>
<td>15%</td>
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<tr>
<td>$CA/Y$</td>
<td>Current account/ GDP ratio</td>
<td>-3.9%</td>
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<tr>
<td>$\lambda$</td>
<td>Share of non-Ricardian household</td>
<td>30%</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Share of worker remittances received by non-Ricardian household</td>
<td>60%</td>
</tr>
<tr>
<td>$\alpha_C$</td>
<td>Share of core consumption in consumption basket</td>
<td>94%</td>
</tr>
<tr>
<td>$\gamma_C$</td>
<td>Share of home good in core consumption basket</td>
<td>65%</td>
</tr>
<tr>
<td>$\gamma_I$</td>
<td>Share of home good in investment</td>
<td>42%</td>
</tr>
<tr>
<td>$\alpha_H$</td>
<td>Share of combination of labor and capital in home production</td>
<td>96%</td>
</tr>
<tr>
<td>$\eta_H$</td>
<td>Share of labor in value added production</td>
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</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation rate of capital</td>
<td>12% (annual)</td>
</tr>
<tr>
<td>$\epsilon_L$</td>
<td>Elasticity of substitution among labor varieties</td>
<td>6</td>
</tr>
<tr>
<td>$\epsilon_H$</td>
<td>Elasticity of substitution among home goods varieties</td>
<td>6</td>
</tr>
<tr>
<td>$\epsilon_F$</td>
<td>Elasticity of substitution among imported goods varieties</td>
<td>6</td>
</tr>
<tr>
<td>Parameter</td>
<td>Prior</td>
<td>Posterior</td>
</tr>
<tr>
<td>-----------------------------------------------</td>
<td>----------------</td>
<td>------------------------------------------</td>
</tr>
<tr>
<td></td>
<td>Mean</td>
<td>S.D</td>
</tr>
<tr>
<td>Persistence of transitory productivity shock</td>
<td>ρ_{aH}</td>
<td>0.7</td>
</tr>
<tr>
<td>Persistence of investment adjustment cost shock</td>
<td>ρ_{sI}</td>
<td>0.7</td>
</tr>
<tr>
<td>Persistence of foreign output shock</td>
<td>ρ_{y*}</td>
<td>0.7</td>
</tr>
<tr>
<td>Persistence of foreign interest rate shock</td>
<td>ρ_{t*}</td>
<td>0.7</td>
</tr>
<tr>
<td>Persistence of foreign inflation shock</td>
<td>ρ_{π*}</td>
<td>0.7</td>
</tr>
<tr>
<td>Persistence of imported price shock</td>
<td>ρ_{F}</td>
<td>0.7</td>
</tr>
<tr>
<td>Persistence of international oil price shock</td>
<td>ρ_{prO}</td>
<td>0.7</td>
</tr>
<tr>
<td>Persistence of remittance shock</td>
<td>ρ_{Z}</td>
<td>0.7</td>
</tr>
<tr>
<td>Persistence of monetary policy shock</td>
<td>ρ_{SM}</td>
<td>0.7</td>
</tr>
<tr>
<td>Persistence of consumption preference shock</td>
<td>ρ_{CC}</td>
<td>0.7</td>
</tr>
<tr>
<td>Persistence of government expenditure shock</td>
<td>ρ_{g}</td>
<td>0.7</td>
</tr>
<tr>
<td>Std deviation of transitory productivity shock</td>
<td>σ_{aH}</td>
<td>0.01</td>
</tr>
<tr>
<td>Std deviation of investment adjustment cost shock</td>
<td>σ_{sI}</td>
<td>0.01</td>
</tr>
<tr>
<td>Std deviation of foreign output shock</td>
<td>σ_{y*}</td>
<td>0.01</td>
</tr>
<tr>
<td>Std deviation of foreign interest rate shock</td>
<td>σ_{t*}</td>
<td>0.01</td>
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<tr>
<td>Std deviation of foreign inflation shock</td>
<td>σ_{π*}</td>
<td>0.01</td>
</tr>
<tr>
<td>Std deviation of imported price shock</td>
<td>σ_{F}</td>
<td>0.01</td>
</tr>
<tr>
<td>Std deviation of international oil price shock</td>
<td>σ_{prO}</td>
<td>0.01</td>
</tr>
<tr>
<td>Std deviation of remittance shock</td>
<td>σ_{Z}</td>
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</tr>
<tr>
<td>Std deviation of monetary policy shock</td>
<td>σ_{SM}</td>
<td>0.01</td>
</tr>
<tr>
<td>Std deviation of consumption preference shock</td>
<td>σ_{CC}</td>
<td>0.01</td>
</tr>
<tr>
<td>Std deviation of government expenditure shock</td>
<td>σ_{g}</td>
<td>0.01</td>
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## Table A.3: Sensitivity of estimation outcome to structural break

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<th>Parameter</th>
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<th>Short sample (2008-2014)</th>
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<td>$\sigma_L$</td>
<td>2.64</td>
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<tr>
<td>$h$</td>
<td>0.93</td>
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</tr>
<tr>
<td>$\eta_C$</td>
<td>0.67</td>
<td>0.78</td>
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<tr>
<td>$\eta_I$</td>
<td>0.81</td>
<td>0.9</td>
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<tr>
<td>$\mu_S$</td>
<td>2.39</td>
<td>2.25</td>
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<tr>
<td>$\phi_L$</td>
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<td>0.63</td>
</tr>
<tr>
<td>$\phi_{HD}$</td>
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<td>0.43</td>
</tr>
<tr>
<td>$\phi_{HR}$</td>
<td>0.90</td>
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</tr>
<tr>
<td>$\phi_F$</td>
<td>0.74</td>
<td>0.72</td>
</tr>
<tr>
<td>$\omega_C$</td>
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<td>0.2</td>
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<tr>
<td>$\omega_H$</td>
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<td>0.24</td>
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<td>0.89</td>
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<tr>
<td>$\varphi_\pi$</td>
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<td>1.36</td>
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<tr>
<td>$\varphi_y$</td>
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</tr>
<tr>
<td>$\varphi_v$</td>
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<td>1.27</td>
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<tr>
<td>$\eta^*$</td>
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<td>0.89</td>
</tr>
<tr>
<td>$\varrho$</td>
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<td>0.005</td>
</tr>
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Figure A.1: Prior and posterior plot- parameters
Figure A.2: Prior and posterior plot- shocks
Figure A.3: Monte Carlo Markov Chain (MCMC) Multivariate diagnostics (Brooks and Gelman, 1998)
Appendix C: Impulse responses

Figure A.4: Responses to import price shock
Figure A.5: Responses to foreign interest rate shock

Figure A.6: Responses to foreign inflation shock
Figure A.7: Responses to foreign output shock

Figure A.8: Responses to investment adjustment cost shock
Figure A.9: Responses to consumption preference shock

Figure A.10: Responses to government expenditure shock
Bibliography


URL: http://dx.doi.org/10.2139/ssrn.2147341


URL: www.dynare.org/documentation-and-support/user guide


Senbeta (2011), ‘How applicable are the new keynesian DSGE models to a typical low-income economy?’, *Munich Personal RePEc Archive (MPRA) Paper* (31043).


