Optimal Monetary Policy and Welfare Analysis: a Case Study for Sri Lanka

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Abstract

This paper determines welfare maximising optimal monetary policy rules for Sri Lanka, based on an open economy New Keynesian DSGE model. I solve the model up to second order accuracy which facilitates welfare computation with alternative policy rules.

I consider a standard Taylor-rule type monetary policy reaction function where the short-term nominal interest rate responds to inflation, output and exchange rate. Welfare associated with the Ramsey policy is used as the benchmark for welfare comparisons. I determine optimal monetary policy rules such that the welfare associated with them are as same as that of the Ramsey optimal allocation, conditional on a particular state of the economy in the initial period. Unconditional expectation of welfare is also determined and compared as a robustness measure. The welfare cost of adopting alternative rules, instead of the optimal, are determined to evaluate the relative importance of the different policy rules.

The main findings are: First, the optimal monetary policy rule suggests an aggressive response to inflation and a moderate response to output-gap. Second, the optimal policy advocates a muted response to exchange rate fluctuations, and further, monetary policy reaction functions with positive response to exchange rate can lead to minor welfare losses even. Third, welfare gains from interest rate smoothing are significant. Fourth, the optimised monetary rules yield a level of welfare, very close to that of Ramsey optimal policy. Finally, the welfare losses associated with the current realised monetary policy rule for Sri Lanka can be mitigated significantly, by responding to inflation stronger.

JEL Classification: C6; E5; E6; I3

Key words: Dynamic Stochastic General Equilibrium (DSGE) Models, Monetary Policy, Welfare, Sri Lanka

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1 INTRODUCTION

The conventional wisdom as stated in statutory mandates of many central banks is that price stability and the full employment are the key objectives of monetary policy. Achieving these twin objectives leads to economic prosperity and better welfare of the general public, in the long run. Hence, determining the optimal monetary policy rules which yields highest lifetime-utility or welfare is an important question to be answered. Over the last three decades, there have been a lot of research efforts on the above concern. Evaluating policy rules with quadratic loss functions has been a popular approach, mainly attributable to its simplicity and transparency. These loss functions essentially minimize inflation and output deviations from their respective targets, with a given upper bound for interest rate variability[1]. The implied assumption (some what arbitrarily) in this approach that minimization of inflation and output variability is as same as maximizing welfare is, however, challenged by many, including Juillard et al. (2006) and Schmitt-Grohé and Uribe (2007). They further criticize the misleading prescriptions that results from the use of exogenous ad-hoc loss functions in welfare analysis, since the policy makers do not care about the welfare of the agents in such an environment. In the backdrop of these criticisms, the use of micro-founded loss functions consistent with the respective models, gradually became prevalent in deriving welfare maximising policy rules, from the beginning of 2000s[2].

Since the influential papers of Kydland and Prescott (1982) and King et al. (1988), approximating the solutions to nonlinear DSGE models using linear techniques started to flourish in communities of academic scholars, research groups and central banks. Consequently, many, including McGrattan (1994), Tesar (1995), Obstfeld and Rogoff (1998), Benigno and Benigno (2001) and Bils and Chang (2003), follow the above innovative idea and extend them to welfare analysis. These initial first-order linear approximation methods have been useful in analysing the dynamics of complicated non-linear process in several important aspects, particularly, such approximations are sufficient to check the local existence of determinacy of equilibrium and the variance of the endogenous variables, given that the shocks driving aggregate fluctuations are small[3]. Among others, Kollmann (2002) and Kim and Kim

[1]For details of initial efforts see for instance, Williams (1999)

[2]Among others, Kim and Kim (2003), Kollmann (2002) and Straub and Tchakarov (2004) started to use micro founded methods in formal welfare analysis.

[3]It is possible to assume directly that nonlinearities are themselves small in certain dimensions as a justification for use of

(2003), however, argue that the first-order approximations are not adequate to estimate welfare effects accurately. In their paper, Schmitt-Grohe and Uribe (2004) derive a second order approximation to the policy function of a general class of DSGE models[4] and stress the point that a correct second-order approximation of the welfare function needs a policy function approximated up to second-order accuracy.[5] In the second-order approximation, they use a perturbation method that includes a scale parameter for the standard deviation of the exogenous shocks, as a policy-function argument.[6] The discussion on approximation of equilibrium solutions to nonlinear rational expectations models continue to grow fruitfully, owing to remarkable contributions of many: Kim et al. (2008) and Lombardo and Sutherland (2007) introduce a algorithm for calculating second-order approximations to the solutions to nonlinear stochastic rational expectation models, Swanson et al. (2006) present an algorithm and software routines for computing n^{th} order Taylor series approximate solutions to dynamic, discrete time rational expectations models, Lan and Meyer-Gohde (2013) propose a nonlinear infinite moving average as an alternative to the standard state space policy function for solving nonlinear DSGE models and Johnston et al. (2014) present a new approach to the approximation of equilibrium solutions to nonlinear rational expectations models that applies to any order of approximation.

Schmitt-Grohé and Uribe (2007) computes welfare-maximizing monetary and fiscal policy rules in a real business model augmented with sticky prices, a demand for money, taxation and stochastic government consumption. They use perturbation methods, in determining welfare up to second-order accuracy. Ehelepola (2014) replicates it in the context of Sri Lanka with necessary modifications, particularly in parameters.

first-order approximations in these contexts. In this regards, Woodford (2002) is an example of making the necessary auxiliary assumptions explicit.

[4]There are several alternative algorithms of the second-order approximate solution method, developed separately. Some of them include Schmitt-Grohe and Uribe (2004), 'Perturbation AIM' by Federal Reserve Board staff (Swanson et al. (2006)), Judd (1998), Dynare codes developed by Juillard et al and the algorithm presented in Kim et al. (2008).

[5]In a related study, Collard and Juillard (2001) investigates the accuracy of a perturbation method in approximating the solution to stochastic equilibrium models under rational expectations. They found that second-order expansions are more efficient than standard linear approximation, as they can account for higher-order moments of the distribution which constitutes a major improvement of this stochastic approach to approximation, compared to other methods that assume certainty equivalence.

[6]The main theoretical contribution of Schmitt-Grohe and Uribe (2004) shows that for any model in the class of general models considered, the coefficients in the linear and quadratic terms in the state vector, with a second-order expansion of the policy rule are independent of the volatility of the exogenous shocks (i.e. the said coefficient are not different in deterministic and stochastic versions of the model.). Hence, the presence of uncertainty affects only the constant term of the decision rules, up to second order.

In the present study, I estimate the welfare maximising optimal policy rules for Sri Lanka in the context of a model closely related to the one specified in Ehelepola (2015). I use welfare derived from the Ramsey optimal policy as the benchmark reference point against which the welfare comparisons of the alternative policies are made. Ramsey plans implied by the optimality conditions of Lagrangian are, however, not directly observable to the policy maker and hence not implementable. Monetary policy reaction function where nominal interest rate usually act as the policy variable, on contrary, is implementable and have been shown to characterize the behaviour of monetary policy satisfactorily. Therefore, I estimate optimal policy rules which ensure welfare as close as possible to that associated with Ramsey optimal policy, by solving the model up to second order accuracy. Further, I compute the possible welfare losses of deviating from the said optimal policy, for few different cases. Finally, I attempt to asses the welfare loss associated with the current monetary policy rule for Sri Lanka and make recommendations as to minimise such losses, by way of proper adjustments to the policy rule coefficients. Welfare analysis for a developing country, with the above methodology is not available, to best of my knowledge, and the present study contributes to fill this gap.

The rest of the paper is organized as follows: in the Section 2, I setup the model and explain the optimization problem. Section 3 discusses the basis of welfare analysis with respect to the model, in the Sri Lankan context. In Section 4, I outline the main results and interpret them. Section 5 offers some concluding remarks.

2 MODEL SPECIFICATION

This section specifies the model in brief. I use a variant of the small open economy model in Ehelepola (2015), however, excluding fiscal variables while focussing on optimal monetary policy and welfare.[7] Accordingly, the structure of the model in this section characterised by heterogeneous agents, households, firms in an open economy environment.

It is also postulated in the paper that if intermediate-good firms are not able to adjust their prices optimally, then their prices will be adjusted (or indexed) fully according to the steady-state domestic inflation rate index. Fully indexing to the steady state inflation

[7]This model is closely related to Lubik and Schorfheide (2007), Del Negro and Schorfheide (2008) and Bhattarai et al. (2012) as well.

(in contrast to past inflation rate) will still result in a purely forward-looking NKPC.[8] Few other important changes in this model with non-linear equations, as opposed to the linearised model in Ehelepola (2015) are as follows:

- I set the *inverse of the inter temporal elasticity of substitution parameter* (σ) to unity, making the welfare analysis easier, without loss of generality.
- In the nonlinear equations below, I still maintain the terms of trade (q_t) as endogenous. In the linearized version of the model, however, it can make the simplifying assumption that q_t is exogenous, as in Del Negro and Schorfheide (2008), to ease the tight restrictions imposed by the DSGE model which is useful in estimating the model parameters.
- I assume, without loss of generality, that there is no trend growth in this model, as in a standard NK business cycle model. Accordingly, the exogenous productivity is now denoted by a_t (this is in line with the production function $y_t{=}a_tN_t$), which is assumed to follow a stationary AR(1) process. A shock to a_t is accordingly, interpreted as a temporary shock to the level of productivity.

The model specified below is a close variant of the model given in Ehelepola (2015) with the symbols denoting the same variables, however, subject to the above additional assumptions.[9] Accordingly, it features with the Calvo-type nominal price rigidities, complete international asset markets, perfectly competitive retailers, monopolistically competitive intermediate good producers, and a perfect exchange rate pass-through mechanism. The same Taylor-rule type monetary policy reaction function where the nominal interest rate responds to inflation, output and exchange rate is also assumed here. The solution methodology is briefly explained in the Appendix 6.

Households

The utility function, U_t, is given by,[10]

$$U_t = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right]$$

[8]This indexation assumption can be easily removed however by removing the steady state inflation in the optimal reset price and the PPI inflation equation.

[9]See Appendix 1 for details.

[10]When the *inverse of the inter temporal elasticity of substitution* is unity (i.e. $\sigma = 1$), I assume that the consumption function takes logarithmic form such that, $U(c_t, N_t) = \frac{log\left(c_t^{1-\sigma}\right)}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} = log\,(c_t) - \frac{N_t^{1+\varphi}}{1+\varphi}$.

The first order conditions of the utility function subject to the budget constraint, yield following relationships,

$$N_t^{\varphi} = c_t^{-\sigma} w_t \tag{1}$$

$$c_t^{-\sigma} = \beta E_t \left[R_t c_{t+1}^{-\sigma} \left(\pi_{t+1} \right)^{-1} \right] \tag{2}$$

$$0 = E_t \left[\left(R_t - R_t^* e_{t+1} \right) \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \left(\pi_{t+1} \right)^{-1} \right] \tag{3}$$

where, $\pi_t = P_t / P_{t-1}$ is the gross inflation rate and $e_t = \varepsilon_t / \varepsilon_{t-1}$ is the gross exchange rate depreciation (or appreciation).

Terms of trade (TOT) and real exchange rate

Terms of trade (q_t) is defined as the price of foreign good in terms of a unit of domestic good, as follows,[11]

$$q_t = P_{H,t} / P_{F,t}$$

The law of one price (LOP) holds for foreign goods,

$$P_{F,t} = \varepsilon_t P_{F,t}^*$$

where $P_{F,t}^*$ is the price of foreign good in the foreign country, in terms of foreign currency. The small open economy assumption imply that domestically produced goods have approximately zero weight in world consumption. Thus, $P_{F,t}^*$ equals to the foreign Consumer Price Index (CPI), P_t^*. Hence, it yields,

$$q_t = P_{H,t} / \left(\varepsilon_t P_t^* \right)$$

The real exchange rate, S_t, can now be defined as,

$$S_t = \varepsilon_t P_t^* / P_t \tag{4}$$

where P_t is the domestic CPI. From the above two relationships, it can be deduced that,

$$P_{H,t} / P_t = q_t S_t$$

A new variable, ν_t is defined such that,

$$\nu_t = P_{H,t} / P_t = q_t S_t$$

$$\nu_t = q_t S_t \tag{5}$$

The price index ratio, ν_t is stationary and now we can avoid possible non-stationariness of $P_{H,t}$ or P_t by rewriting them in terms of ν_t, as required.

[11] Domestic price of home produced goods and foreign produced goods are denoted as $P_{H,t}$ and $P_{F,t}$ respectively

Composite good

From the CPI price level equation, the relationship between TOT and real exchange rate as follows,

$$S_t = \left[(1-\alpha)\, q_t^{1-\eta} + \alpha\right]^{\frac{1}{\eta-1}} \tag{6}$$

The real marginal cost

The real marginal cost is given by,

$$mc_t = \frac{W_t}{P_{H,t}} = \frac{W_t}{P_t}\frac{P_t}{P_{H,t}} = w_t q_t^{-1} S_t^{-1}$$

Therefore,

$$mc_t = w_t q_t^{-1} S_t^{-1} \tag{7}$$

Domestic intermediate good

There are two important assumptions made on the price setting. Firstly, θ is used as the Calvo probability of price fixity and therefore, a fraction of firms, $\theta \in [0,1)$ is not allowed to adjust prices optimally. Secondly, it is assumed that the firms that are not allowed to adjust prices optimally will adjust their prices (or index) fully to the steady-state domestic inflation rate index. For simplicity, it is assumed that the steady-state domestic inflation rate is unity.

Optimal pricing condition is given by,

$$\sum_{k=0}^{\infty} (\beta\theta)^k E_t \left[C_{t+k}^{-\sigma} Y_{t+k} \frac{P_{H,t-k}}{P_{t+k}} \left(\frac{\tilde{P}_{H,t}}{P_{H,t-1}} - \frac{\epsilon}{\epsilon-1} \frac{P_{H,t+k}}{P_{H,t-1}} mc_{t+k} \right) \right] = 0$$

This can be represented in the recursive form as follows,

$$\tilde{p}_{H,t} = \frac{\epsilon}{\epsilon-1}\frac{K_{1,t}}{K_{2,t}} \tag{8}$$

where,

$$K_{1,t} = c_t^{-\sigma} y_t mc_t S_t q_t + \theta\beta E_t K_{1,t+1} \left(\pi_{H,t+1}\right)^{\epsilon} \tag{9}$$

and

$$K_{2,t} = c_t^{-\sigma} y_t S_t q_t + \theta\beta E_t K_{2,t+1} \left(\pi_{H,t+1}\right)^{(\epsilon-1)} \tag{10}$$

The aggregate price level (domestic)

In line with the Calvo price setting mechanism followed here, the dynamics of the domestic price index is given by,

$$P_{H,t} \equiv \left[\theta P_{H,t-1}^{1-\epsilon} + (1-\theta)\,\tilde{P}_{H,t}^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$$

By dividing the equation by $P_{H,t}$ to make the quantities stationary, and rearranging yields,

$$1 = \theta \left(\frac{P_{H,t-1}}{P_{H,t}}\right)^{1-\epsilon} + (1-\theta)\left(\frac{\tilde{P}_{H,t}}{P_{H,t}}\right)^{1-\epsilon}$$

This reduces to,

$$1 = \theta\,(\pi_{H,t})^{\epsilon-1} + (1-\theta)\,(\tilde{p}_{H,t})^{1-\epsilon} \tag{11}$$

where, $\tilde{p}_{H,t} = \frac{\tilde{P}_{H,t}}{P_{H,t}}$

The market clearing, aggregate production function and the rest of the world

World and domestic consumption relationship is given by,

$$c_t = \vartheta c_t^* S_t^{1/\sigma} \tag{12}$$

Market clearing condition for the domestically produced goods implies,

$$y_t = \vartheta c_t^* q_t^{-\eta}\left[(1-\alpha)\,S_t^{1/\sigma-\eta} + \alpha\right] \tag{13}$$

Small economy assumption: when $\vartheta \to 0$, we get,

$$c_t^* = y_t^* \tag{14}$$

Aggregate production function,

$$y_t = N_t \delta_t^{-1} \tag{15}$$

where, $\delta_t = \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon} di$, is a measure of relative price dispersion, which is equal to unity in a fully flexible price setting environment.

Evolution of the price dispersion variable

Dynamics of the price dispersion variable (δ_t) can be elaborated as follows,

$$\delta_t = \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon} di$$

$$\delta_t = \left[\frac{1}{P_{H,t}}\right]^{-\epsilon} \int_0^1 \left[P_{H,t}\left(i\right)\right]^{-\epsilon} di$$

$$\delta_t = \left[\frac{1}{P_{H,t}}\right]^{-\epsilon} \left[\left(1-\theta\right)\tilde{P}_{H,t}^{-\epsilon} + \theta \int_0^1 \left[P_{H,t-1}\left(i\right)\right]^{-\epsilon} di\right]$$

$$\delta_t = \left(1-\theta\right)\left(\frac{\tilde{P}_{H,t}}{P_{H,t}}\right)^{-\epsilon} + \theta\left[\frac{P_{H,t-1}}{P_{H,t}}\right]^{-\epsilon} \int_0^1 \left[\frac{P_{H,t-1}\left(i\right)}{P_{H,t-1}}\right]^{-\epsilon} di$$

which reduces to the following form,

$$\delta_t = \left(1-\theta\right)\tilde{p}_{H,t}^{-\epsilon} + \theta \pi_{H,t}^{\epsilon}\delta_{t-1} \tag{16}$$

Auxiliary equations:

Evolution of the price ratio variable (ν_t) can be derived as follows,

$$\nu_t \equiv \frac{P_{H,t}}{P_t} = \frac{P_{H,t}}{P_{H,t-1}}\frac{P_{H,t-1}}{P_{t-1}}\frac{P_{t-1}}{P_t}$$

$$\nu_t = \pi_{H,t}\nu_{t-1}\pi_t^{-1}$$

$$\pi_{H,t} = \frac{\nu_t}{\nu_{t-1}}\pi_t \tag{17}$$

The exchange rate growth rate (e_t) which is the exchange rate depreciation (or appreciation) is given by,

$$e_t = \frac{\varepsilon_t}{\varepsilon_{t-1}}$$

Now, by combining equation (4) above together with the above relationship, it gives,

$$\frac{S_t}{S_{t-1}} = e_t \frac{\pi_t^*}{\pi_t} \tag{18}$$

Therefore, equation (4) can now be replaced by the equation (18) since equation (4) is redundant otherwise.

Policy rule:

Following Clarida et al. (1998), Ireland (2000), Canova (2009), I use the modified Taylor type monetary policy rule given below,

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi}\right)^{\psi_\pi} \left(\frac{y_t}{y}\right)^{\psi_y} \left(\frac{e_t}{e_{t-1}}\right)^{\psi_e}\right]^{1-\rho_R} \varepsilon_t^R$$

which can equivalently be represented in the logarithm form as follows,

$$ln\left(\frac{R_t}{R}\right) = \rho_R\, ln\left(\frac{R_{t-1}}{R}\right) + \left(1-\rho_R\right)\left[\psi_\pi\, ln\left(\frac{\pi_t}{\pi}\right) + \psi_y\, ln\left(\frac{y_t}{y}\right) + \psi_e\, ln\left(\frac{e_t}{e_{t-1}}\right)\right] + ln\left(\varepsilon_t^R\right) \tag{19}$$

This essentially says that the monetary authority adjusts its policy instrument, the short-term nominal interest rate, in response to any deviation of the current gross inflation π_t and output y_t from their respective steady state values, or to a deviation of the current exchange rate from its value in the previous period. The coefficient parameters ψ_π, ψ_y, and ψ_e represent the strength of the responsiveness to their respective variables in the the policy rule while ρ_R denotes the extent of interest rate smoothing.

Exogenous processes:

There are four exogenous variables: productivity (a_t), world output (y_t^*), world inflation (π_t^*) and world nominal interest rate (R_t^*), which I assume to evolve as follows: Productivity shock,

$$ln(a_t) = \rho_a ln(a_{t-1}) + \varepsilon_t^a \tag{20}$$

World output shock,

$$ln(y_t^*/\bar{y}^*) = \rho_y ln(y_{t-1}^*/\bar{y}^*) + \varepsilon_t^{y^*} \tag{21}$$

World inflation shock,

$$(ln(\pi_t^*) - \gamma_{\pi^*}) = \rho_{\pi^*}\left(ln(\pi_{t-1}^*) - \gamma_{\pi^*}\right) + \varepsilon_t^{\pi_t^*} \tag{22}$$

World nominal interest rate shock

$$ln(R_t^*/\bar{R}^*) = \rho_{R_t^*} ln(R_{t-1}^*/\bar{R}^*) + \varepsilon_t^{R_t^*} \tag{23}$$

The nonlinear rational expectations system of equations given above are then solved with the Ramsey approach specified, for instance, in Khan et al. (2003), Schmitt-Grohé and Uribe (2007) and Johnston et al. 2014. Restrictions are derived on the optimal allocations and solve the system to second order approximation, as described in the Appendices-2,3,4,5 and 6.

3 Welfare Evaluation

3.1 Ramsey-Optimal Policy

At the equilibrium, the system of equations described above can be presented as follows,

$$E_0 \left\{ f_j \left(\mathbf{x_t}, \mathbf{y_t}, \mathbf{x_{t+1}}, \mathbf{y_{t+1}} \right) \right\} = 0 \tag{24}$$

where f_j is the equilibrium condition for $j = 1, ..., (n-1)$ and $\mathbf{x_t}$ is the vector of state of size $(n_x \times 1)$. Similarly, $\mathbf{y_t}$ is the vector of endogenous variables of size $(n_y \times 1)$, such that $n = n_x + n_y$. Thus, I have $n-1$ equilibrium conditions and n variables now and I use the monetary policy rule to close the system in the competitive equilibrium case, making both the number of equations and number of variables equal to n. The Ramsey optimal policy satisfies the system of equations and therefore it yields,

$$\mathbf{y}_t = g^* \left(\mathbf{x}_t, \sigma \right) \tag{25}$$

$$\mathbf{x}_{t+1} = h^* \left(\mathbf{x}_t, \sigma \right) + \eta \sigma \varepsilon_{t+1} \tag{26}$$

The Ramsey optimal policy is explained by the constrained efficient equilibrium (optimized), that can be obtained by maximizing lifetime utility of the representative household subject to the efficiency conditions of the other economic agents of the economy. This is determined by taking the first order conditions of a Lagrangian of the following form,

$$\mathfrak{L} = \overset{\min}{\{\Lambda_t\}_{t=0}^{\infty}}, \overset{\max}{\{d_t\}_{t=0}^{\infty}} \; E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \; E_t \left[U(\mathbf{x_t}, \mathbf{y_t}) + \sum_{j=1}^{n-1} \Omega_{j,t} f_j \left(\mathbf{x_t}, \mathbf{y_t}, \mathbf{x_{t+1}}, \mathbf{y_{t+1}} \right) \right] \right\} \tag{27}$$

where Λ_t and d_t denote the set of Lagrangian multipliers, and the set of endogenous variables, respectively. The period-by-period utility of the household is given by U. The system consists of n number of endogenous variables which, therefore, yields n first-order necessary conditions. Further there are $(n-1)$ Lagrangian multipliers, suggested by the $(n-1)$ efficiency conditions. Accordingly, I have $(2n-1)$ unknowns and the same number of equilibrium conditions to solve the system.

The Lagrange multipliers of the respective equations with forward looking variables are added to the vector of state variables, since these terms appear in the equilibrium conditions in the previous period (i.e. the period, $(t-1)$). The initial values of the Lagrangian multipliers are set to their respective steady state values, eliminating the difficulty of obtaining the value of the lagged Lagrangian multipliers in the initial period. The actual Lagrangian and its First Order Conditions (FOCs) are shown in the Appendices 2, 3 and 4.

3.2 Competitive Equilibrium

With the monetary policy rule, I can close the model with 17 endogenous variables and the same number of efficiency condition equations.[12] In equilibrium, at any time, $t \geqslant 0$, economic agents make decisions to maximize their objectives, given the monetary policy rule and exogenous shock process. These economic agents can access the whole information set for the period from $t = -1$ to $t = t$. Formally, competitive equilibrium in this model is a set of stationary processes of c_t, N_t, w_t, R_t, π_t, mc_t, y_t, δ_t, ν_t, $\tilde{p}_{H,t}$, $K_{1,t}$, $K_{2,t}$, e_t, q_t, S_t, $\pi_{H,t}$ and c_t^* for $t = 0, 1, ..., \infty$, such that, these processes satisfy the optimality conditions given by the sixteen equations above, given the monetary policy rule (equation 19), the initial conditions S_{-1}, e_{-1}, δ_{-1} , ν_{-1}, R_{-1} and the four exogenous stochastic process, z_t, y_t^*, π_t^*, R_t^*.

I attempt to determine the optimal coefficients for the monetary rule in the competitive equilibrium case, such that the welfare associated with the competitive equilibrium is very much closer to that associated with the Ramsey optimal allocation. Moreover, I calculate conditional[13] and unconditional welfare costs of allowing an alternative policy rule, instead of Ramsey policy. This enables me to rank and asses different alternative policy scenarios, under the competitive equilibrium.

3.3 Welfare Cost of implementing an alternative policy rule

The objective of the monetary policy rule specified in this paper is to maximise the lifetime utility of the households, conditional on the initial state of the economy.[14] It is vitally different from unconditional expectations of welfare, when it comes to ranking policy rules optimality. Unconditional welfare calculation does not take in to account the welfare effects of transitioning of the economy from the initial state (deterministic steady state in many cases) to any other stochastic steady state

Accordingly, I compute welfare costs associated with implementing an alternative policy

[12] Note that the foreign consumption (c_t^*) is endogenous in this setup, since the value is determined within the system.

[13] Conditional upon the initial state of the economy being the Ramsey optimal allocation.

[14] This innovative idea of conditional expectations of welfare calculation is pioneered by Lucas (1987), and Schmitt-Grohé and Uribe (2007) implement it with second order approximations.

regime, compared to the time invariant Rmasey policy. Welfare associated with the time-invariant equilibrium implied by the Rmasey policy, conditional on a particular initial state of the economy, is given by,

$$V_0^r = E_0 \sum_{t=0}^{\infty} \beta^t U\left(c_t^r, N_t^r\right) \tag{28}$$

where c_t^r and N_t^r denote contingent plans for consumption and labour hours associated with the Ramsey policy. From direct analogy, the conditional welfare cost related to an alternative police regime, $'a'$, can be denoted as,

$$V_0^a = E_0 \sum_{t=0}^{\infty} \beta^t U\left(c_t^a, N_t^a\right) \tag{29}$$

It is assumed that all state variables take their respective values at Ramsey steady state, when $t = 0$. Thus, the welfare cost of implementing an alternative policy $'a'$, instead of Ramsey policy $'r'$ is given by,

$$V_0^a = E_0 \sum_{t=0}^{\infty} \beta^t U\left(\left(1-\lambda^c\right) c_t^r, N_t^r\right) \tag{30}$$

where, λ^c is the fraction of Ramsey regime's consumption that a consumer would be willing to give up, to be as well off under the alternative regime, as under the Ramsey regime. Therefore, with the logarithmic utility function given above, V_0^a can be rewritten as follows,

$$V_0^a = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ log\left[\left(1-\lambda^c\right) c_t^r\right] - \frac{N_t^{r\,1+\varphi}}{1+\varphi} \right\}$$

or, equivalently,

$$V_0^a = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ log\left(c_t^r\right) - \frac{N_t^{r\,1+\varphi}}{1+\varphi} \right\} + E_0 \sum_{t=0}^{\infty} \beta^t log\left(1-\lambda^c\right)$$

This can be presented in the compact form,

$$V_0^a = V_0^r + \frac{log\left(1-\lambda^c\right)}{1-\beta}$$

Therefore the measure of welfare cost, λ^c can be expressed as,

$$\lambda^c = 1 - exp\left[\left(1-\beta\right)\left(V_0^a - V_0^r\right)\right]$$

I calculate V_0^a and V_0^r up to second order accuracy and hence restrict the approximation of λ^c up to second order, leaving the higher terms of order greater than two. Value of the welfare functions V_0^a and V_0^r depend on the initial state vector, x_0 and σ_ϵ which is the scaling parameter that scale the standard deviation of exogenous shocks. Hence, the welfare cost, conditional on the initial state is given by,

$$\lambda^c = 1 - exp\left[(1-\beta)\left(V^{ac}\left(x_0, \sigma_\epsilon\right) - V^{rc}\left(x_0, \sigma_\epsilon\right)\right)\right] \tag{31}$$

This can be expressed in the compact form as follows,

$$\lambda^c = \Lambda^c\left(x_0, \sigma_\epsilon\right)$$

where, x_0: initial state vector and σ_ϵ: scaling parameter that scale the standard deviation of exogenous shocks (as per Schmitt-Grohe and Uribe (2004)). Then, λ^c can be approximated up to the second order accuracy, as follows[15]

$$\lambda^c \approx \Lambda^c\left(x_0, 0\right) + \Lambda^c_{\sigma_\epsilon}\left(x_0, 0\right)\sigma_\epsilon + \frac{\Lambda^c_{\sigma_\epsilon\sigma_\epsilon}\left(x_0, 0\right)}{2}\sigma_\epsilon^2 \tag{32}$$

Since the deterministic steady state level of welfare is the same for the monetary policy functional form considered here, it implies that the value of λ^c is zero at the point where $(x_0, \sigma_\varepsilon) = (x, 0)$. Therefore,

$$\Lambda^c\left(x, 0\right) = 0. \tag{33}$$

Then I totally differentiating (32) above, w.r.t. σ_ϵ and evaluate it at $(x_0, \sigma_\epsilon) = (x_0, 0)$. Using the fact that the first derivatives of the policy functions w.r.t. σ_ε, evaluated at $(x_0, \sigma_\varepsilon) = (x, 0)$ are zero (i.e. $V^{ac}_{\sigma_\epsilon} = V^{rc}_{\sigma_\epsilon}$), it then implies that,

$$\Lambda^c_{\sigma_\epsilon}\left(x, 0\right) = 0. \tag{34}$$

By taking the second derivative of (32), w.r.t. σ_ϵ and evaluating it at $(x_0, \sigma_\epsilon) = (x_0, 0)$ gives,

$$\Lambda^c_{\sigma_\epsilon\sigma_\epsilon} = -\left(1-\beta\right)\left(V^{ac}_{\sigma_\epsilon\sigma_\epsilon} - V^{rc}_{\sigma_\epsilon\sigma_\epsilon}\right) \tag{35}$$

Therefore, by combining (32), (33), (34) and (35) it follows immediately that,

$$\lambda^c = \left(1-\beta\right)\left(V^{rc}_{\sigma_\epsilon\sigma_\epsilon} - V^{ac}_{\sigma_\epsilon\sigma_\epsilon}\right)\frac{\sigma_\epsilon^2}{2} \tag{36}$$

With direct analogy, an unconditional welfare cost measure (λ^u), up to second order accuracy can be derived as follows,

$$\lambda^u = \left(1-\beta\right)\left(V^{ru}_{\sigma_\epsilon\sigma_\epsilon} - V^{au}_{\sigma_\epsilon\sigma_\epsilon}\right)\frac{\sigma_\epsilon^2}{2} \tag{37}$$

[15] The Taylor expansion for λ^c, up to second order is given by, $\lambda^c = \Lambda^c\left(x_0 = x, \sigma = 0\right) \approx \Lambda^c\left(x_0, 0\right) + \Lambda^c_x\left(x_0, 0\right)\left(x - x\right) + \Lambda^c_{\sigma_\epsilon}\left(x_0, 0\right)\left(\sigma_\epsilon - 0\right) + \frac{\Lambda^c_{xx}(x_0,0)}{2}\left(x - x\right)^2 + \frac{\Lambda^c_{\sigma_\epsilon\sigma_\epsilon}(x_0,0)}{2}\left(\sigma_\epsilon - 0\right)^2$. It is, however, reduced to the above form since I consider a second-order approximation of the function Λ^c around the point $x_0 = x$ and $\sigma_\epsilon = 0$, where x is the deterministic Ramsey steady state of the state vector.

Accordingly, the equations (36) and (37) can be used to calculate the conditional and unconditional welfare cost of implementing an alternative monetary policy rule of the functional form given above, relative to that under Ramsey policy.

4 ANALYSIS AND INTERPRETATION

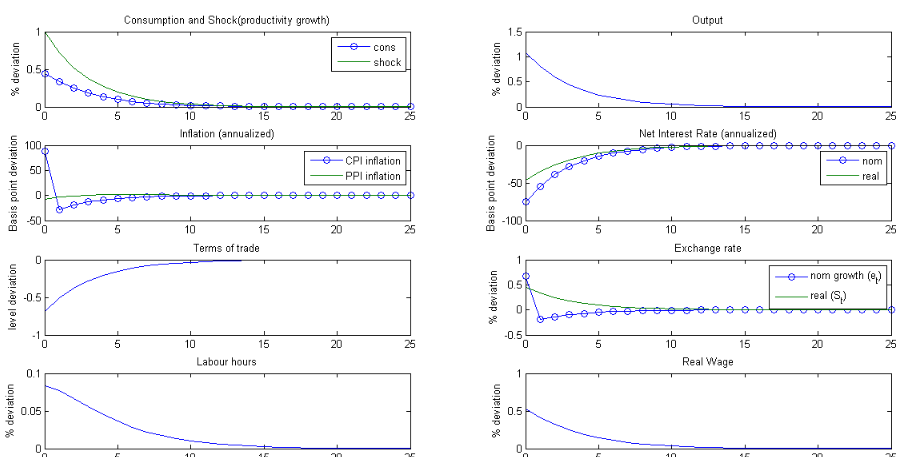
4.1 Model Dynamics under Ramsey Optimal Allocation

The steady state of the Ramsey optimal allocation is used as the reference benchmark in this welfare analysis. Near steady state dynamic effects of a number of macroeconomic variables, in response to a one-time shocks of (1). domestic productivity and (2). foreign output, under the Ramsey optimal allocation are shown in the Figure 3.1 and Figure 3.2 respectively.

Domestic productivity shock

It is observed that a one-percent positive shock to productivity drives up output, consumption, per capita hours worked, real wage and real exchange rate, as shown in the Figure 3.1. They all, however, converges back to the initial steady state gradually within about fifteen quarters.

Figure 3.1: Impulse Response Functions of the Variables to a Productivity Shock of One Standard-Deviation



A productivity shock in the home country is absorbed by an immediate jump in home CPI inflation rate, due to initial high demand. But it soon goes down, owing to the reduced real marginal costs, caused by higher productivity. This tends to lower the prices of domestically produced goods which in turn reduce inflation. The reduction of inflation, however, is dampening away gradually, partly due to the newly enhanced demand and upward wage bargaining.

Thus the monetary authority reduces the domestic interest accommodating the inflation dynamics effected by the productivity shock. The direction of the nominal interest rate movement is consistent with the inflation movement.

Productivity shock leads to a persistent expansion in the output, which gradually decay over a period of 15 quarters, bringing output back to its steady state level. The level of consumption broadly resembles the level of output and its growth rate is negative. The initial jump in consumption in response to the one percent productivity shock is, however, less than one percent due to perfect risk sharing that leads to Uncovered Interest Rate Parity (UIP). Equivalently, it can be viewed as follows: domestic households consume both domestic and foreign goods, thus, a positive domestic productivity shock of one percent increases domestic consumption, which consists of both domestic and foreign goods, less than one percent only. Terms of trade declines with the productivity shock since the domestic prices tend to go down immediately in response to the shock and so does the domestic to foreign price ratio, given the assumption that the global prices cannot be influenced by a productivity shock in a small open economy.

An increase in productivity leads to an economy wise wage hike which in turn lifts real marginal cost to some extent, however, the reduction in the marginal cost, caused by the productivity shock is dominant. Total labour hours also increase, though less than one percent, contributing to the rise of exports. As a consequence of the productivity shock, labour hours can either increase or decrease, depending on the relative importance of the wealth effect or the substitution effect. In the present case, substitution effect seems to dominate, as reflected by the hike in labour hours, in response to the productivity shock.

Exchange rate dynamics reflects a real depreciation of the domestic currency[16] of approximately by one half of a percentage, in response to a one-percent rise in productivity. This is

[16] In line with Lubik and Schorfheide (2007) and Del Negro and Schorfheide (2008), the real exchange rate is defined as $S_t = \varepsilon_t P_t^*/P_t$, where ε_t, P_t^* and P_t are the nominal exchange rate, foreign consumer price index and domestic consumer price index respectively. Thus, an increase in the real exchange rate will reflect an appreciation of the foreign currency as well as a depreciation of the domestic currency and vice-versa.

in line with the lowered interest rate that leads to a decline in the foreign investments leading to a reduced demand for the domestic currency thereby causing a depreciation of the home currency. Further, as reveals from the real exchange rate relationship, the real exchange rate depreciation is partly due to the lowered domestic price level caused by higher productivity, given the fact that foreign price level is not much affected by the productivity shock in the small open economy. Figure 3.1 shows that the nominal exchange rate[17] growth rate, e_t, is getting an immediate positive jump in response to the productivity shock and it confirms the domestic currency depreciation. This depreciation, however, diminish gradually with the diluting effect of the onetime productivity shock. Accordingly, a negative value for e_t in the second period onwards which converges to zero is observed, indicating that the domestic currency returns back to its original value gradually.

Figure 3.1 further suggests that a inflation targeting monetary authority should consider a PPI target as opposed to a CPI target since the fluctuations in the Producer Price Inflation (PPI) is lower, compared to Consumer Price Inflation (CPI), in response to the productivity shock.[18] This is owing to the fact that CPI calculation considers both home and foreign goods in contract to PPI that considered only domestic goods, thus, unexposed to the exchange rate changes.

Foreign output shock

As depicted in the Figure 3.2, a one-percent positive shock to the foreign output leads to an immediate hike in the home consumption, which is, however, less than one percent, owing to the fact that home consumption consists of both domestic and foreign goods. Resulting higher demand for foreign goods lifts inflation up. Accordingly, the central bank is leaning against the wind, by tightening the monetary policy stance to curtail inflationary pressure. Thus, the direction of the nominal interest rate movement is consistent with the inflation movement.

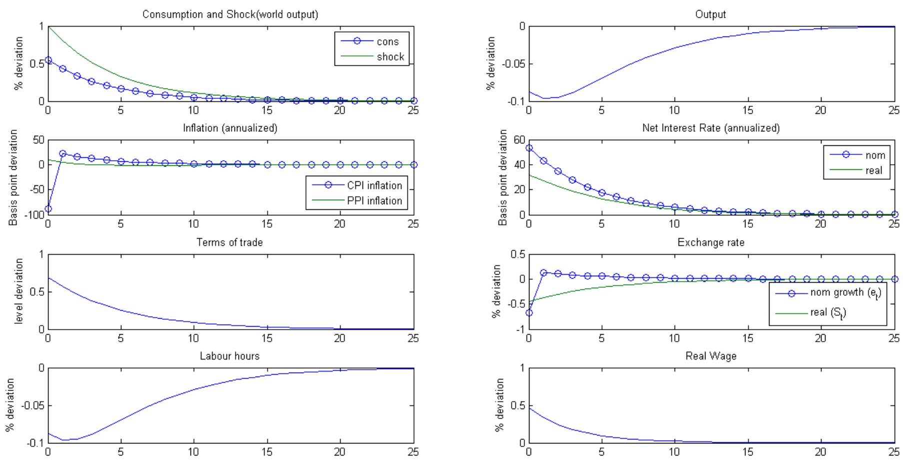
The foreign output shock leads to a persistent contraction in the domestic output , which lasts for over 20 quarters. Improved foreign output that lowered the demand for home goods seems to be attributable to the reduction in home output, consequently. Dynamics

[17]Note that the nominal exchange rate is defined as the amount of domestic currency per one unit of foreign currency

[18]In a study on designing targeting rules for international monetary policy cooperation, Benigno and Benigno (2006) find similar results.

of the labour hours, consequent to the foreign productivity shock resembles the dynamics of the domestic output curve, for the same reasons, reduced demand for domestic goods, consequently lowers labour hours. In line with the price hike wage bargaining takes place resulting a rise in real wages.

Figure 3.2: Impulse Response Functions of the Variables to a World Output Shock of One Standard-Deviation



In line with the increase in interest rate, foreign investments go up which in turn appreciate domestic currency, as evident from the immediate negative value of the both real and nominal exchange rate impulse responses.[19]

4.2 Optimal Monetary Policy Rules

I search for optimal policy coefficients for the monetary policy rule which ensure welfare as close as possible to that of Ramsey optimal allocation, in the light of Schmitt-Grohé and Uribe (2007). The following coefficient values maximise conditional welfare,[20] for the functional form of the monetary policy rule used in the model: the interest rate smoothing parameter (ρ_R)=0.735, the inflation coefficient (ψ_π)=3.000,[21] the output gap coefficient

[19] As mentioned above, a decline in the real exchange rate reflects an appreciation of the domestic currency. Similarly, a decline in the growth rate of the nominal exchange rate reflects an appreciation of the domestic currency, owing to the fact that the nominal exchange rate is defined as the value of one unit of foreign currency in terms of the home currency.

[20] Equivalently, this policy rule with the given coefficients, minimises the welfare cost of deviating from the Ramsey optimal allocation, compared to other alternative policy rules with different coefficient values.

[21] Note that I set an upper bound of 3 for any the coefficient values, for practical implementability considerations. This is a standard practice followed by others, in similar studies.

(ψ_y)=0.677 and the exchange rate coefficient (ψ_e)=0.064. These coefficients therefore represent the optimal values for the monetary policy rule proposed to Sri Lanka, given the model and the set of predetermined parameter values. Thus the optimal monetary policy rule is given by,

$$ln\left(\frac{R_t}{R}\right) = 0.735\ ln\left(\frac{R_{t-1}}{R}\right) + 0.265\left[3\ ln\left(\frac{\pi_t}{\pi}\right) + 0.677\ ln\left(\frac{y_t}{y}\right) + 0.064\ ln\left(\frac{e_t}{e}\right)\right] + ln\left(\ \varepsilon_t^R\right) \tag{38}$$

As expected, it suggests a strong response to inflation, a moderate response to output gap and a weak response to exchange rate and these results are in line with that of Schmitt-Grohé and Uribe (2007). In their closed economy model for the US economy, they find that it is optimal to response inflation aggressively and hence, set ψ_π)=3.000. They, however, advocate a very weak response to output with (ψ_y)=0.01, while having strong policy smoothing with (ρ_R)=0.84. In the following section I study the welfare implications of few alternative policy scenarios to better understand the how welfare varies with different policy co-efficient values.

4.3 Welfare Analysis

4.3.1 Case 1: Monetary policy rule with interest rate smoothing and response to exchange rate

I first consider a case where monetary policy does systematically respond to exchange rate (*i.e.* $\psi_e \neq 0$). In this exercise, the exchange rate coefficient (ψ_e) and the interest rate smoothing parameter (ρ_R) are set to 0.064 and 0.735, which are as same as their optimal values, obtained above. Then I numerically study the impact on welfare when the response to inflation coefficient (ψ_π) and the response to output coefficient (ψ_y), deviate from their respective optimal values.[22]

[22]This enables one to understand the *welfare sensitivity of coefficients*, as well.

Figure 3.3: Conditional welfare response to inflation and output coefficient variations

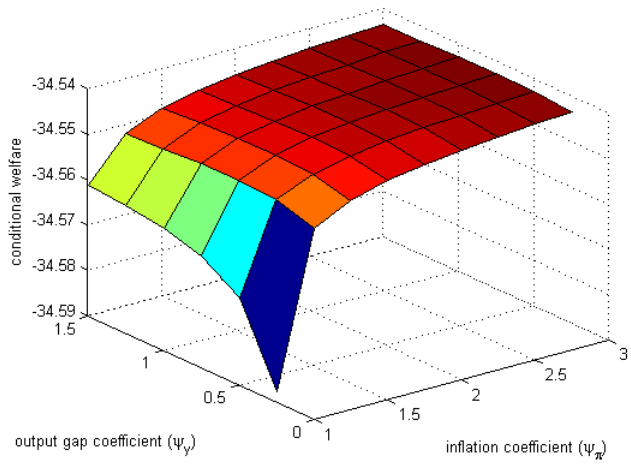
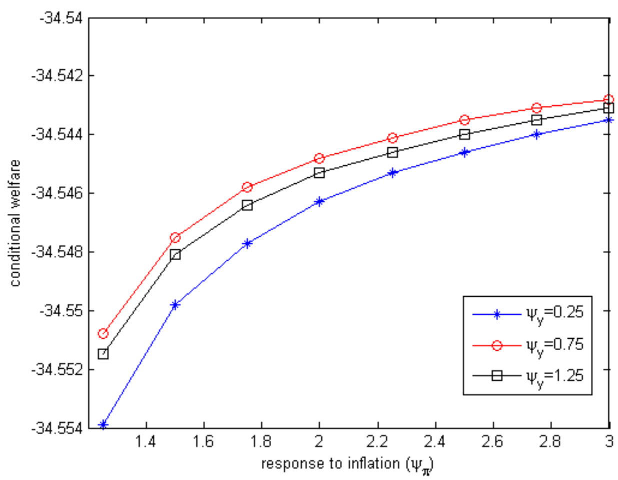


Figure 3.2 shows that for any given value of the output gap coefficient (ψ_y) in the search grid, welfare is strictly increasing in the inflation coefficient (ψ_π). This fact is clearly visible in the Figure 3.3, where the marginal welfare gain of responding to inflation is plot against three different values for output gap coefficient.

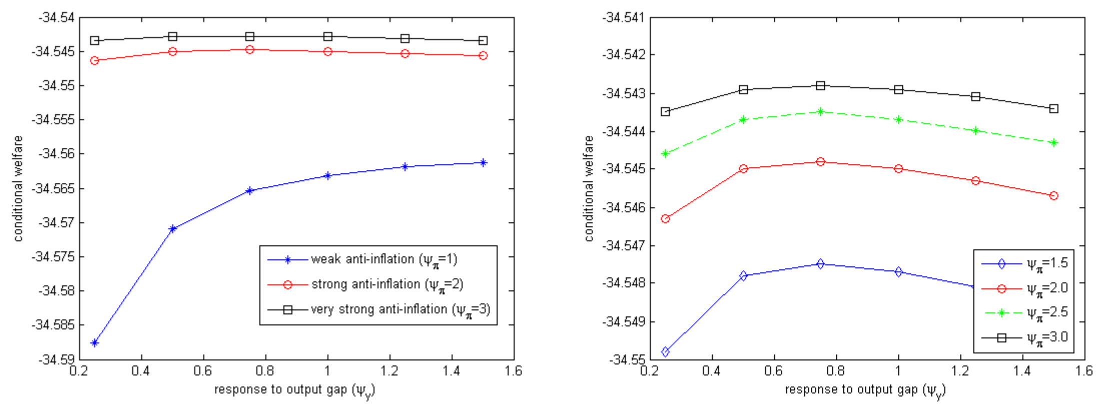
Figure 3.4: Conditional welfare response to output, for given inflation coefficients



It is further evident that the marginal welfare gain declines with the increasing aggressiveness of anti-inflation policy (i.e. raising the value of ψ_π). This is attributable to the fact that the curves are converging and getting flattened out with increasing ψ_π which is consistent with the findings of Schmitt-Grohé and Uribe (2007). For the practical constraints on implementability and for the fact that the raising ψ_π beyond 3 results only a very little welfare gain, restricting the upper bound of ψ_π to 3 seems to be reasonable. Moreover, it is interesting to note that out of the three curves in the figure, $\psi_y = 0.75$ corresponds to the one with highest welfare for any given value of ψ_π, in the interval considered.[23] This confirms that the optimal value for the response to output should lie somewhere near 0.75.

Another important result is shown in the Figure 3.4 given below. It illustrates how the marginal welfare gain changes when response to output (ψ_y) vary, for different monetary policy stances against inflation. The first panel of the figure implies that response to output, ψ_y, can influence conditional welfare considerably under the weak anti-inflationary monetary policy (i.e. smaller ψ_π, closer to unity). When the anti-inflationary monetary policy is sufficiently large, on contrary, a change in response to output ψ_y does not make much difference in welfare, as demonstrated by both the moderate and strong anti-inflation curves in the first panel.

Figure 3.5: Conditional welfare response to inflation, for given output gap coefficients



The second panel of the Figure 3.4 is as same as the first panel, however, excluding the weak anti-inflationary monetary policy curve while including two more curves corresponding to intermediate inflation values. This figure clearly shows that stronger the anti-inflationary action, weaker the response to output and vies-versa, as the curves are getting flatter when ψ_π goes up. Further, the peak of each of the curves occurs near $\psi_y = 0.75$ suggesting that the optimal value for the response to output gap should lie closer to 0.75 and it matches well with the corresponding optimal policy rule coefficient given above.[24]

[23]Note that both $\psi_y = 0.25$ and $\psi_y = 1.25$ curves are below the curve $\psi_y = 0.75$, in Figure 3.3 above.

[24]The optimal value $\psi_y = 0.677$ in equation 40 is below 0.75 for the fact that each of the curves in Figure 3.4 are constructed by using only six discrete data points. If a large number of data points are used, on contrary, one can get a much smoother curve having a peak closer to 0.677.

This emphasize the point that ψ_y should neither be too high nor be too low. If ψ_y is too large, it hampers economic growth, as it recommends a larger than required interest rate in response slight upward movements of the output gap. If ψ_y is too small, on contrary, policy reaction will fail to stabilise output while causing inflationary pressure in the economy. Thus, it is important to identify an appropriate value for ψ_y, and implement the monetary rule in agreement with it.

Welfare cost of implementing alternative rules

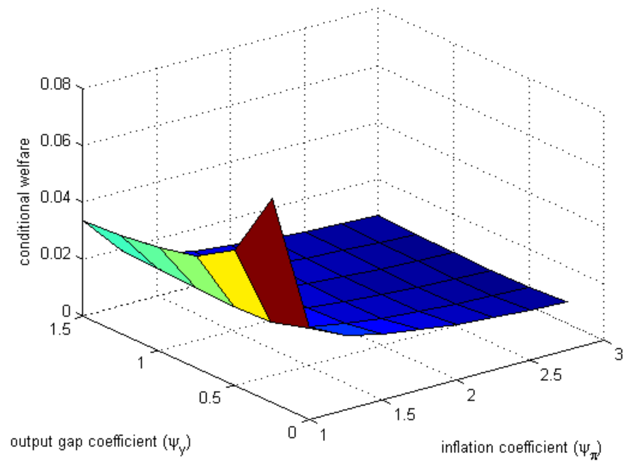
For different alternative policy rules, I calculate the conditional welfare cost of implementing such rules, instead of the optimal rule, using the equation (38) above (i.e. $\lambda^c = (1-\beta)\left(V^{rc}_{\sigma_\epsilon\sigma_\epsilon} - V^{ac}_{\sigma_\epsilon\sigma_\epsilon}\right)\frac{\sigma_\epsilon^2}{2}$), and the results are summarised in the Table 3.1 and the Figure 3.4 given below:

Table 3.1: Conditional welfare cost of implementing alternative policy rules ($\lambda^c \times 100$)

Responsiveness	Responsiveness to output (ψ_y)					
to inflation (ψ_π)	0.25	0.50	0.75	1.0	1.25	1.50
1.00	0.0718	0.0475	0.0395	0.0361	0.0343	0.0334
1.25	0.0227	0.0188	0.0181	0.0185	0.0191	0.0198
1.50	0.0167	0.0137	0.0133	0.0136	0.0142	0.0150
1.75	0.0136	0.0112	0.0109	0.0112	0.0117	0.0124
2.00	0.0116	0.0097	0.0094	0.0096	0.0101	0.0107
2.25	0.0101	0.0086	0.0083	0.0086	0.0090	0.0095
2.50	0.0090	0.0077	0.0075	0.0077	0.0081	0.0086
2.75	0.0082	0.0071	0.0069	0.0071	0.0075	0.0079
3.00	0.0075	0.0066	0.0064	0.0066	0.0069	0.0073

Notes: In the optimized rules policy parameters are restricted to lie in the interval [0,3] for practical convenience. Conditional and unconditional welfare costs, $\lambda^c \times 100$ and $\lambda^u \times 100$ denote the percentage decrease in Ramsey optimal consumption process necessary to equate the level of welfare under Ramsey policy as same as to that under the alternative policy considered. Hence, a positive figure implies welfare is higher under Ramsey policy than under the alternative considered.

Figure 3.6: Conditional welfare cost of implementing alternative monetary policy rules



The qualitative responses implied by both the Table 3.1 and Figure 3.6 are in well agreement with the previous analysis. It is revealed that the welfare costs are declining with the increasing aggressiveness of response to inflation, for any given value of the output gap coefficient. The marginal improvement, however, dies out with the increasing values of ψ_π.

Resembling the previous result, this clearly shows that stronger the anti-inflationary action, weaker the response to output and vies-versa, as revealed from both the grid and the table. Moreover, the figures suggest that the best alternative policy rule which is associated with the lowest conditional welfare cost is the one bearing the coefficients, $\psi_\pi = 3.00$ and $\psi_y = 0.75$ and this matches well with the above result.[25] This refers to a welfare cost equivalent to 0.0064 percent of the steady state consumption per capita, per quarter, compared to the welfare associated with the Ramsey optimal allocation.

This result can be interpreted in a more sensible way by expressing it in dollar terms. Per capita consumption in Sri Lanka in 2014 is 2,588 US Dollars[26] and therefore, the welfare cost of following the above optimal rule, as opposed to following the Ramsey allocation, is approximately 0.16 US Dollars, or equivalently, 21.05 Sri Lankan Rupees,[27] per person, quarterly.[28] With the 20.6 million population in the country, the total welfare cost turns out

[25] This is none other than the above optimal monetary policy rule and I am getting slightly different coefficient values here since I have used only a limited number of discrete data points in the table/ graph. One can obtain the same results as in the optimal policy rule case, by simply increasing the number of data points in the table/graph.

[26] Source: Annual Report, Central Bank of Sri Lanka, 2014.

[27] The exchange rate for Sri Lankan Rupee (LKR) to US Dollar (USD) on 31 December 2014 was LKR 131.58/USD.

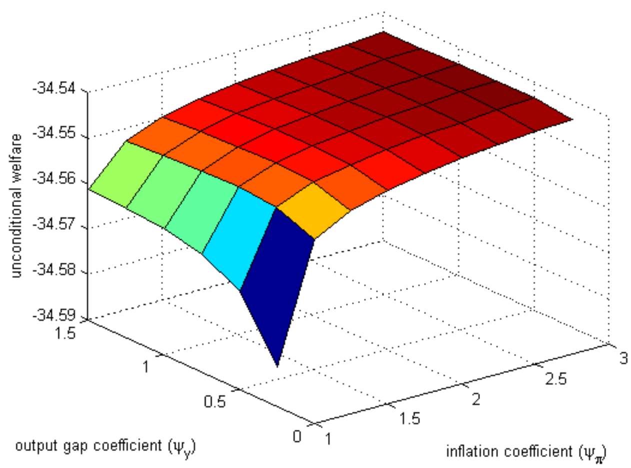
[28] Computed as 2,588 USD $\times$0.0064/100.

to be significantly large, particularly with the current policy rule followed by Sri Lanka.[29]

Unconditional welfare effects.

So far the discussion is limited to the conditional welfare analysis. In particular, it is assumed the condition that the economy starts with the Ramsey steady state and evolve as per the dynamics of the shock process. I now relax this assumption and observe the unconditional welfare effects.

Figure 3.7: Unconditional welfare response to inflation and output coefficient variations



Unconditional welfare effects of the policy rule for the same set of coefficient values are demonstrated in the Figure 3.7. It resembles the Figure 3.1 above and share the same qualitative properties. Even quantitatively, the figures are much similar except the fact that welfare responsiveness of the frontier is slightly lesser now, for the lower values of ψ_π and ψ_y.

The unconditional welfare cost of implementing alternative monetary policy rules is displayed in the Table 3.2 below.

[29]This is discussed in the next section.

Table 3.2: Unconditional welfare cost of implementing alternative policy rules ($\lambda^u \times 100$)

Responsiveness	Responsiveness to output (ψ_y)					
to inflation (ψ_π)	0.25	0.50	0.75	1.0	1.25	1.50
1.00	0.0626	0.0437	0.0376	0.0351	0.0340	0.0334
1.25	0.0246	0.0199	0.0192	0.0195	0.0202	0.0210
1.50	0.0188	0.0152	0.0146	0.0149	0.0156	0.0164
1.75	0.0155	0.0127	0.0122	0.0125	0.0131	0.0138
2.00	0.0133	0.0110	0.0107	0.0109	0.0115	0.0121
2.25	0.0116	0.0098	0.0095	0.0098	0.0103	0.0109
2.50	0.0103	0.0089	0.0086	0.0089	0.0094	0.0099
2.75	0.0093	0.0081	0.0079	0.0082	0.0086	0.0092
3.00	0.0085	0.0075	0.0074	0.0076	0.0080	0.0085

As in the conditional case, coefficients for the optimal rule occur near the point such that $\psi_\pi = 3.00$ and $\psi_y = 0.75$. The welfare cost of the corresponding optimal policy is, however, marginally higher now. The least welfare cost is 0.0074 in the unconditional case, compared to 0.0066 in the conditional case (this can equivalently be viewed as an unconditional welfare cost of 0.19 US Dollars, as opposed to 0.16 US Dollars, in the conditional case).

4.3.2 Case 2: Monetary policy rule with interest rate smoothing but with no response to exchange rate

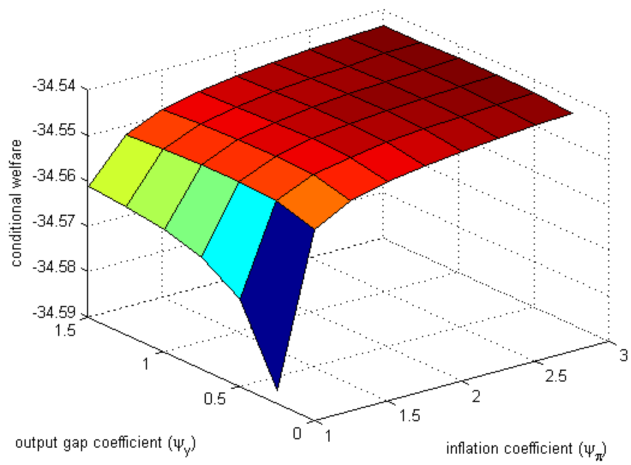
In the previous section, it is found that the optimal policy prescribes a very small value for the exchange rate response coefficient (*i.e.* $\psi_e = 0.064$), for the functional form including exchange rate. In this exercise, I am attempting to assess the welfare impact of omitting the exchange rate response coefficient (ψ_e) fully, from the monetary policy reaction function. Accordingly, I set $\psi_e = 0$ and the interest rate smoothing parameter (ρ_R) to 0.677, which is as same as its optimal value, obtained for this particular functional form of the policy rule.[30] Then I numerically study the impact on welfare when the coefficients ψ_π and ψ_y deviate from their respective optimal values, as in the case 1 above.

Findings suggest that the welfare effects of removing ψ_e from the policy rule are negligibly small and it still produces all the graphs under the case 1 above, with exactly the same qualitative properties. For comparison, I present the graph of *welfare response to inflation*

[30]Note that the optimal value of ρ_R in case 1 and case 2 are the same, up to three decimal places (i.e. they are 0.735364 and 0.734789 respectively.)

and output coefficient variations, here and detailed results of the analysis can be found in the Appendix 7.

Figure 3.8: Conditional welfare response to inflation and output coefficient variations (when $\psi_e = 0$)



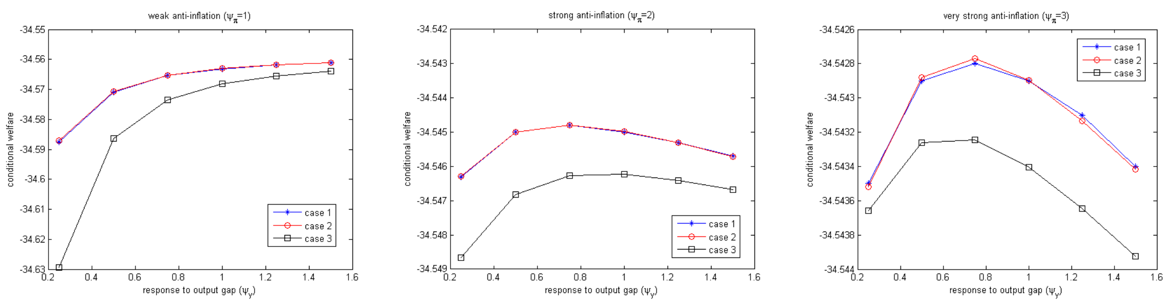
The main finding of case 2 is that the nominal interest rate respond to the fluctuations in exchange rate only very weakly. Among others, Benigno and Benigno (2006), Lubik and Schorfheide (2007), Corsetti et al. (2010) and Justiniano and Preston (2010) report similar results empirically, arguing that optimal monetary policy prescriptions are not influenced by the exchange rate fluctuations to a significant level. This study confirms it in the Sri Lankan context, in line with the previous studies in the country, including Perera and Jayawickrema (2013) and Karunaratne and Pathberiya (2014).

4.3.3 Case 3: Monetary policy rule without interest rate smoothing but with response to exchange rate

In this section, I attempt to study the significance of interest rate smoothing in the monetary policy reaction function. Hence, I set ρ_R to zero in this analysis and compare the resulting welfare implications against that of the previous two cases where ρ_R was at its respective optimal values. It is observed that the conditional welfare associated with the policy rule and the welfare cost of implementing an alternative rule are qualitatively similar in this case and the corresponding graphs takes the same form as in the two cases above. The values are, however, different to some extent in case 3, compared to the previous two scenarios.[31]

The Figure 3.9 given below displays the variation of conditional welfare against the responsiveness to the output gap, for the three different cases we discussed above. The three panels of the figure denote different strengths of anti-inflationary action. In each of the three panels, the first two curves denote case 1 and case 2 and they are nearly identical, as expected. This reconfirms the irrelevance of exchange rate in the monetary policy rule.[32]

Figure 3.9: Conditional welfare response to inflation and output coefficient variations in the three cases



More importantly, the results indicate that interest rate smoothing has a considerable bearing on welfare as depicted in the first panel, in contrast to the exchange rate fluctuations. The impact of excluding the smoothing parameter (i.e. case 3) is more evident when inflation response (ψ_π) and output gap response (ψ_y) are both weak. That fact is visible from the obvious deviation of the case 3 curve from the other two curves in the first panel. When anti-inflationary action (ψ_π) is getting stronger, on the other hand, the relative importance of interest rate smoothing becomes significant in the large values of ψ_y, as shown in the third panel.

Including interest rate smoothing in the monetary policy rule proved to be welfare improving as found in this analysis. It is elaborated in the Table 3.3, given below. The welfare cost of implementing an alternative rule, instead of the optimal policy is significantly large when smoothing is excluded (i.e. case 3), particularly when ψ_π and ψ_y are both small.[33]

[31] One main result indicating the importance of interest rate smoothing is described below and the detailed results are included in the Appendix 7.

[32] Note that case 2 differs from case 1 only for the fact that the latter does not include response to exchange rate variation. The observation that the two curves are similar implies that response to exchange rate does not have any welfare effect.

[33] The complete tables of results for the three cases are given in the Appendix 7.

Table 3.3: Conditional welfare cost of implementing alternative policy rules, in the three cases ($\lambda^c \times 100$)

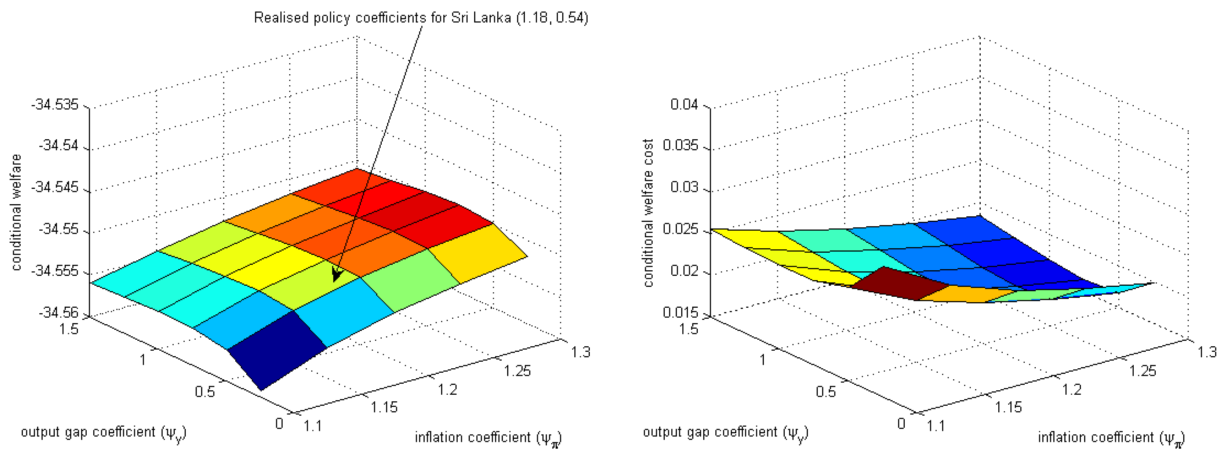
Responsiveness to inflation (ψ_π)		Responsiveness to output (ψ_y) 0.25	0.50	0.75	1.0	1.25	1.50
1.00	case 1	0.0718	0.0475	0.0395	0.0361	0.0343	0.0334
	case 2	0.0712	0.0473	0.0394	0.0360	0.0343	0.0333
	case 3	0.1130	0.0701	0.0514	0.0436	0.0396	0.0373
2.00	case 1	0.0116	0.0097	0.0094	0.0096	0.0101	0.0107
	case 2	0.0115	0.0096	0.0093	0.0096	0.0101	0.0107
	case 3	0.0150	0.0123	0.0115	0.0114	0.0117	0.0121
3.00	case 1	0.0075	0.0066	0.0064	0.0066	0.0069	0.0073
	case 2	0.0075	0.0066	0.0064	0.0066	0.0069	0.0073
	case 3	0.0077	0.0071	0.0071	0.0073	0.0077	0.0081

4.4 Welfare implications of the realised monetary policy rule for Sri Lanka

In the chapter 4 above, I estimate a monetary policy rule for Sri Lanka with Bayesian methods, in an open economy DSGE framework, using quarterly data for the period 1996:Q1 to 2014:Q2. The results suggest the following coefficient values: $\rho_R = 0.80$, $\psi_\pi = 1.18$, $\psi_y = 0.54$ and $\psi_e = 0.05$, for the realised monetary policy rule. Evidently, these values are different to that of optimal policy coefficients discussed above and it is important to asses the extent of welfare loss caused by following this rule, deviating from the Ramsey optimal policy rule.

It is widely known that ψ_π, ψ_y are the dominant coefficients in a monetary policy rule and it is evident from the previous analysis as well. Accordingly, I set ρ_R and ψ_e at their empirically obtained values for Sri Lanka (i.e. 0.80 and 0.05 respectively) and then study how welfare changes, when ψ_π and ψ_y deviate from the local vicinity of their realised values for Sri Lanka and the results are given in the Figure 3.10 and in 3.4, given below.

Figure 3.10: Conditional welfare analysis for Sri Lanka



The first panel of the Figure 3.10 given below shows that the welfare response to inflation and output coefficient variations while the second panel displays the welfare loss[34] as in the above cases. From the first panel, it is clear that welfare can be improved significantly by raising the responsiveness to inflation (ψ_π), above the prevailing value of 1.18. Welfare can only be improved marginally, however, by lifting the responsiveness to output (ψ_y).

Table 3.4: Conditional welfare cost analysis for Sri Lanka ($\lambda^u \times 100$)

Responsiveness	Responsiveness to output (ψ_y)					
to inflation (ψ_π)	0.25	0.50	0.75	1.0	1.25	1.50
1.10	0.0306	0.0262	0.0251	0.0250	0.0252	0.0256
1.15	0.0261	0.0225	0.0218	0.0221	0.0226	0.0231
1.20	0.0233	0.0200	0.0195	0.0199	0.0205	0.0212
1.25	0.0214	0.0182	0.0178	0.0182	0.0189	0.0196
1.30	0.0199	0.0168	0.0164	0.0169	0.0176	0.0183

Raising ψ_y should be done with caution, since it will be counter productive to lift ψ_y above 0.68. In agreement with the first panel, the second panel also indicates that the welfare cost of not-implementing the *Ramsey optimal allocation* can be mitigated considerably by raising ψ_π and it also shows that changing ψ_y affect the level of welfare only to a smaller extent.

As depicted from the second panel of the Figure 3.10 and the Table 3.4, there is a considerable welfare cost associated with the current monetary policy rule practised in Sri Lanka. The per capita welfare cost is computed to be 0.0206 percent for a quarter or equivalently,

[34]Following the same procedure as above, the welfare loss is computed with reference to the Ramsey optimal allocation.

an amount of 0.53 US Dollars per capita for a quarter. This is clearly high compared to the previous optimal policy scenarios considered and therefore, Sri Lanka can significantly mitigate welfare cost by responding to inflation more aggressively.

4.5 Summary of the policy rules in the above cases

In the Table 3.4, I summarise the key features of the optimal policies in the above cases, as in Schmitt-Grohé and Uribe (2007). There are seven monetary policies: three constrained optimized rules and four non-optimized rules, out of which one denotes the realised monetary policy for Sri Lanka. In the constrained optimized rule labelled as case 1, I search for all four policy coefficients ρ_R, ψ_π, ψ_y and ψ_e while in case 2, I set ψ_e to zero to asses the impact of omitting response to exchange rate, in the rule. In case 3, ρ_R is fixed at zero, forbidding interest rate inertia to evaluate the impact of interest rate smoothing.[35]

It is found that all three optimised policy rules call for an aggressive response to inflation[36] and a muted response or very weak response to exchange rate. The case 1 and case 2 under optimized rules clearly shows that, exchange rate fluctuations are totally irrelevant as the omission of ψ_e in the policy rule does not affect either conditional or unconditional welfare cost, even up to the fourth decimal place. As revealed in the case 3 under the optimised policy rules, interest rate smoothing is, however, important and exclusion of ρ_R leads to a significantly large hike in both the conditional or unconditional welfare costs. Interestingly, the optimal policy advocates moderate response to output gap, in all three cases.

[35]Note that I have not restricted ψ_e in case 3, thought the optimal rule itself proposes $\psi_e = 0.000$.

[36]In the light of Schmitt-Grohé and Uribe (2007), I set the upper bound of policy parameters to 3. Removing this constraint leads to a much higher optimal inflation policy coefficients with only marginal high welfare effects than that associated with the restricted coefficient.

Table 3.5: Key features of the policy rules

General form of the policy rule: $ln\left(\frac{R_t}{R}\right) = \rho_R\, ln\left(\frac{R_{t-1}}{R}\right) + (1-\rho_R)\left[\psi_\pi ln\left(\frac{\pi_t}{\pi}\right) + \psi_y ln\left(\frac{y_t}{y}\right) + \psi_e ln\left(\frac{e_t}{e_{t-1}}\right)\right]$

Coefficients		ρ_R	ψ_π	ψ_y	ψ_e	Conditional welfare cost ($\lambda^c \times 100$)	Unconditional welfare cost ($\lambda^u \times 100$)
Ramsey policy		-	-	-	-	0.0000	0.0000
Optimized rules	case 1	0.7354	3.0000	0.6767	0.0638	0.0064	0.0073
	case 2	0.7348	3.0000	0.6840	-	0.0064	0.0073
	case 3	-	3.0000	0.6175	0.0000	0.0070	0.0084
Selected alternative rules	case 1	0.7354	1.5000	0.5000	0.0638	0.0137	0.0152
(Taylor type)	case 2	0.7348	1.5000	0.5000	-	0.0136	0.0150
	case 3	-	1.5000	0.5000	0.0000	0.0194	0.0232
Realised policy rule for SL		0.8000	1.1800	0.5400	0.0500	0.0206	0.0204

Notes: In the optimized rules policy parameters are restricted to lie in the interval [0,3] for practical convenience. Conditional and unconditional welfare costs, $\lambda^c \times 100$ and $\lambda^u \times 100$ denote the percentage decrease in Ramsey optimal consumption process necessary to equate the level of welfare under Ramsey policy as same as to that under the alternative policy considered.

In the three cases under selected alternative rules, I set ρ_R and ψ_e as in their corresponding optimized rules but set ψ_π and ψ_y to two widely tested coefficient values in literature which are 1.5 and 0.5 respectively. For these non-optimized rules, welfare cost is considerably large (for instance, an annual per capita welfare cost of 0.35 US Dollars in the case 1 of the optimised rules, compared to 0.16 US Dollars in the case 1 of the non optimised rules), however, the muted response to exchange rate and importance of including interest rate inertia are still valid for these rules. Moreover, it is observed that including response to exchange rate can even be harmful as it slightly increase the welfare cost further.[37] The realised monetary policy rule for Sri Lankan is having a fairly large welfare cost which is mainly attributable to the weak response to inflation (i.e. fairly small value of ψ_π=1.18).

[37] Note that the welfare cost of case 2 where it excludes exchange rate in the rule, is having a marginally lower welfare cost compared to that of case 1.

5 CONCLUSION

In this study I evaluate the stabilization properties of monetary policy rules in a DSGE framework, with special reference to Sri Lankan economy. The welfare level of the private agents is used as the measure of stabilisation, in line with Schmitt-Grohé and Uribe (2007). Accordingly, a *simple policy rule* where nominal interest rate responds only to a few number of observables, namely, inflation, output and exchange rate is used in the exercise. Further, restrictions are imposed by requiring that the rules to be implemented are to have a unique rational expectations equilibrium and zero lower bound on the nominal interest rate. I attempt to determine optimal monetary policy rules such that the welfare associated with them are as much as close to that of the Ramsey optimal allocation. Unconditional expectation of welfare is also determined and compared as a robustness measure. The welfare cost of adopting alternative policy rules, instead of the optimal policy rule are determined to evaluate the relative importance of the coefficients in the policy rules.

Within the class of above simple and implementable policy rules, I found that: First, optimal monetary policy rule suggests an aggressive response to inflation and a moderate response to output-gap. Second, optimal policy advocate a muted response to exchange rate fluctuations, and importantly, monetary policy reaction functions with positive response to exchange rate could lead to minor welfare losses even. Third, optimized interest rate rule features significantly strong interest rate smoothing and the welfare gains associated therewith are substantial. Fourth, the optimized simple monetary rules attain a level of welfare, very closer to that of Ramsey optimal policy. Finally, the *welfare loss* associated with the current realised monetary policy rule in Sri Lanka can be mitigated significantly by responding to inflation more aggressively.

According to best of my knowledge, optimal monetary policy studies which employ welfare maximising Ramsey approach are limited to the US economy only. This is for the first time Ramsey approach is used in analysing optimal monetary policy rules in a developing or emerging country context. Accordingly, the modelling contribution of the study in welfare maximising policy rules for a small open economy (SOE), in a DSGE environment, is also important. The current study can be further extended in number of ways: The restriction made on the *inverse of the inter temporal elasticity of substitution parameter* (σ), setting it to unity[38], can be relaxed, the model can be enriched further by incorporating it with more realistic features such as a fiscal policy rule, incomplete exchange rate pass through, habit formation, nominal wage stickiness, enabling the model to explain Sri Lankan business cycles more realistically.

[38]This was made to make the welfare analysis simpler and easier.

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A Appendix 1: The list of variables and parameters in the model

Table A1: Endogenous Variables

No.	Variable	Description
1	c_t	domestic consumption
2	N_t	labour hours
3	w_t	real wage
4	R_t	domestic nominal interest rate
5	π_t	CPI inflation (P_t/P_{t-1})
6	mc_t	real marginal cost
7	y_t	domestic output
8	δ_t	price dispersion variable
9	ν_t	The price index ratio $(P_{H,t}/P_t = q_t S_t)$
10	$\tilde{p}_{H,t}$	optimal, domestic price level (home prod. goods)
11	$K_{1,t}$	optimal, domestic price index - axillary parameter
12	$K_{2,t}$	optimal, domestic price index - axillary parameter
13	e_t	nominal exchange rate growth rate
14	q_t	terms of trade
15	S_t	real exchange rate
16	$\pi_{H,t}$	domestic producer price inflation
17	c_t^*	foreign consumption

Table A2: Exogenous Variables

No.	Variable	Description
1	a_t	productivity
2	y_t^*	foreign output
3	R_t^*	foreign interest rate
4	π_t^*	foreign inflation

Table A3: Parameters

Variable	Description
σ	inverse of the inter temporal elasticity of substitution
β	subjective discount factor
φ	labour elasticity
α	export share
η	elasticity of substitution between domestic and foreign goods
θ	price stickiness parameter
ϑ	relative size of domestic consumption
ϵ	elasticity of substitution of domestic intermediate goods
ρ_R	interest rate persistence parameter
ρ_a	productivity shock persistence parameter
ρ_{y^*}	world output persistence parameter
ρ_{π^*}	world inflation persistence parameter
ρ_{R^*}	world interest rate persistence parameter
γ_{π^*}	world inflation s.s parameter
ψ_π	inflation coefficient of the monetary rule
ψ_y	output coefficient of the monetary rule
ψ_e	exchange rate coefficient of the monetary rule

Summary of the equations:

There are 16 efficiency condition equations in the system[39]. I use these 16 orthogonal equations in constricting the Lagrangian to determine the Ramsey optimal allocation. This yields to the 16 constraints[40] in the Lagrangian which is meant to maximize lifetime utility of the representative households. The model consists of 17 endogenous variables and hence 17 First Order Conditions (FOCs) can be derived. Additionally, I have 4 exogenous variables and corresponding 4 exogenous dynamic processes related to them. Therefore, the system contains of 37 equations and 37 variables, altogether[41], which can be solved following the methodology specified in the Appendix-6.

[39]The breakdown of the 16 equations is as follows: equation (1), equations (3) to (17) and equation (18). Note that the equation (4) is replaced by (18), thus, it amounts to a sum of 16 equations.

[40]These are the Ω_t multipliers in the Lagrangian shown in the Appendices 2 and 3.

[41]Note that the policy rule (equation (19)) is not a part of the Ramsey optimal allocation and it will be used in the competitive equilibrium case and welfare analysis.

B Appendix 2: The Standard Lagrangian

The standard Lagrangian for the optimal policy problem is as follows. The aim is to maximise the objective function which is the life time utility, subject to the given constraints. In this Lagrangian[42], d_t is the vector of endogenous variables[43] at time t while Λ_t is the vector of Lagrange multipliers chosen at time t.

$$
\begin{aligned}
&\mathfrak{L} = \overset{\min}{\{\Lambda_t\}_{t=0}^{\infty}}, \overset{\max}{\{d_t\}_{t=0}^{\infty}} \left\{\sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}\right]\right. \\
&+\Omega_{1,t}\left[N_t^{\varphi} - c_t^{-\sigma} w_t\right] \\
&+\Omega_{3,t}\left[E_t\left[\left(R_t - R_t^{*} e_{t+1}\right)\frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}}\left(\pi_{t+1}\right)^{-1}\right]\right] \\
&+\Omega_{4,t}\left[\frac{S_t}{S_{t-1}} - e_t\frac{\pi_t^{*}}{\pi_t}\right] \\
&+\Omega_{5,t}\left[\nu_t - q_t S_t\right] \\
&+\Omega_{6,t}\left[S_t - \left[\left(1-\alpha\right) q_t^{1-\eta} + \alpha\right]^{\frac{1}{\eta-1}}\right] \\
&+\Omega_{7,t}\left[mc_t - (w_t/a_t) q_t^{-1} S_t^{-1}\right] \\
&+\Omega_{8,t}\left[\tilde{p}_{H,t} - \frac{\epsilon}{\epsilon-1}\frac{K_{1,t}}{K_{2,t}}\right] \\
&+\Omega_{9,t}\left[K_{1,t} - c_t^{-\sigma} y_t mc_t S_t q_t - \theta\beta E_t K_{1,t+1}\left(\bar{\pi}_H\right)^{-\epsilon}\left(\pi_{H,t+1}\right)^{\epsilon}\right] \\
&+\Omega_{10,t}\left[K_{2,t} - c_t^{-\sigma} y_t S_t q_t - \theta\beta E_t K_{2,t+1}\left(\bar{\pi}_H\right)^{1-\epsilon}\left(\pi_{H,t+1}\right)^{(\epsilon-1)}\right] \\
&+\Omega_{11,t}\left[1 - \theta\left(\bar{\pi}_H\right)^{1-\epsilon}\left(\pi_{H,t}\right)^{\epsilon-1} - \left(1-\theta\right)\left(\tilde{p}_{H,t}\right)^{1-\epsilon}\right] \\
&+\Omega_{12,t}\left[c_t - \vartheta c_t^{*} S_t^{1/\sigma}\right] \\
&+\Omega_{13,t}\left[y_t - \left(1-\alpha\right)\left(S_t q_t\right)^{-\eta} c_t - \alpha\vartheta q_t^{-\eta} c_t^{*}\right] \\
&+\Omega_{14,t}\left[y_t^{*} - c_t^{*}\right] \\
&+\Omega_{15,t}\left[y_t - a_t N_t \delta_t^{-1}\right]\}. \\
&+\Omega_{16,t}\left[\delta_t - \left(1-\theta\right)\tilde{p}_{H,t}^{-\epsilon} - \theta\left(\bar{\pi}_H\right)^{-\epsilon}\pi_{H,t}^{\epsilon}\delta_{t-1}\right] \\
&+\Omega_{17,t}\left[\pi_{H,t} - \frac{\nu_t}{\nu_{t-1}}\pi_t\right]
\end{aligned}
$$

where,

$$\Lambda_t = \left[\Omega_{1,t}\ \Omega_{3,t}\ \Omega_{4,t}\ \Omega_{5,t}\ \Omega_{6,t}\ \Omega_{7,t}\ \Omega_{8,t}\ \Omega_{9,t}\ \Omega_{10,t}\ \Omega_{11,t}\ \Omega_{12,t}\ \Omega_{13,t}\ \Omega_{14,t}\ \Omega_{15,t}\ \Omega_{16,t}\ \Omega_{17,t}\right]'$$

$$d_t = \left[c_t\ N_t\ w_t\ R_t\ \pi_t\ mc_t\ y_t\ \delta_t\ \nu_t\ \tilde{p}_{H,t}\ K_{1,t}\ K_{2,t}\ e_t\ q_t\ S_t\ \pi_{H,t}\ c_t^{*}\right]'$$

[42]Domestic stochastic consumption Euler equation (eq.2) is excluded from the Lagrangian since this equation is redundant, once we assume complete financial markets and impose perfect risk sharing condition in (eq.14).

[43]Note that under perfect risk sharing condition, $y^{*} = c^{*}$. Therefore, c^{*} is endogenous since it's value is determined within the system.

C Appendix 3: The Recursive Augmented Lagrangian

The standard Lagrangian is augmented for the optimal policy problem, expressing it recursively, as follows.

$$
\begin{aligned}
&U^{*}\left(\tilde{s}_t,\zeta_t\right)=\overset{\min}{\{\Lambda_t\}_{t=0}^{\infty}},\overset{\max}{\{d_t\}_{t=0}^{\infty}}\ \{U_t\left(c_t,N_t\right)+\beta E_t U^{*}\left(\tilde{s}_{t+1},\zeta_{t+1}\right)\\
&+\Omega_{1,t}\left[N_t^{\varphi}-c_t^{-\sigma}w_t\right]\\
&+\Omega_{3,t-1}\left[\left(R_{t-1}-R_{t-1}^{*}e_t\right)\tfrac{c_t^{-\sigma}}{c_{t-1}^{-\sigma}}\left(\pi_t\right)^{-1}\right]\\
&+\Omega_{4,t}\left[\tfrac{S_t}{S_{t-1}}-e_t\tfrac{\pi_t^{*}}{\pi_t}\right]\\
&+\Omega_{5,t}\left[\nu_t-q_tS_t\right]\\
&+\Omega_{6,t}\left[S_t-\left[\left(1-\alpha\right)q_t^{1-\eta}+\alpha\right]^{\frac{1}{\eta-1}}\right]\\
&+\Omega_{7,t}\left[mc_t-(w_t/a_t)q_t^{-1}S_t^{-1}\right]\\
&+\Omega_{8,t}\left[\tilde{p}_{H,t}-\tfrac{\epsilon}{\epsilon-1}\tfrac{K_{1,t}}{K_{2,t}}\right]\\
&+\Omega_{9,t}\left[K_{1,t}-c_t^{-\sigma}y_tmc_tS_tq_t\right]+\Omega_{9,t-1}\left[-\theta K_{1,t}\left(\bar{\pi}_H\right)^{-\epsilon}\pi_{H,t}^{\epsilon}\right]\\
&+\Omega_{10,t}\left[K_{2,t}-c_t^{-\sigma}y_tS_tq_t\right]+\Omega_{10,t-1}\left[-\theta K_{2,t}\left(\bar{\pi}_H\right)^{1-\epsilon}\pi_{H,t}^{\epsilon-1}\right]\\
&+\Omega_{11,t}\left[1-\theta\left(\bar{\pi}_H\right)^{1-\epsilon}\left(\pi_{H,t}\right)^{\epsilon-1}-\left(1-\theta\right)\left(\tilde{p}_{H,t}\right)^{1-\epsilon}\right]\\
&+\Omega_{12,t}\left[c_t-\vartheta c_t^{*}S_t^{1/\sigma}\right]\\
&+\Omega_{13,t}\left[y_t-\left(1-\alpha\right)\left(S_tq_t\right)^{-\eta}c_t-\alpha\vartheta q_t^{-\eta}c_t^{*}\right]\\
&+\Omega_{14,t}\left[y_t^{*}-c_t^{*}\right]\\
&+\Omega_{15,t}\left[y_t-a_tN_t\delta_t^{-1}\right]\\
&+\Omega_{16,t}\left[\delta_t-\theta\left(\bar{\pi}_H\right)^{-\epsilon}\pi_{H,t}^{\epsilon}\delta_{t-1}-\left(1-\theta\right)\tilde{p}_{H,t}^{-\epsilon}\right]\\
&+\Omega_{17,t}\left[\pi_{H,t}-\tfrac{\nu_t}{\nu_{t-1}}\pi_t\right]\}.
\end{aligned}
$$

where, the vector of state variables ($\tilde{s}_t$) at time t is given by:

$$\tilde{s}_t=[\ \Omega_{9,t-1}\ \Omega_{10,t-1}\ S_{t-1}\ e_{t-1}\ \delta_{t-1}\ \nu_{t-1}\ R_{t-1}\ \ a_t\ y_t^{*}\ \pi_t^{*}\ R_t^{*}\ ]'$$

and the vector of exogenous shock vector (ζ_t) is as follows:

$$\zeta_t=\left[\ \varepsilon_t^{a_t}\ \varepsilon_t^{y_t^{*}}\ \varepsilon_t^{\pi_t^{*}}\ \varepsilon_t^{R_t^{*}}\ \right]'$$

To solve for the Ramsey optimal allocation, I obtain the FOCs of the Lagrangian w.r.t. d_t and Λ_t.[44] This yields a total of 17+16=33 FOCs which summarize the optimal Ramsey allocation, given the state variables.

[44]Taking FOCs w.r.t. Λ_t gives back the private sector efficiency conditions.

D Appendix 4: The First Order Conditions w.r.t. non-policy endogenous variables

The first order conditions w.r.t. each of the endogenous variables are as follows,

(1). c_t :

$$0 = c_t^{-\sigma} + \Omega_{1,t}\left[\sigma c_t^{-\sigma-1} w_t\right]$$
$$+\Omega_{3,t-1}\left[(R_{t-1} - R^*_{t-1}e_t)(\pi_t)^{-1}\tfrac{1}{c_{t-1}^{-\sigma}}(-\sigma)c_t^{-\sigma-1}\right] + \Omega_{3,t}\left[\beta(R_t - R^*_t e_{t+1})(\pi_{t+1})^{-1}c_{t+1}^{-\sigma}(\sigma)c_t^{\sigma-1}\right]$$
$$+\Omega_{9,t}\left[\sigma c_t^{-\sigma-1} y_t mc_t S_t q_t\right] + \Omega_{10,t}\left[\sigma c_t^{-\sigma-1} y_t S_t q_t\right] + \Omega_{12,t}\left[1\right] + \Omega_{13,t}\left[-\left(1-\alpha\right)\left(S_t q_t\right)^{-\eta}\right]$$

(2). N_t :

$$0 = -N_t^{\varphi} + \Omega_{1,t}\left[\varphi N_t^{\varphi-1}\right] + \Omega_{15,t}\left[-a_t \delta_t^{-1}\right]$$

(3). w_t :

$$0 = \Omega_{1,t}\left[-c_t^{-\sigma}\right] + \Omega_{7,t}\left[-(a_t q_t S_t)^{-1}\right]$$

(4). R_t :

$$0 = \Omega_{3,t}\left[\beta\tfrac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}}(\pi_{t+1})^{-1}\right]$$

(5). π_t :

$$0 = \Omega_{3,t-1}\left[-(R_{t-1} - R^*_{t-1}e_t)\tfrac{c_t^{-\sigma}}{c_{t-1}^{-\sigma}}\pi_t{}^{-2}\right] + \Omega_{4,t}\left[\tfrac{e_t \pi^*_t}{\pi_t^2}\right] + \Omega_{17,t}\left[\tfrac{-\nu_t}{\nu_{t-1}}\right]$$

(6). mc_t :

$$0 = \Omega_{7,t}\left[1\right] + \Omega_{9,t}\left[-c_t^{-\sigma} y_t S_t q_t\right]$$

(7). y_t :

$$0 = \Omega_{9,t}\left[-c_t^{-\sigma} mc_t S_t q_t\right] + \Omega_{10,t}\left[-c_t^{-\sigma} S_t q_t\right] + \Omega_{13,t}\left[1\right] + \Omega_{15,t}\left[1\right]$$

(8). δ_t :

$$0 = \Omega_{15,t}\left[a_t N_t \delta_t^{-2}\right] + \Omega_{16,t}\left[1\right] + \Omega_{16,t+1}\left[-\beta\theta\left(\bar{\pi}_H\right)^{-\epsilon}\pi_{H,t+1}^{\epsilon}\right]$$

(9). ν_t :

$$0 = \Omega_{5,t}\left[1\right] + \Omega_{17,t}\left[\tfrac{-\pi_t}{\nu_{t-1}}\right] + \Omega_{17,t+1}\left[\beta\tfrac{\nu_{t+1}}{\nu_t^2}\pi_{t+1}\right]$$

(10). $\tilde{p}_{H,t}$:

$$0 = \Omega_{8,t}\left[1\right] + \Omega_{11,t}\left[-(1-\theta)(1-\epsilon)\tilde{p}_{H,t}^{-\epsilon}\right] + \Omega_{16,t}\left[-(1-\theta)(-\epsilon)\left(\tilde{p}_{H,t}\right)^{-\epsilon-1}\right]$$

(11). $K_{1,t}$:

$$0 = \Omega_{8,t}\left[\left(\tfrac{-\epsilon}{\epsilon-1}\right)\tfrac{1}{K_{2,t}}\right] + \Omega_{9,t}\left[1\right] + \Omega_{9,t-1}\left[-\theta\left(\bar{\pi}_H\right)^{-\epsilon}\pi_{H,t}^{\epsilon}\right]$$

(12). $K_{2,t}$:

$$0 = \Omega_{8,t}\left[\left(\tfrac{\epsilon}{\epsilon-1}\right)\tfrac{K_{1,t}}{K_{2,t}^2}\right] + \Omega_{10,t}\left[1\right] + \Omega_{10,t-1}\left[-\theta\left(\bar{\pi}_H\right)^{1-\epsilon}\pi_{H,t}^{\epsilon-1}\right]$$

(13). e_t :

$$0 = \Omega_{3,t-1}\left[-R_{t-1}^{*}\tfrac{c_t^{-\sigma}}{c_{t-1}^{-\sigma}}(\pi_t)^{-1}\right] + \Omega_{4,t}\left[\tfrac{-\pi_t^{*}}{\pi_t}\right]$$

(14). q_t :

$$0 = \Omega_{5,t}\left[-S_t\right] + \Omega_{6,t}\left[\tfrac{-1}{\eta-1}\left[\left(1-\alpha\right)q_t^{1-\eta}+\alpha\right]^{\frac{2-\eta}{\eta-1}}\left(1-\alpha\right)\left(1-\eta\right)q_t^{-\eta}\right] + \Omega_{7,t}\left[(w_t/a_t)S_t^{-1}q_t^{-2}\right]$$
$$+\Omega_{9,t}\left[-c_t^{-\sigma}y_t mc_t S_t\right] + \Omega_{10,t}\left[-c_t^{-\sigma}y_t S_t\right] + \Omega_{13,t}\left[\left(1-\alpha\right)\eta\left(q_t S_t\right)^{-\eta-1}c_t S_t + \alpha\eta\vartheta q_t^{-\eta-1}c_t^{*}\right]$$

(15). S_t :

$$0 = \Omega_{4,t}\left[1/S_{t-1}\right] + \Omega_{4,t+1}\left[-\beta S_{t+1}S_t^{-2}\right] + \Omega_{5,t}\left[-q_t\right] + \Omega_{6,t}\left[1\right] + \Omega_{7,t}\left[(w_t/a_t)q_t^{-1}S_t^{-2}\right]$$
$$+\Omega_{9,t}\left[-c_t^{-\sigma}y_t mc_t q_t\right] + \Omega_{10,t}\left[-c_t^{-\sigma}y_t q_t\right] + \Omega_{12,t}\left[-\vartheta c_t^{*}S_t^{1/\sigma-1}/\sigma\right]$$
$$+\Omega_{13,t}\left[\left(1-\alpha\right)\eta c_t\left(S_t q_t\right)^{-\eta-1}q_t\right]$$

(16). $\pi_{H,t}$:

$$0 = \Omega_{9,t-1}\left[-\theta\left(\bar{\pi}_H\right)^{-\epsilon}K_{1,t}\epsilon\pi_{H,t}^{\epsilon-1}\right] + \Omega_{10,t-1}\left[-\theta\left(\bar{\pi}_H\right)^{1-\epsilon}K_{2,t}(\epsilon-1)\pi_{H,t}^{\epsilon-2}\right]$$
$$+\Omega_{11,t}\left[-\left(\epsilon-1\right)\theta\left(\bar{\pi}_H\right)^{1-\epsilon}\pi_{H,t}^{\epsilon-2}\right] + \Omega_{16,t}\left[-\theta\left(\bar{\pi}_H\right)^{-\epsilon}\delta_{t-1}\epsilon\pi_{H,t}^{\epsilon-1}\right] + \Omega_{17,t}\left[1\right]$$

(17). c_t^{*} :

$$0 = \Omega_{12,t}\left[-\vartheta S_t^{1/\sigma}\right] + \Omega_{13,t}\left[-\alpha\vartheta q_t^{-\eta}\right] + \Omega_{14,t}\left[-1\right]$$

E Appendix 5: The Ramsey Steady State (s.s)

As per Gali and Monacelli (2005), I let domestic net producer price inflation rate be zero at the steady state and by symmetric equilibrium assumed, net foreign inflation rate is also zero at the s.s. Accordingly, the corresponding gross inflation rates are unity at the steady state (i.e. $\pi_H = \pi^* = 1$). I set the parameter ϑ, which denotes the relative size of the domestic economy, to 0.01, letting the size of the domestic economy, just 1/100 of the world economy. Gross productivity (a) is also set to unity at the s.s.

Domestic nominal interest rate at the s.s. is given by the consumption Euler equation as, $R = 1/\beta$. Again, by symmetry, foreign nominal interest rate which is an exogenous variable at s.s. is assumed to be same as that of domestic value (i.e. $R^* = 1/\beta$). The exchange rate growth rate, e_t (i.e. depreciation or appreciation of the exchange rate) is unity at the s.s. since the exchange rate is given by the ratio of domestic to foreign inflation, in the steady state. The real exchange rate, S_t, is set to unity as in Del Negro and Schorfheide (2008). Terms of trade in the s.s. is then given by, $q = ((1-\alpha)^{-1}(S^{\eta-1} - \alpha))^{1/(1-\eta)}$, which reduces to unity. The foreign consumption in the s.s. is explained by the relationship $c_t^* = (1/\vartheta) * (S^(- 1/\sigma)) * c$. Then the foreign output is immediately given by, $y_t^* = c_t^*$.

The marginal cost at the s.s. can be expressed as, $mc = (\epsilon - 1)/\epsilon$. The wage rate and labour hours at the s.s. are then specified as $w = amcqS$ and $N = (\delta y)/a$ respectively. The Ramsey s.s. is characterized with no inflation dispersion across sectors and therefore, relative prices remain set at unity (δ=1). Domestic consumption is finally explained by, $c = (N^\phi/w)^{-1/\sigma}$. Domestic consumption, wage rate and domestic consumer price index at the s.s are determined numerically by using Gauss-Newton method, which are found to be 0.915, 0.875 and 1.000 respectively.

F Appendix 6: The Solution Methodology

The set of nonlinear equilibrium conditions of the model can be summarised as follows:

$$E_t f\left(y_{t+1}, y_t, x_{t+1}, x_t\right) = 0 \tag{39}$$

where, E_t denotes mathematical expectation operator, conditional on information at time t, The vector y_t denotes the vector of non-predetermined endogenous variables of size $n_y \times 1$. The state vector x_t, is of size $n_x \times 1$, that can be partitioned in to two such that $x_t = [x_{1,t}, x_{2,t}]$, where first argument $x_{1,t}$ is the vector of predetermined endogenous variables while second argument argument $x_{2,t}$ is the vector of exogenous variables, that follows a stochastic process:

$$x_{2,t+1} = \Lambda x_{2,t} + \tilde{\eta}\sigma\varepsilon_{t+1} \tag{40}$$

where, both the vector $x_{2,t}$ and the innovation ε_t are of size $n_\varepsilon \times 1$. The scalar $\sigma \geqslant 0$ and the matrix $\tilde{\eta}$ of size $n_\varepsilon \times 1$ contains known parameters. ε_t denotes the innovations, s.t. $\varepsilon_t \sim i.i.d.(0, 1)$. The coefficient matrix Λ is supposed to have eigen values less than unit in absolute terms. In line with Schmitt-Grohe and Uribe (2004), the solution of the model takes the following form;

$$y_t = g\left(x_t, \sigma\right) \tag{41}$$

$$x_{t+1} = h\left(x_t, \sigma\right) + \eta\sigma\varepsilon_t \tag{42}$$

where g is a function that maps $R^{n_x} \times R^+$ into R^{n_y} and h is a function that maps $R^{n_x} \times R^+$ into R^{n_x} while η is a matrix of size $n_x \times n_\varepsilon$ is such that, $\eta = \begin{bmatrix} \emptyset \\ \tilde{\eta} \end{bmatrix}$. The above two equations describe the policy function and the transition function respectively. Then, the deterministic steady state of the model is defined as,

$$f\left(\bar{y}, \bar{y}, \bar{x}, \bar{x}\right) = 0 \tag{43}$$

The solution method is to find a second order accurate approximation of the functions g and h around the non-stochatic steady state, $x_t = \bar{x}$ and $\sigma = 0$.

Schmitt-Grohe and Uribe (2004) show that the first-order approximations suffers from a crucial limitation of 'certainty equivalence property', which is the first-order approximation to the unconditional means of the endogenous variables coincide with their respective steady state values. They, however, show that in a second-order approximation of the model, the expected value of any variable differs from its deterministic steady state by a constant term.[45] Hence, the unconditional mean of endogenous variables can considerably be different from their corresponding non-stochastic steady state values, in contrast to the case of first-order approximations. This is important in the present study as it can capture important effects of uncertainty on the average level of consumer welfare.

Accordingly, the conditional expected welfare can be expressed as follows;

$$\Pi_{0,t} = \bar{\Pi}_0 + \frac{1}{2}\, g_{\sigma\sigma}\, \Pi_0 \tag{44}$$

where $\Pi_{0,t} = \frac{U(\bar{c},\bar{N})}{(1-\beta)}$ is the welfare (lifetime utility) of households, evaluated at the non-stochastic steady state, $g_{\sigma\sigma}$ is a vector that captures how non-predetermined variables, y_t reacts to stochastic volatility of the second-order approximation of the policy function.

[45] For details see Schmitt-Grohe and Uribe (2004)

G Appendix 7: Results of the analysis: Cases 1, 2 and 3

Table A4: Conditional and unconditional welfare of alternative policy rules (Cases 1)

Responsiveness to inflation (ψ_π)	Responsiveness to output (ψ_y) 0.25	0.50	0.75	1.0	1.25	1.50
1.00	-34.5876	-34.5709	-34.5654	-34.5631	-34.5619	-34.5612
1.25	-34.5539	-34.5513	-34.5508	-34.5510	-34.5515	-34.5519
1.50	-34.5498	-34.5478	-34.5475	-34.5477	-34.5481	-34.5486
1.75	-34.5477	-34.5461	-34.5458	-34.5460	-34.5464	-34.5469
2.00	-34.5463	-34.5450	-34.5448	-34.5450	-34.5453	-34.5457
2.25	-34.5453	-34.5443	-34.5441	-34.5443	-34.5446	-34.5449
2.50	-34.5446	-34.5437	-34.5435	-34.5437	-34.5440	-34.5443
2.75	-34.5440	-34.5432	-34.5431	-34.5433	-34.5435	-34.5438
3.00	-34.5435	-34.5429	-34.5428	-34.5429	-34.5431	-34.5434
1.00	-34.5813	-34.5683	-34.5641	-34.5624	-34.5617	-34.5613
1.25	-34.5553	-34.5520	-34.5515	-34.5518	-34.5522	-34.5528
1.50	-34.5513	-34.5488	-34.5484	-34.5486	-34.5491	-34.5496
1.75	-34.5490	-34.5471	-34.5468	-34.5470	-34.5474	-34.5479
2.00	-34.5475	-34.5459	-34.5457	-34.5459	-34.5463	-34.5467
2.25	-34.5463	-34.5451	-34.5449	-34.5451	-34.5455	-34.5459
2.50	-34.5455	-34.5445	-34.5443	-34.5445	-34.5448	-34.5452
2.75	-34.5448	-34.5439	-34.5438	-34.5440	-34.5443	-34.5447
3.00	-34.5442	-34.5435	-34.5434	-34.5436	-34.5439	-34.5442

Notes: The upper panel refers to the conditional case while the lower panel refers to the unconditional case.

Table A5: Conditional and unconditional welfare cost of implementing alternative policy rules, instead of the optimal policy rule (Cases 1)

Responsiveness to inflation (ψ_π)	Responsiveness to output (ψ_y) 0.25	0.50	0.75	1.0	1.25	1.50
1.00	0.0718	0.0475	0.0395	0.0361	0.0343	0.0334
1.25	0.0227	0.0188	0.0181	0.0185	0.0191	0.0198
1.50	0.0167	0.0137	0.0133	0.0136	0.0142	0.0150
1.75	0.0136	0.0112	0.0109	0.0112	0.0117	0.0124
2.00	0.0116	0.0097	0.0094	0.0096	0.0101	0.0107
2.25	0.0101	0.0086	0.0083	0.0086	0.0090	0.0095
2.50	0.0090	0.0077	0.0075	0.0077	0.0081	0.0086
2.75	0.0082	0.0071	0.0069	0.0071	0.0075	0.0079
3.00	0.0075	0.0066	0.0064	0.0066	0.0069	0.0073
1.00	0.0626	0.0437	0.0376	0.0351	0.0340	0.0334
1.25	0.0246	0.0199	0.0192	0.0195	0.0202	0.0210
1.50	0.0188	0.0152	0.0146	0.0149	0.0156	0.0164
1.75	0.0155	0.0127	0.0122	0.0125	0.0131	0.0138
2.00	0.0133	0.0110	0.0107	0.0109	0.0115	0.0121
2.25	0.0116	0.0098	0.0095	0.0098	0.0103	0.0109
2.50	0.0103	0.0089	0.0086	0.0089	0.0094	0.0099
2.75	0.0093	0.0081	0.0079	0.0082	0.0086	0.0092
3.00	0.0085	0.0075	0.0074	0.0076	0.0080	0.0085

Notes: The upper panel refers to the conditional case while the lower panel refers to the unconditional case.

Table A6: Conditional and unconditional welfare of alternative policy rules (Cases 2)

Responsiveness to inflation (ψ_π)	Responsiveness to output (ψ_y) 0.25	0.50	0.75	1.0	1.25	1.50
1.00	-34.5871	-34.5708	-34.5654	-34.5630	-34.5619	-34.5612
1.25	-34.5538	-34.5512	-34.5508	-34.5510	-34.5514	-34.5519
1.50	-34.5497	-34.5477	-34.5474	-34.5477	-34.5481	-34.5486
1.75	-34.5477	-34.5461	-34.5458	-34.5460	-34.5464	-34.5468
2.00	-34.5463	-34.5450	-34.5448	-34.5450	-34.5453	-34.5457
2.25	-34.5453	-34.5442	-34.5441	-34.5442	-34.5445	-34.5449
2.50	-34.5446	-34.5437	-34.5435	-34.5437	-34.5440	-34.5443
2.75	-34.5440	-34.5432	-34.5431	-34.5433	-34.5433	-34.5438
3.00	-34.5435	-34.5429	-34.5428	-34.5429	-34.5431	-34.5434
1.00	-34.5807	-34.5681	-34.5640	-34.5623	-34.5616	-34.5612
1.25	-34.5551	-34.5520	-34.5515	-34.5517	-34.5522	-34.5527
1.50	-34.5512	-34.5487	-34.5484	-34.5486	-34.5491	-34.5496
1.75	-34.5489	-34.5470	-34.5467	-34.5470	-34.5474	-34.5478
2.00	-34.5474	-34.5459	-34.5457	-34.5459	-34.5463	-34.5467
2.25	-34.5463	-34.5451	-34.5449	-34.5451	-34.5454	-34.5458
2.50	-34.5455	-34.5444	-34.5443	-34.5445	-34.5448	-34.5452
2.75	-34.5448	-34.5439	-34.5438	-34.5440	-34.5440	-34.5447
3.00	-34.5442	-34.5435	-34.5434	-34.5436	-34.5439	-34.5442

Notes: The upper panel refers to the conditional case while the lower panel refers to the unconditional case.

Table A7: Conditional and unconditional welfare cost of implementing alternative policy rules, instead of the optimal policy rule (Cases 2)

Responsiveness to inflation (ψ_π)	Responsiveness to output (ψ_y) 0.25	0.50	0.75	1.0	1.25	1.50
1.00	0.0712	0.0473	0.0394	0.0360	0.0343	0.0333
1.25	0.0225	0.0187	0.0181	0.0184	0.0191	0.0198
1.50	0.0166	0.0137	0.0132	0.0136	0.0142	0.0149
1.75	0.0135	0.0112	0.0108	0.0111	0.0117	0.0123
2.00	0.0115	0.0096	0.0093	0.0096	0.0101	0.0107
2.25	0.0101	0.0086	0.0083	0.0085	0.0090	0.0095
2.50	0.0090	0.0077	0.0075	0.0077	0.0081	0.0086
2.75	0.0082	0.0071	0.0069	0.0071	0.0071	0.0079
3.00	0.0075	0.0066	0.0064	0.0066	0.0069	0.0073
1.00	0.0618	0.0434	0.0374	0.0350	0.0339	0.0333
1.25	0.0244	0.0198	0.0191	0.0194	0.0201	0.0209
1.50	0.0186	0.0151	0.0145	0.0149	0.0156	0.0163
1.75	0.0154	0.0126	0.0122	0.0125	0.0131	0.0138
2.00	0.0132	0.0110	0.0106	0.0109	0.0115	0.0121
2.25	0.0115	0.0098	0.0095	0.0098	0.0103	0.0109
2.50	0.0103	0.0088	0.0086	0.0089	0.0094	0.0099
2.75	0.0093	0.0081	0.0079	0.0082	0.0082	0.0092
3.00	0.0085	0.0075	0.0073	0.0076	0.0080	0.0085

Notes: The upper panel refers to the conditional case while the lower panel refers to the unconditional case.

Table A8: Conditional and unconditional welfare of alternative policy rules (Cases 3)

Responsiveness to inflation (ψ_π)	Responsiveness to output (ψ_y) 0.25	0.50	0.75	1.0	1.25	1.50
1.00	-34.6295	-34.5864	-34.5736	-34.5682	-34.5655	-34.5639
1.25	-34.5658	-34.5574	-34.5548	-34.5539	-34.5537	-34.5537
1.50	-34.5564	-34.5516	-34.5502	-34.5498	-34.5498	-34.5500
1.75	-34.5516	-34.5487	-34.5478	-34.5476	-34.5477	-34.5480
2.00	-34.5487	-34.5468	-34.5463	-34.5462	-34.5464	-34.5467
2.25	-34.5467	-34.5455	-34.5452	-34.5452	-34.5454	-34.5457
2.50	-34.5454	-34.5446	-34.5444	-34.5445	-34.5447	-34.5450
2.75	-34.5444	-34.5438	-34.5437	-34.5439	-34.5441	-34.5444
3.00	-34.5437	-34.5433	-34.5432	-34.5434	-34.5436	-34.5439
1.00	-34.6383	-34.5910	-34.5769	-34.5710	-34.5680	-34.5663
1.25	-34.5714	-34.5608	-34.5575	-34.5563	-34.5559	-34.5558
1.50	-34.5602	-34.5543	-34.5524	-34.5518	-34.5517	-34.5519
1.75	-34.5544	-34.5508	-34.5497	-34.5494	-34.5494	-34.5497
2.00	-34.5508	-34.5485	-34.5479	-34.5478	-34.5479	-34.5482
2.25	-34.5484	-34.5470	-34.5466	-34.5466	-34.5468	-34.5471
2.50	-34.5468	-34.5458	-34.5456	-34.5457	-34.5459	-34.5463
2.75	-34.5456	-34.5449	-34.5448	-34.5450	-34.5452	-34.5456
3.00	-34.5446	-34.5442	-34.5442	-34.5444	-34.5447	-34.5450

Notes: The upper panel refers to the conditional case while the lower panel refers to the unconditional case.

Table A9: Conditional and unconditional welfare cost of implementing alternative policy rules, instead of the optimal policy rule (Cases 3)

Responsiveness to inflation (ψ_π)	Responsiveness to output (ψ_y) 0.25	0.50	0.75	1.0	1.25	1.50
1.00	0.1330	0.0701	0.0514	0.0436	0.0396	0.0373
1.25	0.0400	0.0278	0.0240	0.0227	0.0223	0.0224
1.50	0.0262	0.0194	0.0172	0.0166	0.0167	0.0170
1.75	0.0192	0.0150	0.0137	0.0135	0.0137	0.0140
2.00	0.0150	0.0123	0.0115	0.0114	0.0117	0.0121
2.25	0.0122	0.0104	0.0099	0.0100	0.0103	0.0107
2.50	0.0102	0.0090	0.0087	0.0089	0.0092	0.0096
2.75	0.0088	0.0079	0.0078	0.0080	0.0084	0.0088
3.00	0.0077	0.0071	0.0071	0.0073	0.0077	0.0081
1.00	0.1459	0.0768	0.0562	0.0476	0.0432	0.0407
1.25	0.0482	0.0327	0.0278	0.0261	0.0255	0.0254
1.50	0.0319	0.0232	0.0204	0.0196	0.0195	0.0197
1.75	0.0234	0.0181	0.0164	0.0160	0.0161	0.0165
2.00	0.0182	0.0148	0.0138	0.0137	0.0139	0.0143
2.25	0.0147	0.0125	0.0119	0.0120	0.0123	0.0127
2.50	0.0122	0.0108	0.0105	0.0106	0.0110	0.0115
2.75	0.0105	0.0095	0.0094	0.0096	0.0100	0.0105
3.00	0.0091	0.0085	0.0084	0.0087	0.0092	0.0096

Notes: The upper panel refers to the conditional case while the lower panel refers to the unconditional case.