Role of Financial Intermediation in Capital Formation and Economic Growth: A Search Equilibrium Model

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Abstract:

This paper presents a model of search and matching equilibrium in financial economics by extending some ideas in the search and matching literature, such as Diamond (1982a) in trade, and Shapiro and Stiglitz (1984) and Masters (1998) in labor economics. The basic idea is that a lender owns capital and an entrepreneur owns entrepreneurial skill. Output is produced only when search generates a successful meeting between a lender and an entrepreneur. The autarkic equilibrium under bargaining-without-search and bargaining-with-search has been examined. It is shown that the autarkic investment in capital and skill is sub-optimal and inefficient. The inefficiency in autarky is due to search frictions and externality problems. The outcome under bargaining-with-search is better than that of bargaining-without-search though it is still inefficient. The introduction of competitive financial intermediation, however, removes most of the inefficiencies.

The outcome under asymmetric information – moral hazard on the part of entrepreneurs – is not efficient even in the in presence of financial intermediation. Any attempt that reduces market frictions and eases the search and matching process induces more investment, output and economic growth. Introduction of financial intermediation lowers monitoring cost and raises the probability of detecting shirking, thus creating opportunities for higher investment and output.

Keywords: Financial Intermediation, Entrepreneur, Capital, Search and Matching Equilibrium, Moral Hazard and Economic Growth.

JEL Classification: C78, G14, G21, O16

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1. Introduction
Growth theorists both classical, such as Adam Smith (1776) and Thomas Malthus (1798), and endogenous, such as Romer (1986 and 1990), Grossman and Helpman (1991), Aghion and Howitt (1992), Lucas (1988), and Rebelo (1991), have been trying to explain different mechanics of economic growth. Some well-known sources from which economic growth is derived are infrastructure, socio-economic and political institutions, technology, research and development (R&D), physical capital and human capital. Neoclassical growth theorists led by Robert Solow (1956) suggest that physical capital and technology are the main sources of economic growth. Endogenous growth models, such as Romer (1986 and 1990) and Lucas (1988) emphasize physical and human capital and their productivity, and assume that technological innovations are endogenous and derive from externalities in the accrual of capital. In the mainstream growth literature, however, the role of finance in capital accumulation and growth is largely overlooked.

Recent theoretical and empirical work, such as Wang and Williamson (1998), D. Diamond (1984), Fama (1983), Collin (1997), Bencivenga and Smith (1991 & 1998), Hansson and Jonung (1997), King and Levine (1993a & 1993b), and Levine (1997) suggest that financial intermediation is an important part of the story of economic growth. The focus of these studies is the link between financial development and economic growth. A well-known paper by Ross Levine (1997) explains how financial intermediary solves the market frictions and creates opportunities for economic growth. He argues that financial intermediaries mobilize savings, allocate resources, exert corporate control, facilitate risk management, and ease trades and contracts by solving market frictions that generate higher production and growth.

The role of financial intermediation in achieving higher economic growth by identifying two channels through which financial intermediaries stimulate economic growth has been examined. One is the usual capital accumulation channel where the introduction of a financial intermediary generates higher investment and thus higher economic growth. The other one is the efficiency gains, the impact of financial intermediation on economic growth that derives from sources other than increased capital accumulation. In order to see the usual capital accumulation channel, a model of search and matching equilibrium in a financial economies based on some ideas in the search and matching literature – Diamond (1982a), Shapiro and Stiglitz (1984) and Masters (1998) in trade and labor economics – is developed. The basic idea is that a lender owns capital and looks for an entrepreneur who needs capital, and an entrepreneur has production skills and searches for a lender. When a successful meeting occurs output is produced which in turn generates future investable funds, leading to capital accumulation and growth.

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2 This paper also presents a very important list of literature regarding finance-growth nexus.
Peter Diamond (1981) and Mortensen (1982) used search and matching concepts to explain the mechanisms of bilateral trade and their effects on decentralized market equilibrium. Following their work, theoretical as well as empirical studies of search and matching problems mushroomed in the various fields of economics. There is now a sizeable literature on search and matching equilibrium in labor economics and trade (e.g., Albrecht and Axell 1984, Lindeboom et al., 1994, Berman 1997, Andolfatto 1996, Coulson et al. 2001, P. Diamond 1982b, 1984 and 1990, Coles and Muthoo 1998, Rubinstein and Wolinsky 1985 & 1987, Rubinstein 1985). The matching process between buyers and sellers in trade or between workers and firms in the job markets is the main focus of these papers. Peter Diamond (1982a), Bester (1995) and Masters (1998) use search and matching approach to justify intermediaries. They try to model the role of intermediaries in matching two parties, such as workers and employers in Masters, buyers and sellers in Diamond, and lenders and borrowers in Bester. It has been shown in their papers that the solution in presence of an intermediary is Pareto superior in terms of higher employment and transactions. Another literature, such as Wang and Williamson (1998), D. Diamond (1984), Fama (1983), Collin (1996), Bencivenga and Smith (1991 & 1998), Chan (1983), Allen and Santomero (1998) examine the role of intermediation without applying search and matching concepts. Their investigation concerning the role of intermediaries in different economic scenarios produce convincing evidence that intermediaries play a positive role in various economic activities.

Bester (1995) is the only paper, to our knowledge, that investigates the role of financial intermediation using search and matching techniques. The author uses financial intermediary as a delegate for the investors. This generates a commitment advantage and induces more risky investment projects. The bargaining model of the current paper is completely different from that paper in the following ways. The current study uses completely different setup where bilateral search takes place; the above paper uses only unilateral search. We use financial intermediary as an independent agent to investigate possible sources of inefficiency, whereas the other paper uses intermediary as a delegation for investors only and does not investigate the reasons for inefficiency. Besides, our paper incorporates growth implications in the model and extends the basic model to include information asymmetry.

To facilitate exposition, we briefly describe the key ideas in the two background papers – P. Diamond (1982a) and Masters (1998). Peter Diamond develops a search and matching model to analyze the equilibrium in a barter economy with identical risk-neutral agents where trade is coordinated by a stochastic process. Because of the difficulty of successful trading, he shows that there are multiple steady state rational expectations equilibria where all non-corner solutions are inefficient. He then concludes that in a many-person and many-goods world representing a complex modern economy there will always remain unrecognized and unrealized trading opportunities. A central policy coordinator, by affecting individual production and trade incentives, could influence the outcomes of bilateral trades.
Masters’s (1998) assumes two kinds of infinitely lived individuals: workers who are restricted to invest in human capital, and employers (or firms) who are restricted to invest in physical capital. He shows in his Rubinstein bargaining model that the presence of search frictions has adverse impact on employment and investment in physical capital. In this case, the bargaining solution is sub-optimal for both workers and employers. Any attempt that makes matching process easier, such as intermediation, is Pareto improving in terms of higher investment, employment and output.

2. Model

Basic Assumptions

1. There are total \( N (=N_1+N_2) \) number of infinitely lived agents that are exogenously divided into two groups: \( N_1 \) and \( N_2 \). Here \( N_1 \) = number of lenders and \( N_2 \) = number of entrepreneurs. The fraction of lenders and entrepreneurs are given by \( n_1 = N_1/N \) and \( n_2 = N_2/N \) respectively.

2. The production function \( F(k,s) \) is assumed to be strictly increasing, strictly concave, twice differentiable and exhibits constant return to scale that is solely dependent on physical capital, denoted by \( k \), and production plan, design or skills, denoted by \( s \). Only lenders can save and invest capital \( k \), and only entrepreneurs have production or entrepreneurial skills \( s \). Assume that both inputs are completely durable. There is a onetime cost to acquire inputs, where \( C(k) \) and \( C(s) \) are the onetime investment cost functions for lender and entrepreneur respectively. The cost function is assumed strictly monotonic and convex. When a lender with capital \( k \) and an entrepreneur with skill \( s \) match, output is produced which is given by the production function \( F(k,s) \). Assume that the share of output that goes to lender is given by \( \phi \) and the remaining share \((1-\phi)\) goes to entrepreneur. All lenders and entrepreneurs are identical, and one lender needs a single entrepreneur to produce.

3. All agents derive utility from their output share, which is given by the discounted value of search minus cost. There is no consumption in this model.\(^3\)

4. There are two distinct states for each individual: matched and unmatched, referring to a match between a lender and an entrepreneur. If a pair – a lender and an entrepreneur – is matched, output is produced instantly after which both agents separate. The arrival rate of opportunities is as follows: an entrepreneur finds an unmatched lender at rate \( (n_1) \) and a lender finds an unmatched entrepreneur at rate \( (n_2) \).

In order to explain the role of financial intermediation, we examine agents’ maximization problem under the following two situations:

1. Autarkic Solution and
2. Intermediated Solution

Autarkic Solution

Utility Functions

Suppose \( u_1 \) represents present value of expected lifetime utility of a lender, \( v_{1e} \) and

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\(^3\) Just to keep the model simple. Inclusion of consumption will make the model very complicated as it will impose leakages in the process with varying degrees depending on the magnitudes of marginal propensity to consume (MPC).
\(v_{1u}\) are present value of a matched and unmatched lender respectively, and \(C(k)\) is the onetime cost. Then \(u_1\) is given by

\[(1) \quad u_1 = v_{1e} + v_{1u} - C(k)\]

Similarly, the present value of expected utility for a typical entrepreneur is

\[(2) \quad u_2 = v_{2e} + v_{2u} - C(s)\]

**Asset Value Equations**

The asset value equation for an unmatched lender is given by

\[(3) \quad r v_{1u} = n_2 [v_{1e} - v_{1u}]\]

This means an unmatched lender finds an entrepreneur at rate \(n_2\) — the fraction of population that has entrepreneurial skills.\(^4\) Collecting terms gives

\[(4) \quad v_{1u} = \left[\frac{n_2}{r + n_2}\right] v_{1e}\]

The asset value equation for a matched lender with her output share \(\phi\) is given by

\[(5) \quad r v_{1e} = \phi F + [v_{1u} - v_{1e}]\]

This assumes that production takes place instantaneously and the lender then becomes unmatched. Collecting terms in \(v_{1e}\) gives

\[(6) \quad v_{1e} = \frac{1}{r + n_2} [\phi F + v_{1u}]\]

Equation \(4\) and \(6\) can be solved simultaneously as

\[(7) \quad v_{1e} = \left[\frac{r + n_2}{r + n_2 + 1}\right] \phi F_d\]

and

\[(8) \quad v_{1u} = \left[\frac{n_2}{r + n_2 + 1}\right] \phi F_d\]

Here \(F_d\) is the discounted value of production. Substituting \(v_{1u}\) and \(v_{1e}\) into equation \(1\) gives

\[(9) \quad u_1 = \left[\frac{r + 2n_2}{r + n_2 + 1}\right] \phi F_d - C(k)\]

Similarly, the utility function for a typical entrepreneur who has production skill \(s\) is given by

\[(10) \quad u_2 = \left[\frac{r + 2n_1}{r + n_1 + 1}\right] (1 - \phi) F_d - C(s)\]

The bargaining process postulated in this model is similar to that of Rubinstein and Wolinsky (1985) and Masters (1998), where an agent is chosen randomly by nature to look for a partner. The other agent does the same. Acceptance of an offer ends bargaining with the proposed share of output implemented. If the proposal is rejected, a new offer will be made to a new partner in the same fashion. This bargaining game will be repeated until an offer is accepted. As in Trejos and Wright (1995) and Masters (1998), two cases of bargaining equilibrium will be studied in this paper under autarky. One is 'bargaining-without-search' where bargaining will not take place after the first

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\(^4\) For what follows, we could set \(n_1 = n_2 = \frac{1}{2}\) without any qualitative change in results. Also, we could model the transition probability as a function of \(n_2\); this would not change our conclusions.
round. The other is 'bargaining-with-search' that allows continuous negotiations.

**Bargaining-Without-Search**

When there is no search during bargain, the equilibrium solution can be obtained by maximizing respective expected lifetime utility for each agent. A lender and an entrepreneur will choose their capital \(k^a\) and skill \(s^a\) such that

\[
(11) \quad k^a = \arg \max_{k} u_1(k^a, s^a) \quad \text{and} \quad \phi = \frac{r + 2n_2}{r + n_2 + 1}
\]

\[
(12) \quad s^a = \arg \max_{s} u_2(k^a, s^a)
\]

Where 'a' denotes autarky. The first order conditions for this equilibrium are obtained by maximizing lifetime discounted expected utility functions \([equations (9) and (10)]\) with respect to \(k\) and \(s\) respectively

\[
(13) \quad \frac{r + 2n_2}{r + n_2 + 1} \phi f^k(k^a, s^a) = c^k(k^a) \quad \text{and} \quad \phi = \frac{r + 2n_1}{r + n_1 + 1} f^s(k^a, s^a) = c^s(s^a)
\]

Here \(f^k(.)\) and \(c^k(.)\) are the marginal product and marginal cost of capital, and \(f^s(.)\) and \(c^s(.)\) are marginal product and marginal cost of skill respectively. With equal bargaining power for lender and entrepreneur, it can be shown that the equilibrium output shares for both parties are also equal\(^5\), i.e., \(\phi = (1-\phi) = \frac{1}{2}\). Therefore, \([\frac{r + 2n_2}{r + n_2 + 1}]\phi < 1\) and

\[\frac{r + 2n_1}{r + n_1 + 1}(1-\phi) < 1,\]

and equilibrium investment of capital and skill under autarky would be such that \(f^k(k^a, s^a) > c^k(k^a, s^a)\) and \(f^s(k^a, s^a) > c^s(k^a, s^a)\). That is, the equilibrium pair of investments \((k^a, s^a)\) in 'bargaining-without-search' under autarky is inefficient. If \(k^*\) and \(s^*\) are investments that solve the above problem where marginal product and marginal cost are equal, i.e., \(f^k(k^*, s^*) = c^k(k^*, s^*)\) and \(f^s(k^*, s^*) = c^s(k^*, s^*)\), then \(k^*\) and \(s^*\) would be the efficient investment for capital and skill. We obviously have \(k^* > k^a\) and \(s^* > s^a\).

Figure-1 compares the investment of capital under 'bargaining-without-search' with the efficient amount of investment. A similar explanation is applicable for the other input, entrepreneurial skills. This inefficiency can be attributed to an externality – each agent ignores the fact that higher action raises the welfare of the other agent. This is variously known as coordination problem, sharecropper’s problem, and commitment problem. Inefficiency in autarky follows because any increase in investment of either input is Pareto improving (Proposition 1: proof in Appendix-A).

**Figure 1: Solution under autarky (where \(k^* > k^a\))**

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\(^5\) Let \(\phi^1\) and \(\phi^2\) be the offer made by the lender and entrepreneur respectively. Given voluntary participation of both agents in bargaining, the solution can be written as: \(\phi = \frac{1}{2}[\phi^1 + \phi^2]\), that is the offer for an output share made by an entrepreneur, in equilibrium, would be equal to average offer made by the lender and entrepreneur. Setting, \(\phi^1 = \phi^2 = \phi\) and given that \((\phi^1 + \phi^2 = 1)\), it immediately follows that: \(\phi^1 = \phi^2 = \phi = \frac{1}{2}\).
Bargaining-With-Search

In order to complete the bargaining model, we need to compare the equilibrium outcome of bargaining-with-search with those of bargaining-without-search as well as the outcome under intermediated equilibrium. Assume a Rubinstein bargaining model where \( \phi^1 \) and \( \phi^2 \) are the offers made by the lender and entrepreneur respectively, with \( \delta \) length of time between the rounds of bargaining. Given that offers arrive at interval \( \delta \), and assuming \( n_1=n_2=n=\frac{1}{2} \) for simplification, the value of lender’s share in the search market when the lender is stuck with a single entrepreneur is given by \( \phi^2 F_d + r \phi^2 F_d \). Under bargaining-with-search, the lender can continue to search for potential entrepreneurs with the possible output share = \( \frac{1}{2}(\phi^1 + \phi^2)F_d + \delta n [v_1(.) - \frac{1}{2}(\phi^1 + \phi^2)F_d] \). Where \( v_1 = v_{1e} + v_{1w} \).

In equilibrium, the lender will be indifferent between the two options and the Bellman’s equation is

\[
\phi^2 F_d + r \phi^2 F_d = \frac{1}{2}(\phi^1 + \phi^2)F_d + \delta n [v_1(.) - \frac{1}{2}(\phi^1 + \phi^2)F_d]
\]

or,

\[
\frac{\delta}{\frac{1}{2}(\phi^1 + \phi^2)F_d - \phi^2 F_d} = \frac{1}{r \phi^2 F_d - \frac{1}{2} n(\phi^1 + \phi^2)F_d - \delta n [v_1(.) - \frac{1}{2}(\phi^1 + \phi^2)F_d]}
\]

Similarly, the Bellman’s equation for an entrepreneur gives us

\[
\frac{\delta}{\frac{1}{2}(\phi^1 + \phi^2)F_d - \phi^1 F_d} = \frac{1}{r \phi^1 F_d - \frac{1}{2} n(\phi^1 + \phi^2)F_d - \delta n [v_1(.) - \frac{1}{2}(\phi^1 + \phi^2)F_d]}
\]
Where \( v_2 = v_{2e} + v_{2u} \). Since \( \phi^1 = (1 - \phi^2) \), equation (17) could be written as

\[
\frac{\delta}{1/2[(1-\phi^2)] + (1-\phi^1)]F_d - (1-\phi^2)F_d} = \frac{1}{r(1-\phi^2)F_d - n v_2 + \frac{1}{2} n [(1-\phi^2) + (1-\phi^1)]F_d}
\]

Limiting \( \delta \to 0 \) and setting \( \phi^1 = \phi^2 = \phi \), it can be shown from equation (16) and (18) that \(^6\)

\[
\phi = \frac{(r + n)F_d + n v_1 - n v_2}{(2r + 1)F_d}
\]

Recall that

\[
v_1 = v_{1e} + v_{1u} = \left[ \frac{r + 2n}{r + n} + 1 \right] \phi F_d
\]

Substituting (19) in to (20) gives

\[
v_1 = \frac{(r + 2n)(r + n)F_d}{(r + n + 1)(2r + 1) - n(r + 2n)} - \frac{(r + 2n)(r + n)v_2}{(r + n + 1)(2r + 1) - n(r + 2n)}
\]

Similarly,

\[
v_2 = \frac{(r + 2n)(r + n)F_d}{(r + n + 1)(2r + 1) - n(r + 2n)} - \frac{(r + 2n)(r + n)v_1}{(r + n + 1)(2r + 1) - n(r + 2n)}
\]

Substituting (21) into the lifetime utility function for lender, we get

\[
u_1 = \frac{(r + 2n)(r + n)F_d}{(r + n + 1)(2r + 1) - n(r + 2n)} - \frac{(r + 2n)(r + n)v_2}{(r + n + 1)(2r + 1) - n(r + 2n)} - C
\]

Similarly, the utility function for an entrepreneur can be written as

\[
u_2 = \frac{(r + 2n)(r + n)F_d}{(r + n + 1)(2r + 1) - n(r + 2n)} - \frac{(r + 2n)(r + n)v_1}{(r + n + 1)(2r + 1) - n(r + 2n)} - C
\]

Now, the lender’s problem is to choose \( k^a \) and the entrepreneur’s problem is to choose \( s^a \) such that \( u_1 \) and \( u_2 \) are maximized respectively. The first order conditions for these maximization problems are

\[
f^k(k^a, s^a) = c^k(k^a) \quad \text{and} \quad f^s(k^a, s^a) = c^s(k^a)
\]

Here \( k^a \) and \( s^a \) are the equilibrium investments in capital and skill under bargaining-with-search. Given that \( n_1 = n_2 = n = \frac{1}{2} \), it is readily observable that

\(^6\) Limiting \( \delta \to 0 \), setting \( \phi^1 = \phi^2 = \phi \), and using equation (16) and (18), we can write

\[
r \phi F_d - n v_1 + n \phi F_d = r F_d - r \phi F_d - n v_2 + n F_d - n \phi F_d
\]

or, \( \phi (2r + 2n) F_d = (r + n) F_d + n v_1 - n v_2 \), (replacing \( 2n \) by \( 1 \))

or, \( \phi = \frac{(r + n) F_d + n v_1 - n v_2}{(2r + 1) F_d} \)
Again, the autarkic solution results in inefficient investment of both capital and skill. The autarkic solutions, therefore, are inefficient regardless of the nature of bargaining—bargaining-without-search or bargaining-with-search. However, the outcome under bargaining-with-search is better than bargaining-without search. This is because,

\[
1 > \frac{(r+2n)(r+n)}{(r+n+1)(2r+1) - n(r+2n)} \Rightarrow f^k(.) > c^k(.) \quad \text{and} \quad f^s(.) > c^s(.)
\]

For example, if we assume that \( r = 0 \), \( \phi = \frac{1}{2} \) and \( n = \frac{1}{2} \), it can be shown that

\[
\frac{(r+2n)(r+n)}{(r+n+1)(2r+1) - n(r+2n)} = \frac{1}{2} \quad \text{and} \quad \frac{r+2n}{r+n+1} \phi = \frac{1}{3}.
\]

### Intermediated Solution

Financial intermediation can take different forms depending on what the intermediary does. We specify two types of intermediary. First, financial intermediary is a matchmaker, where she matches lenders with borrowers. Other examples of matchmaker are employment agencies that match workers with firms, real estate brokers that match home buyers with sellers. Second, financial intermediary is an owner-manager where she is both a manager and the residual profit claimant. General examples of this type of intermediary are investment banks, stock-brokers, and used-car dealers.

### When Financial Intermediary is a Matchmaker

Let us consider a situation where financial intermediaries, such as banks and other financial institutions just facilitate the matching process between lenders and entrepreneurs by taking deposits from lenders and providing loans to entrepreneurs in a perfectly competitive environment. Competition among intermediaries, lenders, entrepreneurs, and financial intermediaries take the rental rate of capital\(^7\), \( R_1 \), as given. Lender and entrepreneur then lend and borrow at that rate via intermediaries. We assume that the entrepreneur manages and keeps residual profits. Therefore, the problem for the entrepreneur is to choose \( k \) and \( s \) to maximize

\[
(27) \ u_2 = F_d - k \cdot R_{1d} - C(s)
\]

Where, \( R_{1d} \) is discounted value of the rental rate. In presence of financial intermediation, we assume neither borrower nor lender is subject to any search friction. In this case, a lender could deposit her capital with a financial intermediary and get return \( R_1 \). Therefore, a lender’s value in the search market would be

\[
(28) \ rv_l = r(v_{le} + v_u) = kR_i
\]

or,

\[
\Phi_{new} = \Phi_{old} + \Delta \Phi
\]

\(^7\) Financial intermediary in our model do not charge any fee, an assumption that can be changed. Here we assume long run zero profit condition.
Substituting \( v_i = k . \frac{R_i}{r} = k . R_{1d} \) into equation (1), we get

\[
(30) \ u_1(k, S) = k . R_{1d} - C(k)
\]

A lender chooses capital \((k)\) such that

\[
(31) \ \text{Max}_{(k)} u_1(k, S) = k . R_{1d} - C(k)
\]

If \( \hat{k}(= \hat{k}_s = \hat{k}_d) \) and \( \hat{s} \) are equilibrium investments for capital and skill by the lender and the entrepreneur respectively, the first order conditions from equations (27) and (31) are

\[
(32) \ f^k(\hat{k}_d, \hat{s}) = R_{1d}
\]

\[
(33) \ f^s(\hat{k}_d, \hat{s}) = c^s(\hat{s}) \text{ and}
\]

\[
(34) \ R_{1d} = c^k(\hat{k}_s)
\]

Substitution equation (34) into (32) yields

\[
(35) \ f^k(\hat{k}_d, \hat{s}) = c^k(\hat{k}_s)
\]

Equation (33) and (35) are market-clearing conditions where investments in both inputs are efficient. Therefore, in presence of financial intermediaries \( \hat{k} = k^* > k^a \) and \( \hat{s} = s^* > s^a \).

A perfectly competitive market for financial intermediation is crucial in generating the efficient allocation condition. If that market is not perfectly competitive, a different kind of inefficiency arises. Here we do not explore distortions that occur due to intermediaries’ having market power. Figure 2 compares the investment in capital under autarky with the amount under intermediation. Unlike the solution under autarky, it can be shown that in the equilibrium increased investment by any agent is no longer Pareto improving (Proposition 2: proof in Appendix-A).
When Financial Intermediary is a Manager

Here financial intermediary manages and pays the lender and entrepreneur some predetermined rates of return $R_1$ and $R_2$ for capital and skill respectively, and is the residual profit claimant. Both rates are offered in perfectly competitive markets where all parties take these rates as given. Given that the intermediary’s problem is to choose $k$ and $s$, such that

$$
\text{(36) Max}_{(k,s)} [F_d - k \cdot R_{1d} - s \cdot R_{2d}]
$$

In a frictionless world, the lender and the entrepreneur will choose capital and skill respectively such that

$$\text{(37) Max}_{(k)} u_1 = k \cdot R_{1d} - C(k)$$

$$\text{(38) Max}_{(s)} u_2 = s \cdot R_{2d} - C(s)$$

If $\hat{k} = \hat{k}_d$ and $\hat{s} = \hat{s}_d$ are the equilibrium amounts, the first order conditions are

$$\text{(39) } f^k(\hat{k}_d, \hat{s}) = R_{1d}$$

$$\text{(40) } f^s(\hat{k}_d, \hat{s}_d) = R_{2d}$$

$$\text{(41) } R_{1d} = c^k(\hat{k})$$

$$\text{(42) } R_{2d} = c^s(\hat{s})$$
Replacing \( R_{1d} \) and \( R_{2d} \), we get

\[
(43) \quad f^k(\hat{k}_d, \hat{s}) = c^k(\hat{k}_e) \quad \text{and} \\
(44) \quad f^s(\hat{k}_e, \hat{d}_e) = c^s(\hat{s}_e)
\]

Equations (43) and (44) are identical to those that hold when the financial intermediary is a matchmaker. These conditions imply that \( \hat{k} = k^* > k^a \) and \( \hat{s} = s^* > s^a \) i.e., investment of both inputs under financial intermediation are optimal and higher than in autarky. As the production function is strictly increasing in capital and skill, the output under financial intermediation would also be higher. Therefore, the solution in presence of financial intermediation is superior to the solution under autarky in terms of higher investment and output via capital accumulation channel.

Law of Motion (LOM) for Capital

As it is assumed that all agents derive utility from their output share with no consumption, the equilibrium law of motion for capital under autarky and financial intermediation would be, respectively, as follows:

\[
(45) \quad k_{t+1}^a = \frac{r + 2n_2}{r + n_2 + 1} \phi F(k_t^a, s_t^a) \\
(46) \quad \hat{k}_{t+1} = R_t \hat{k}_t = \phi F(\hat{k}_t, \hat{s}_t)
\]

Setting \( k_{t+1}^a = k_t^a \) and \( \hat{k}_{t+1} = \hat{k}_t \) in the above equations, we can get steady state capital under autarky (\( k^{a*} \)) and financial intermediation (\( \hat{k}^* \)) respectively. The equilibrium law of motions and respective levels of steady state capital in both situations are shown in Figure 3 where, \( \hat{k}^* > k^{a*} \). As long as the production function \( F(k, s) \) is strictly increasing, strictly concave and subject to Inada Condition, there would be a unique and stable steady-state equilibrium. Therefore, the level of output produced under financial intermediation \([F(\hat{k}, \hat{s})]\) would be greater than the level of output in autarky \([F(k^a, s^a)]\).

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\(^8\) Here, \( \phi F(\hat{k}_t, \hat{s}_t) \) is the equivalent amount of production shares.
3. **Some Extensions**

**When Lenders and Entrepreneurs are Heterogeneous**

Initially, we assumed that both lenders and entrepreneurs are homogeneous where one lender (entrepreneur) needs just one entrepreneur (lender) for a successful meeting. Now, suppose there are two types of lenders and entrepreneurs: type-A and type-B. In order to have a successful meeting, lender type-A must meet an entrepreneur of type-A. This heterogeneity can be sector specific or based on specialization. For example, a lender (entrepreneur) who is living in a mega city might not be interested in any rural based lending. Assume that lending (borrowing) contract will not take place, unless a lender (entrepreneur) finds an entrepreneur (lender) of the same kind. In this case, the probability of a successful match would be significantly lower. If ‘j’ is the index of heterogeneity, this could be extended up to ‘N’ types of lenders and entrepreneurs. Where, j= 1, 2, 3 … N. When j=1, all of the lenders and entrepreneurs are considered to be homogeneous. Assuming two types of lenders and entrepreneurs: type-A and type-B, we can write the following:

\[ N_1 = \text{Number of lenders} = N_{1A} + N_{1B} \]  
\[ N_2 = \text{Number of entrepreneurs} = N_{2A} + N_{2B} \]

Now, if we assume that \( N_{1A} = N_{1B} \) and \( N_{2A} = N_{2B} \), then the ratio of lender type-A = the ratio of lender type-B = \( \frac{1}{2} n_1 \), and the ratio of entrepreneur type-A = the ratio of entrepreneur type-B = \( \frac{1}{2} n_2 \). To see the role of financial intermediation, given the assumption of heterogeneity, we need to solve the problem of utility maximization under the following two situations:
1. In autarky and
2. In presence of financial intermediary.

Solution under Autarky

Utility Functions

The utility functions for type-j lender and entrepreneur can be written as

\( u_{1j} = v_{1j} - C(k_j) \)  
(47)

\( u_{2j} = v_{2j} - C(s_j) \)
(48)

Where \( u_{1j} \) is the present value of expected lifetime utility of type-j lender, \( v_{1j} \) is the value of a j-type lender in the search markets, \( C(k_j) \) is the cost function for a j-type lender who invests \((k_j)\), \( u_{2j} \) is the present value of expected lifetime utility of type-j entrepreneur, \( v_{2j} \) is the value of a j-type entrepreneur in the search markets and \( C(s_j) \) is the cost function for a j-type entrepreneur who invests \((s_j)\). Incorporating new arrival rates for lenders and entrepreneurs into the original utility functions, it can be shown that

\( u_1 = \left[ \frac{r + n_2}{2} \right] \phi F_d - C(k_j) \)  
(49)

\( u_{2j} = \left[ \frac{r + n_1}{2} \right] (1 - \phi) F_d - C(s_j) \)  
(50)

Assume that the cost of capital is same for all lenders i.e., \( C(k_j) = C(k) \) and the cost of skill is also same for all entrepreneurs i.e., \( C(s_j) = C(s) \). If \((k^{aj})\) and \((s^{aj})\) are equilibrium amount of investment, the following first order conditions can be obtained

\( \frac{r + n_2}{2} \phi f^{k} = c^k (k^{aj}) \equiv c(k) \)  
(51)

\( \frac{r + n_1}{2} (1 - \phi) f^{s} = c(s^{aj}) \equiv c(s) \)  
(52)

Comparing these first order conditions with equations (13) and (14), it can be shown that investments by both agents under heterogeneity are less than that of homogeneity assumption \(^9\) i.e., \( k^{aj} < k^a \) and \( s^{aj} < s^a \). Where, \((k^a)\) and \((s^a)\) are the equilibrium investments by lender and entrepreneur respectively under the assumption of homogenous lender and entrepreneur. Figure 4 compares the equilibrium investments made by lender under the homogeneity and heterogeneity assumptions. The above outcome is under bargaining-without-search. As we observe earlier that the solution under both bargaining-with-search and bargaining-without-search are inefficient and inferior to the intermediated outcome, no discussion on bargaining without search is done.

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\(^9\) Because \( \frac{r + n_2}{r + n_2/2 + 1} \phi < \frac{r + 2n_2}{r + n_2 + 1} \phi \) and \( \frac{r + n_1}{r + n_1/2 + 1} (1 - \phi) < \frac{r + 2n_2}{r + n_2 + 1} (1 - \phi) \)
Figure 4: Solution under autarky (where $k^* > k^a > k_{aj}$)

Solution in Presence of Financial Intermediation

We have seen that the intermediated solution can be solved in two ways: when a financial intermediary is a matchmaker and when a financial intermediary is a manager. Under the assumption of identical agents, we have shown earlier that investments in both inputs are higher and efficient in the intermediated solution. Since the outcome under homogenous agents is better than that of heterogeneous agents and the outcome in presence of intermediaries is better than that of homogenous agents, then intermediated solution must be better than the solution under heterogeneous agents.

When Information is Asymmetric

In our analysis, so far we assumed away information asymmetry, which deals with the study of decisions in transactions where one party has more or better information than the other. This creates an imbalance of power in transactions which can sometimes cause the transactions to go awry, a kind of market failure in the worst case. Examples of this problem are adverse selection, moral hazard, and information monopoly. Most commonly, information asymmetries are studied in the context of principal–agent problems. Information asymmetry causes misinforming and is essential in every communication process. In 2001, the Nobel Prize in Economics was awarded to George Akerlof, Michael Spence, and Joseph E. Stiglitz for their “analyses of markets with asymmetric information.” In order to introduce the second channel through which financial intermediation influences economic growth, let us assume that there are some uncertainties to the realization of contracted output. Consider the following utility function for a typical lender:

$$u_1 = v_{1c} + v_{1u} - C(k)$$
Suppose, p is the probability that the output would be exactly same as contracted and (1-p) is the probability that output would be zero\(^{10}\).

**Autarkic Solution under Uncertainty**

Asset value equations for a lender:

\[(54) \ r \ v_{1u} = n_2 [v_{1e} - v_{1u}] \]

\[(55) \ r v_{1e} = [p \phi F_d + (v_{1u} - v_{1e})] \]

Equation (54) and (55) can be solved simultaneously as:

\[(56) \ v_{1e} = \left[\frac{r + n_2}{r + n_2 + 1}\right] p \phi F_d \]

\[(57) \ v_{1u} = \left[\frac{n_2}{r + n_2 + 1}\right] p \phi F_d \]

Substituting \(v_{1u}\) and \(v_{1e}\) into (53) gives:

\[(58) \ u_i = \left[\frac{r + 2n_2}{r + n_2 + 1}\right] p \phi F_d - C(k) \]

Similarly, the utility function for a typical entrepreneur who has production skill is given by:

\[(59) \ u_2 = \left[\frac{r + 2n_1}{r + n_1 + 1}\right] p (1 - \phi) F_d - C(s) \]

Lender’s problem is to choose \(k\), such that

\[(60) \ \text{Max} \ (k) \ u_i = \left[\frac{r + 2n_2}{r + n_2 + 1}\right] p \phi F_d - C(k) \]

First order condition, yields:

\[(61) \ \frac{r + 2n_2}{r + n_2 + 1} p \phi f^k(\cdot) = c^k(\cdot) \]

Clearly, \(\frac{r + 2n_2}{r + n_2 + 1} \phi p < 1 \implies f^k(\cdot) > c^k(\cdot)\) and therefore, investment in capital is inefficient. If we compare equation (61) with (13), it is quite obvious that the autarkic outcome under uncertainty would be inferior to the outcome under full information.\(^{11}\)

Now, we need to find intermediated outcome with uncertainty.

**Intermediated Solution under Uncertainty**

We know that a financial intermediary can play its role in two ways: as a matchmaker and as a manager. It has been shown earlier that the introduction of an intermediary leads to efficient investment for both inputs regardless the type of roles it plays. In this section of the paper, we will consider financial intermediary just as a matchmaker. Suppose, entrepreneur manages and a residual claimant, pays a rental rate \(R_1\) for capital in a perfectly competitive environment. Because of the competitive nature of the market, all agents take \(R_1\) as given. A lender will choose capital and an

---

\(^{10}\) Zero output is assumed for simplicity, any other output level (less than \(F_d\)) with the probability \((1-p)\) will not change our conclusion.

\(^{11}\) Because, \(\frac{r + 2n_2}{r + n_2 + 1} \phi p < 1 \implies f^k(\cdot) > c^k(\cdot)\)
entrepreneur will choose both capital and skill such that
(62) \( \max_{k} u_{i}(k, S) = k.R_{1d} - C(k) \)
(63) \( \max_{k,s} [ p.F_{d} - k.R_{1d} - s.R_{2d} ] \)

If the solution to this problem gives \( \hat{k} \) and \( \hat{s} \) as the equilibrium investment for capital and skill then the first order conditions for this problem would be
(64) \( p.f^{k}(.) = R_{1d} \)
(65) \( p.f^{s}(.) = R_{2d} \)
(66) \( R_{1d} = c^{k}(.) \)
Replacing \( R_{1d} \), we can write
(67) \( p.f^{k}(.) = c^{k}(.) \)
Note that \( p<1 \Rightarrow f^{k}(.) > c^{k}(.) = R_{1d} \) and \( f^{s}(.) > R_{2d} \), which means inefficient investment in both inputs under asymmetric information even in the presence of financial intermediation. Inefficiencies, in this case, are only due to production uncertainty (p). If we set \( p=1 \), it’s immediately observable that all the inefficiencies disappear. Given the fact that solution under asymmetric information is not efficient, a model of entrepreneurial moral hazard is developed. Our model is similar to that of Shapiro and Stiglitz (1984) where they consider moral hazard on the part of workers. Under a costly monitoring scenario, our paper attempts to outline two conditions – voluntary monitoring condition and no shirking condition – to examine the role of financial intermediation.

**Voluntary Monitoring Condition (VMC)**

Voluntary Monitoring Condition (VMC) is a condition for a lender where monitoring is voluntary. In order to derive VMC for a lender, let us consider two production levels: high = \( F^{H} \) and low = \( F^{L} \) where an entrepreneur is responsible for production activities. Also, assume that \( m \) is the monitoring cost, which is an increasing function of matched borrower per lender, and \( b_{1} \) is an exogenous project termination rate.

Given the possible outcome of the contract, a lender may or may not prefer to monitor an entrepreneur. If a lender monitors, \( F^{H} \) will be produced with probability \( p_{1} \) and \( F^{L} \) will be produced with probability \( (1-p_{1}) \). On the other hand, if he decides not to monitor, output \( F^{H} \) will be produced with probability \( p_{2} \) and output \( F^{L} \) will be produced with probability \( (1-p_{2}) \). Thus, the asset value equations for a lender under both situations are respectively given by

(68) \( r_{1e}^{m} = \phi(p_{1}F^{H} + (1-p_{1})F^{L}) - m + b_{1}(v_{1u} - v_{1e}^{m}) \)
(69) \( r_{1e}^{m} = \phi(p_{2}F^{H} + (1-p_{2})F^{L}) + b_{1}(v_{1u} - v_{1e}) \)

Assuming \( y_{1}^{H} = \phi(p_{1}F^{H} + (1-p_{1})F^{L}) \) and \( y_{1}^{L} = \phi(p_{2}F^{H} + (1-p_{2})F^{L}) \), equation (68) and (69) can be rewritten as

(70) \( v_{1e}^{m}(. \ ) = \frac{1}{r + b_{1}} \left[ y_{1}^{H} - m + b_{1}v_{1u} \right] \)
(71) \( v_{1e}(. \ ) = \frac{1}{r + b_{1}} \left[ y_{1}^{L} + b_{1}v_{1u} \right] \)
Applying VMC i.e., \( v_{ie}^m(.) \geq v_{ie}(.) \) gives

\[
(72) \quad y_H^v \geq m + y_L^L
\]

Here monitoring cost ‘m’ is an increasing function of matched borrowers per lender, say \( N_{2m} \). Therefore, equation (72) can be rewritten as

\[
(73) \quad y_H^v \geq m(N_{2m}^+) + y_L^L
\]

Equation (73) is an algebraic expression of VMC, which is shown in Figure 5.

Figure 5: Voluntary Monitoring Condition (VMC)

Voluntary Monitoring Condition (VMC)

Monitoring is profitable

Monitoring is not profitable

\[
\begin{align*}
0 \quad & \quad 100\% \\
\% \text{ of matched borrowers}
\end{align*}
\]

No Shirking Condition (NSC)

In order to derive an incentive compatible condition for an entrepreneur, let us suppose that an entrepreneur exerts costly effort ‘e’ in order to generate high output \( F_H \). We assume that high output occurs with probability \( q_1 \) and low output occurs with probability \( 1-q_1 \). When an entrepreneur shirks (\( e=0 \)), however, she gets output share \( (1-\phi)F_H \) with probability \( (q_2<q_1) \) and output share \( (1-\phi)F_L(.) \) with probability \( (1-q_2) \). In this case, assume that the rate of project termination is given by \( (d_2+g_2) \). Here \( d_2 \) is the natural termination rate and \( g_2 \) is the probability that an entrepreneur will be caught while shirking, which is a decreasing function of matched borrowers per lender. Given that the project acquisition rate for an entrepreneur is ‘\( a_2 \)’, we can write the following asset value equations

\[
(74) \quad r_{2e} = (1-\phi)[q_1 F_H + (1-q_1) F_L] - e + d_2(v_{2u} - v_{2e})
\]

\[
(75) \quad r_{2e} = (1-\phi)[q_2 F_H + (1-q_2) F_L] + (d_2 + g_2)(v_{2u} - v_{2e})
\]
Equation (74) is discounted lifetime utility of a non-shirker, equation (75) is discounted lifetime utility of a shirker and equation (76) is the discounted utility for an unmatched entrepreneur. The incentive compatibility condition for an entrepreneur not to shirk (NSC) is given by \( v^N_n \geq v^S_n \).

Assuming \( y^H_2 = (1-\phi)\left[q_1 F^H + (1-q_1)F^L\right] \) and \( y^L_2 = (1-\phi)\left[q_2 F^H + (1-q_2)F^L\right] \), equation (74), (75) and (76) could be rewritten as

\[
\begin{align*}
(77) \quad v^N_{2e}(.) & = \frac{1}{r + d_2} \left[ y^H_2 - e + d_2 v_{2u} \right] \\
(78) \quad v^S_{2e}(.) & = \frac{1}{r + d_2 + g_2} \left[ y^L_2 + (d_2 + g_2) v_{1u} \right] \\
(79) \quad v^L_{2e}(.) & = \frac{r + a_2}{a_2} v_{2u}
\end{align*}
\]

Applying (NSC) i.e., \( v^N_n \geq v^S_n \) and simplifying, we get

\[
(80) \quad y^H_2 \geq e + \left[ \frac{a_2 + d_2 + r}{a_2 + d_2 + g_2 + r} \right] y^L_2
\]

Since \( g_2 \) is an decreasing function of matched borrowers per lender, equation (80) can be rewritten as

\[
(81) \quad y^H_2 \geq e + \left[ \frac{a_2 + d_2 + r}{a_2 + d_2 + g_2 (N^m_2) + r} \right] y^L_2
\]

Equation (81) is an expression for no shirking condition. The graphical representation of this NSC is shown in Figure 6.

Figure 6: No Shirking Condition (NSC)
In order to see the role of financial intermediation on investment and output, the outcome with and without intermediary needs to be compared. An introduction of a financial intermediary will bring following changes:

**A change in the monitoring cost (m)**

D. Diamond (1984), Williamson (1996), and Wang and Williamson (1998) argue that a financial intermediary has monitoring and screening cost advantage over individuals. That means with the introduction of financial intermediary the monitoring cost (m) would be lower and as a result the VMC line will shift down creating opportunities for higher output.

**A change in the probability of getting caught (g₂) while shirking**

As a financial intermediary faces lower monitoring cost, it is also feasible to argue that the introduction of financial intermediation will increase the probability of catching entrepreneur (g₂) while shirking (Shapiro and Stiglitz, 1984). As a result, the NSC line will also shift down. Putting both changes together, as shown in Figure 7, the new equilibrium will bring in more opportunities for production.

Therefore, the introduction of financial intermediation creates opportunities for more output and economic growth because of better allocation of resources, lower monitoring cost and higher probability of catching shirking. This explains the second channel, the efficiency gains, through which financial intermediation generates higher economic growth.

Figure 7: VMC and NSC with Financial Intermediation
4. Summary and Conclusion

The present study is an attempt to explain the role of financial intermediation in capital formation and economic growth. This study uses a model of matching and bargaining to investigate the welfare effects of financial intermediation. This study also explores the role of financial intermediation under entrepreneurial moral hazard. We show that the introduction of financial intermediaries stimulates economic growth because of higher capital accumulation as well as efficiency gains. The main results of this paper are:

Under the assumptions of identical agents, a perfectly competitive environment and no information problem, the amount of investment and output are sub-optimal in autarky. This sub-optimality in investment and output in autarky remain even if repeated search is allowed. However, when repeated search is allowed, the investment and output are better. This is mainly due to search frictions and an externality in the matching process. The autarkic solution under heterogeneous agents is inferior to that of homogeneous agents in terms of lower investment and output. This is because the assumption of heterogeneous agents induces more frictions in the matching process. In the absence of information asymmetry, the introduction of financial intermediary in a perfectly competitive environment ensures payment to each factor according to their marginal product. Therefore, investment of capital and skill are Pareto efficient, leading to higher steady state output and economic growth. This is the usual capital accumulation channel where output is higher because of higher investment.

Under entrepreneurial moral hazard, investment is not efficient even in the presence of financial intermediation. Any attempt that reduces the market frictions eases the search and matching process for both parties and makes possible more investment, output and economic growth. Specifically, the introduction of financial intermediaries reduces market frictions by lowering the monitoring cost and by increasing the probability of detecting shirking. As a result, financial intermediaries create opportunities for more output even in the presence of information asymmetry. This is the efficiency gains where output is higher not because of higher investment. This derives from financial intermediary creating better allocation of investment resources, possibly via improved matching, monitoring and organization between the lender and borrower. Therefore, the intermediated outcome is always better than the autarkic outcome in terms of higher investment and output regardless the information problem and other market conditions.

The paper, thus, identifies two channels through which financial intermediaries stimulate economic growth. One is the usual capital accumulation channel where the introduction of financial intermediary generates higher capital and thus higher economic growth. The other one is the efficiency gains where the introduction of financial intermediary reduces frictions, information asymmetry, and monitoring cost, eases contracts and trades and increases the probability of catching shirking, productivity and the quality of investments. Thus, the efficiency effect is the impact of financial intermediation on economic growth that comes from sources other than increased capital accumulation.
References:


Appendix A

Propositions

**Proposition 1:** An increase in investment of either input in autarky is Pareto improving.

**Proof:** The utility functions for the lender and the entrepreneur in equilibrium under autarky are given by

(A1) \[ u_1 = \left( \frac{r + 2n_2}{r + n_2 + 1} \right) \phi F_d (k, s) - C(k) \]

(A2) \[ u_2 = \left( \frac{r + 2n_1}{r + n_1 + 1} \right) (1 - \phi) F_d (k, s) - C(s) \]

Assume that there is an infinitesimal positive change in the equilibrium levels of investment i.e., \((dk, ds)\geq (0,0)\). Taking total differentiation with respect to \(k\) and \(s\), it can be shown\(^{12}\) that

(A3) \[ \frac{du_1(\cdot)}{ds} = \left( \frac{r + 2n_2}{r + n_2 + 1} \right) \phi f^s (k^a, s^a) > 0 \]

(A4) \[ \frac{du_2(\cdot)}{dk} = \left( \frac{r + 2n_1}{r + n_1 + 1} \right) (1 - \phi) f^k (k^a, s^a) > 0 \]

These two conditions imply that any positive action by an entrepreneur (lender) increases the utility of lender (entrepreneur). Since, an increase in lender’s (or entrepreneur’s) input does not increase their own utility but their partner’s, the solution is one where investments in capital and skill are sub-optimal.

**Proposition 2:** Increased investment \((k\text{ or } s)\) is no longer Pareto improving in the intermediated equilibrium.

**Proof:** Under financial intermediation, the utility and the profit functions for lender and entrepreneur are given by

(A5) \[ \text{Max}(k) \quad u_1 = k \cdot R_1d - C(k) \]

(A6) \[ \text{Max}(k, s) \quad u_2 = Fd - k \cdot R_1d - C(s) \]

Taking total differentiation with respect to \((k)\) and \((s)\), it can be shown\(^{13}\) that

\[ \frac{du_1(\cdot)}{dk} = R_1d - c_1^k(\cdot) \]

\[ \frac{du_2(\cdot)}{ds} = R_1d - c_1^s(\cdot) \]

\(^{12}\) From equation (A1), \(du_1(\cdot) = \left[ \frac{r + 2n_2}{r + n_2 + 1} \right] \phi f^s (k^a, s^a) \cdot [R_1d - c_1^k(\cdot)]dk\).

Using F.O.C. (equation 13), \(du_1(\cdot) = \left[ \frac{r + 2n_2}{r + n_2 + 1} \right] \phi f^s (k^a, s^a) > 0\).

From equation (A2), \(du_2(\cdot) = \left[ \frac{r + 2n_1}{r + n_1 + 1} \right] (1 - \phi) f^k (k^a, s^a) \cdot [R_1d - c_1^s(\cdot)]ds\).

Using F.O.C. (equation 14), \(du_2(\cdot) = \frac{r + 2n_1}{r + n_1 + 1} (1 - \phi) f^k (k^a, s^a) > 0\).

\(^{13}\) From (A5), \(du_1(\cdot) = R_1d - c_1^k(\cdot) \).

Using F.O.C (26), \(R_1d - c_1^k(\cdot) = 0\).
(A7) \[ \frac{du_1(.)}{ds} = 0 \]

(A8) \[ \frac{du_2(.)}{dk} = 0 \]

Any action by an entrepreneur (lender) does not have any impact on either utility. The above conditions imply that the investment of capital as well as skill under financial intermediation is efficient and optimal.