An Estimated Open Economy New Keynesian DSGE Model for Sri Lanka with Monetary and Fiscal Rules

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January 31, 2016

Abstract

This paper develops a New Keynesian (NK), Small Open-Economy, Dynamic Stochastic General Equilibrium (SOE-DSGE) model for Sri Lanka and estimates it using Bayesian technique, for the period 1996:Q1 to 2014:Q2. The paper is different from most of the other SOE studies available, as it contains an explicit fiscal sector in the model. I use the model framework proposed by Lubik and Schorfheide (2007), as the baseline model and then enhance it with a fiscal block. The model features with Calvo-type nominal price rigidities, complete international asset markets, perfectly competitive retailers, monopolistically competitive intermediate good producers, government debt, distortionary taxes and a perfect exchange rate pass-through mechanism. A standard Taylor-rule type monetary policy reaction function where the nominal interest rate responds to inflation, output and exchange rate, and an analogues fiscal policy rule where tax revenue responds to the government expenditure, output gap and debt, are considered. An attempt is made to accommodate specific features of the Sri Lankan economy in to the model, by carefully selecting priors. Estimation results suggest that the Central Bank of Sri Lanka conducts moderately strong anti-inflationary monetary policy while paying substantial attention to output stabilisation, however, with negligibly small concerns for exchange rate movements. Findings imply high degree of interest rate persistence as well. Further, the fiscal policy rule respond to debt level and government expenditure to a moderately small extent, while stabilizing output, to a smaller extent.

JEL Classification: C15; C32; E52; F41; H30

Key words: Small Open Economy, DSGE Models, Monetary Policy, Fiscal Policy, Bayesian analysis, Sri Lanka

*I wish to thank Dr. Denny Lie and Dr. Aarti Singh for their guidance. I thankfully acknowledge Prof. Thomas A. Lubik for providing me with the original data used for Lubik and Schorfheide (2007), enabling me to replicate their work, as a preliminary exercise. the author wishes to thank the two anonymous referees, discussant of the paper Dr. Chandranath Amarasekara and other participants of the *8th Central Bank of Sri Lanka International Research Conference, 2015*, for their valuable comments and suggestions. All remaining errors and omissions are, however, of my own. Mail to: kehelepola@gmail.com, ehelepola@cbsl.lk, kithsiri.ehelepola@uni.sydney.edu.au

1 INTRODUCTION

DSGE framework gradually rises as the main modelling tool in contemporary macroeconomics, receiving towering attention from researchers, academics and policy makers worldwide. Central banks all over the world increasingly use DSGE models in their operations, as they provide a coherent framework for policy discussion and analysis (see Kremer et al. (2006) and Tovar (2009)). Even in the South Asian region, few central banks have already initiated use of DSGE models for the said purpose recently (Ahmed et al. (2013), for instance). In Sri Lanka, there is a growing awareness of DSGE literature, particularly among the central bankers and academics, withstanding to the fact that there are only a limited number of studies available yet¹.

Sri Lanka being a small developing country with a fairly high debt stock with persistently negative fiscal balances², plans to implement flexible inflation targeting framework in the medium term, deviating from the existing monetary targeting framework. The medium scale SOE-DSGE model specified herein is an attempt to incorporate both fiscal and monetary policy rules in an open economy set up in the Sri Lankan context. The model could therefore be useful as a guiding tool in policy analysis and forecasting in this transitory period and thereafter.

The rest of the paper is arranged as follows: Section 2 reviews literature, Section 3 explains the small open economy model in detail, Section 4 presents the empirical analysis with results and finally, Section 5 concludes.

¹To best of my knowledge, there are only three NK-DSGE studies available for Sri Lanka so far: Anand et al. (2011) develop a practical model-based forecasting and policy analysis system (FPAS), facilitating the expected transition to an inflation targeting regime in Sri Lanka, in the near future. Ehelepola (2014) conducts an optimal monetary and fiscal policy analysis for Sri Lanka, closely following Schmitt-Grohé and Uribe (2007) and Karunaratne and Pathberiya (2014) estimate a SOE-DSGE model to Sri Lanka, which abstract from a fiscal sector, however.

 $^{^{2}}$ Sri Lanka introduced Fiscal Management (Responsibility) Act in 2003, aiming at curtailing the budget deficit while maintaining a sustainable debt level. Though the targets could not be achieved as per the original plan, a considerable progress has been made in achieving the twin objectives over the years.

2 LITERATURE REVIEW

Over the last three decades modelling tools of macroeconomic fluctuations have evolved dramatically. In the early 1970's, aftermath of the failure of large scale conventional macroeconomic models rooted in Keynesian economic theory, a need emerged to find a novel modelling technique which is immune to $Lucas \ critique^3$. In the backdrop of the severe criticism on the existing mainstream macroeconomic models⁴, Kydland and Prescott (1982) made the anticipated paradigm shift in macroeconomic modelling, by proposing an innovative model where economic agents optimize their behaviour incorporating rational expectations in a Dynamic Stochastic General Equilibrium (DSGE) framework. This family of models are widely known as Real Business Cycle (RBC) models⁵, owing to the fact that they are essentially the neoclassical growth models under fully flexible prices where real shocks are embedded in to the model, in order to create business cycle fluctuations. Complexity of these RBC models started to grow rapidly ever since, as they relax strong assumptions made in the initial models, focussing on various specific applications. Amidst of these important changes, the model framework continue to shares the microfoundations and DSGE structure inherited from the RBC tradition and accordingly, some authors, such as Goodfriend and King (1997), referred the new paradigm as the New Neoclassical Synthesis.

Enhancing the first generation RBC framework by incorporating various imperfections and rigidities together with more realistic assumptions lead to the tradition of New-Keynesian (NK) Macroeconomics. In the influential paper, Rotemberg and Woodford (1997) augment these DSGE models by introducing nominal rigidities in the spirit of Calvo (1983) price setting behaviour and monopolistic competition motivated by Dixit and Stiglitz (1977). These two crucial modifications facilitated bringing in short-run non-neutrality of money in to the model. Accordingly, Galí (2009), argues that monopolistic competition, nominal rigidities and short run non-neutrality of money are the three most important elements which distinguish NK tradition from the previous RBC models ⁶.

 $^{^{3}}$ The Lucas critique states that the econometric policy evaluation procedures should be able to identify the corresponding variations in optimal decision rules of economic agents, with changes in policy (Lucas Jr (1976)).

⁴More details can be found in Lucas Jr (1976); Sims (1980) and Sargent (1981).

⁵The contribution made by Long Jr and Plosser (1983) is also identified as important, in the development of the early RBC models. They demonstrate how certain very ordinary economic principles lead maximizing individuals to choose consumption-production plans that display many of the characteristics commonly associated with business cycles.

 $^{^{6}}$ A rich discussion on this is available in Mankiw and Romer (1991).

In contrast to the pure RBC models, the inclusion of nominal rigidities and the implied non-neutrality of monetary policy, in the NK-DSGE models, allowed monetary authority to make possible welfare improving interventions by minimizing such distortions⁷. This desirable property highly influenced the wide usage of NK-DSGE models in the central banks, since the banks can now include the monetary policy reaction functions in the model, connecting its objectives to the monetary policy instruments, effectively. Conduct of monetary policy under the NK school of thought is characterized with maintaining low and stable inflation while making output as close as possible to its potential level (see for instance; Clarida et al. (1997, 1998, 1999, 2001), and Svensson (2000, 2002, 2003) among others). Development of the NK-DSGE models with explicit theoretical foundations, facilitated counter factual policy experiments (for instance, Christiano et al. (2005), Smets and Wouters (2003, 2007)) and explained transmission of various shocks across different sectors of the economy as well. This remarkable feature attributed to the popularity of NK-DSGE models immensely, as Galí and Gertler (2007) state that a tell-tale sign which these frameworks possess, has been a key reason for their widespread use at central banks, in the process of monetary policy implementation.

With the introduction of the novel model framework *open-economy macroeconomics*, Obstfeld and Rogoff (1995), further expanded the scope of DSGE literature, as they first applied the DSGE framework to a two-country open economy study⁸. In consequence, Gali and Monacelli (2005) layout a *small open economy* version of the Calvo sticky price model, and show how the equilibrium dynamics can be reduced to a simple representation in domestic inflation and the output gap. They further use the resulting framework to analyse the macroeconomic implications of alternative regimes, based on different policy rules, for the small open economy, extending the benchmark NK-DSGE model described in Woodford and Walsh (2005), to the SOE setting⁹. In this background, Lubik and Schorfheide (2007) estimate a variant of a SOE-DSGE model proposed by Gali and Monacelli (2005), using

⁷Several early empirical studies including Cecchetti (1986), Kashyap and Stein (1995), Taylor (1993) and Woodford (2001) for example, conclude that there is ample evidence of price stickiness.

⁸Further extensions to their model framework can be found in their subsequent work: Obstfeld (1996) and Obstfeld and Rogoff (1998).

⁹An analogous analysis of the canonical Clavo model in a closed economy set up an be found in King and Wolman (1996), Yun (1996) and Woodford and Walsh (2005), for example

Bayesian methods¹⁰, instead of depending on parameter calibrations¹¹, heavily.

Fiscal policy has been widely used as one of the main instruments in stabilizing the economic fluctuations and in redistributing resources efficiently and equitably in the society, over many decades. The Fiscal Theory of the Price Level (FTPL), an unorthodox theory, pioneered by Leeper (1991), Sims (1994), Woodford (1996) and Woodford (2001), which essentially says that the government fiscal policy can influence the price level, explains another important dimension of the fiscal policy¹². Further, the zero lower bound, where monetary policy cannot provide sufficient stimulus, necessitate wider use of fiscal tools in stabilisation, in coordination with monetary policy¹³. Amidst these obvious importance of the fiscal policy, surprisingly not many SOE-DSGE studies explicitly include fiscal rules in their models. The state of the art SOE-DSGE studies available to date, such as Gali and Monacelli (2005), Lubik and Schorfheide (2007), Del Negro and Schorfheide (2008) and Justiniano and Preston (2010), abstract from a fiscal policy rule in their models. There are many country specific SOE-DSGE studies, for instance Liu (2006) for New Zealand, Nimark (2009) for Australia, Ahmed et al. (2013) for Pakistan and Karunaratne and Pathberiya (2014) for Sri Lanka, all of which abstract from a fiscal sector in their models. Two exceptions are however, Fragetta and Kirsanova (2010) and Cebi $(2012)^{14}$, where authors use DSGE models with inbuilt fiscal rules.

The present study attempts to contribute to fill this gap by setting up a New-Keynesian SOE-DSGE model with both fiscal and monetary policy rules embedded in to it explicitly. Then it is estimated in the context of Sri Lanka, by employing Bayesian methods and the dynamics of major macroeconomic variables, such as output, inflation and the level of debt, are assessed in response to various innovations.

¹⁰Revolutionary advancement and cost-affordability of the computer technology began in 1990s together with the adoption and introduction of new econometric techniques, particularly the Bayesian methods, vastly expanded the toolboxes available for the applied macroeconomists, making the estimation of DSGE models more accessible. Lubik and Schorfheide (2006), introduced the path-breaking application of Bayesian techniques to estimate a SOE-DSGE model. A fine review of the Bayesian methods developed to estimate and evaluate DSGE models is also available in An and Schorfheide (2007).

¹¹Selecting parameter values on the basis of microeconomic evidence or long-run data properties is referred to as calibration (see for example Karagedikli et al. (2010)).

¹²Muscatelli et al. (2002) argue that FTPL does not necessarily contrast with the usual monetary theory of the price level, where the price level is primarily or exclusively determined by supply of money, but rather compliment it.

¹³A rich discussion on fiscal, monetary policy mix at zero lower bound can be found in Sims (2010).

¹⁴Some features of the model and the functional forms of the policy rules used in their studies are, however, different from that of mine.

3 THE MODEL

The canonical SOE-DSGE framework proposed by Gali and Monacelli (2005) presents a model calibrated to the US economy. Lubik and Schorfheide (2007) use Bayesian method to estimate their model which is a variant of Gali and Monacelli (2005). Both of these frameworks, however, abstract from a fiscal sector in their models and in the present paper, I attempt to introduce a fiscal block to the model proposed by Lubik and Schorfheide (2007) and estimate it with the Sri Lankan data. The fiscal block used in this paper where government budget constraint and the fiscal rule act as the two main ingredients, is analogues to that of Bhattarai et al. (2012).

In line with Gali and Monacelli (2005), I assume that the world economy is a continuum of SOEs represented in a unit interval. None of the individual SOEs can therefore make any influence on the world economy. I further assume that decisions at time t are made after observing all current period shocks. All the variables with subscripts t-s, where $s \ge 0$, are therefore known at time t.

3.1 Households

The economy consists of a continuum of infinitely lived identical households, each of which seek to maximize their lifetime utility function, subject to a sequence of inter-temporal budget constraints. The corresponding utility function, U_t , is given by,

$$U_t = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t/Z_t)^{1-\sigma}}{1-\sigma} + \chi \frac{(G_t/Z_t)^{1-\sigma}}{1-\sigma} - \frac{(N_t)^{1+\varphi}}{1+\varphi} \right]$$
(1)

where, E_0 is the expectations operator, conditional on information available at time 0 (in the beginning of time), $\beta \in (0, 1)$ is the subjective discount factor, σ is the inverse of the intertemporal elasticity of substitution in consumption, φ is the inverse of the labour supply elasticity with respect to real wage and χ denotes the relative weight on consumption of public goods and Z_t is the world technology process. Preferences depend on the aggregate variables which are private consumption, C_t , government spending, G_t , and labour hours, N_t . The period utility function is strictly concave and strictly increasing with the first two arguments, C_t/Z_t and G_t/Z_t and strictly decreasing with the third argument, N_t . Domestic households maximize the above utility function, subject to the following budget constraint,

$$P_t C_t + B_t + E_t \left[Q_{t+1|t} D_{t+1} + \varepsilon_t Q_{t+1|t}^* D_{t+1}^* \right] \le \left(1 - \tau_t^D \right) W_t N_t + D_t + \varepsilon_t D_t^* + R_{t-1} B_{t-1}$$
(2)

where, P_t is the nominal price level of the composite good, D_{t+1} is the holding of a security that pays one unit of domestic currency in period t + 1, $Q_{t+1|t}$ is the period t price of the security in domestic currency, ε_t is the nominal exchange rate, expressed as the amount of domestic currency per one unit of foreign currency¹⁵, τ_t^D is the distortionary tax rate, R_t denotes one period gross nominal risk free interest rate in the domestic economy, B_t is the amount of one period riskless nominal government bond held by the households and W_t denotes the nominal wage. All the symbols with a superscript '*' represent corresponding foreign variables.

I redefine the de-trended consumption and nominal wages as $c_t = C_t/Z_t$ and $w_t = W_t/(P_tZ_t)$ and then take the first order conditions of the utility function subject to the budget constraint, which yields following relationships¹⁶,

$$N_t^{\varphi} = c_t^{-\sigma} w_t \left(1 - \tau_t^D \right) \tag{3}$$

$$c_t^{-\sigma} = \beta E_t \left[R_t c_{t+1}^{-\sigma} \left(z_{t+1} \pi_{t+1} \right)^{-1} \right]$$
(4)

$$0 = E_t \left[(R_t - R_t^* e_{t+1}) \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} (z_{t+1} \pi_{t+1})^{-1} \right]$$
(5)

where, $z_t = Z_t/Z_{t-1}$ is the gross growth rate of the technology process, $\pi_t = P_t/P_{t-1}$ is the gross inflation rate and $e_t = \varepsilon_t/\varepsilon_{t-1}$ is the gross exchange rate depreciation (or appreciation).

3.2 Terms of Trade (TOT) and Real Exchange Rate

I denote domestic price of home produced goods and foreign produced goods as $P_{H,t}$ and $P_{F,t}$ respectively. Accordingly, terms of trade is defined as the price of foreign good in terms of a unit of domestic good, as follows,

$$q_t = P_{H,t} / P_{F,t} \tag{6}$$

Further, I postulate that law of one price (LOP) holds for foreign goods. Sri Lanka being a SOE, can not influence the world prices, instead it acts as a price taker. Therefore it gives,

$$P_{F,t} = \varepsilon_t P_{F,t}^* \tag{7}$$

 $^{^{15}}$ Accordingly, for the Sri Lankan case, I use the amount of Sri Lankan rupees per one US dollar (SLR/USD), as the exchange rate.

¹⁶Detrended consumption is expressed in real terms already and accordingly wage is also expressed in real terms here.

where $P_{F,t}^*$ is the price of foreign good in the foreign country, in terms of foreign currency. The SOE assumption further imply that domestically produced goods have approximately zero weight in world consumption¹⁷. Thus, $P_{F,t}^*$ equals to the foreign Consumer Price Index (CPI), P_t^* . Therefore, it gives,

$$q_t = P_{H,t} / \left(\varepsilon_t P_t^* \right) \tag{8}$$

This implies that either an exchange rate depreciation or foreign inflation can reduce the TOT, making imports more expensive. Further, a decline in TOT could also be due to a reduction in international competitiveness of domestic goods, attributable to an increase of domestic good prices. The real exchange rate can now be defined as,

$$S_t = \varepsilon_t P_t^* / P_t \tag{9}$$

where P_t is the domestic CPI. From the above two relationships, it can be deduced that,

$$P_{H,t}/P_t = q_t S_t \tag{10}$$

3.3 Composite Goods

There exist domestic retailer firms that purchase domestic and foreign produced goods, in quantities $C_{H,t}$ and $C_{F,t}$ respectively and package them as a composite good which is consumed by the households. These firms operate in a perfectly competitive environment maximizing profits as follows,

$$max_{C_{t},C_{H,t},C_{F,t}} [P_{t}C_{t} - P_{H,t}C_{H,t} - P_{F,t}C_{F,t}]$$

s.t. $C_{t} = \left[(1-\alpha)^{1/\eta} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{1/\eta} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$ (11)

where, $\alpha \in [0, 1]$ measures the degree of openness of the economy and $\eta > 0$ is the elasticity of substitution between domestic and foreign goods. First order conditions of the above maximization problem implies that,

$$C_{H,t} = (1 - \alpha) \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t$$

$$C_{F,t} = \alpha \left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t$$
(12)

¹⁷Being a small open economy, this is valid for Sri Lanka as well

Then, the zero profit condition, $P_tC_t - P_{H,t}C_{H,t} - P_{F,t}C_{F,t} = 0$, yields the following,

$$P_t = \left[(1 - \alpha) P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}$$
(13)

By dividing the above equation by P_t and rearranging the terms yields the relationship between TOT and real exchange rate as follows,

$$S_{t} = \left[(1 - \alpha) q_{t}^{1 - \eta} + \alpha \right]^{\frac{1}{\eta - 1}}$$
(14)

I assume that the aggregate quantity of the domestically produced good Y_t comprises a composite good itself, produced by a continuum of domestic intermediate goods. Thus,

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}}$$
(15)

There are also perfectly competitive firms that buy the domestic intermediate goods, package them and resell the composite good to the firms that aggregate $C_{H,t}$ and $C_{F,t}$. Accordingly, these firms solve the problem,

$$max_{Y_{t},Y_{t}(i)} \quad \left[P_{H,t}Y_{t} - \int_{0}^{1} P_{H,t}\left(i\right)Y_{t}\left(i\right)di \right]$$

$$s.t. \quad Y_{t} = \left[\int_{0}^{1} Y_{t}\left(i\right)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}} \tag{16}$$

First order conditions together with zero profits assumption then yields,

$$Y_t(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon} Y_t$$
$$P_{H,t} = \left[\int_0^1 P_{H,t}(i)^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}}$$
(17)

3.4 Domestic Intermediate Goods

Domestic intermediate goods are produced by monopolistically competitive firms. In line with Calvo (1983) and Yun (1996), prices are assumed to be sticky such that a portion of the firms, $\theta \in [0, 1)$, is not allowed to adjust price of the goods it produces, in each period. The remaining $(1 - \theta)$ firms, thus, selects prices optimally. It is assumed that the firms who are unable to re-optimise prices in each period raise the price $P_{H,t}(i)$ according to the steady state inflation rate, π_H . In the continuum, the i^{th} firm produces differentiated good $Y_t(i)$, using a production function linear in labour,

$$Y_t\left(i\right) = Z_t N_t\left(i\right) \tag{18}$$

where Z_t is common for all producer firms and its growth rate is given by, $z_t = Z_t/Z_{t-1}$ follows a first order autoregressive process (AR(1) process) as follows:

$$(ln(z_t) - \gamma) = \rho_z (ln(z_{t-1}) - \gamma) + \epsilon_t^z$$
(19)

where γ growth rate of productivity at the steady state. Accordingly, the optimization condition for firm i is given by:

$$max_{\tilde{P}_{H,t}(i)} \quad E_t \left[\sum_{k=0}^{\infty} \theta^k Q_{t,t+k} Y_{t+k} \left(i \right) \left(\tilde{P}_{H,t} \left(i \right) \pi_H^k - MC_{t+k}^n \right) \right]$$
$$s.t. \quad Y_{t+k} \left(i \right) \le \left(\frac{\tilde{P}_{H,t} \left(i \right) \pi_H^k}{P_{H,t+k}} \right)^{-\epsilon} Y_{t+k}$$
(20)

where $MC_{t+k}^n = W_{t+k}/Z_{t+k}$ is the nominal marginal cost, $Q_{t,t+k}$ is the time t price of a security that pays one unit of domestic currency (SLR), in period t + k and $\tilde{P}_{H,t}$ is the optimal price in period t. Accordingly, the first order condition, after some rearrangements together with the zero steady state inflation assumption, yields,

$$\sum_{k=0}^{\infty} \theta^k E_t \left[Q_{t,t+k} \left(\tilde{P}_{H,t} - \frac{\epsilon}{\epsilon - 1} M C_{t+k}^n \right) \right] = 0$$
(21)

Then using the fact that $Q_{t,t+k} = \beta^k (C_{t+k}/C_t)^{-\sigma} (P_t/P_{t+k})$ the above relationship can be further modified as:

$$\sum_{k=0}^{\infty} \left(\beta\theta\right)^{k} E_{t} \left[P_{t+k}^{-1} C_{t+k}^{-\sigma} Y_{t+k} \left(\tilde{P}_{H,t} - \frac{\epsilon}{\epsilon - 1} M C_{t+k}^{n} \right) \right] = 0$$

which is equivalent to;

$$\sum_{k=0}^{\infty} \left(\beta\theta\right)^{k} E_{t} \left[C_{t+k}^{-\sigma} Y_{t+k} \frac{P_{H,t-k}}{P_{t+k}} \left(\frac{\tilde{P}_{H,t}}{P_{H,t-1}} - \frac{\epsilon}{\epsilon - 1} \Pi_{t-1,t+k}^{H} M C_{t+k} \right) \right] = 0$$
(22)

where,

$$MC_t = \frac{MC_t^n}{P_{H,t}} = \frac{(1-\tau)W_t}{Z_t P_{H,t}} = \frac{(1-\tau)W_t}{Z_t P_t} \frac{P_t}{P_{H,t}} = (1-\tau)w_t q_t^{-1} S_t^{-1}$$
(23)

and, $\Pi_{t-1,t+k}^{H} \equiv \frac{P_{H,t+k}}{P_{H,t-1}}$. The above relationship can now be loglinearized around the zero inflation steady state to arrive at the following result,

$$\tilde{p}_{H,t} = p_{H,t-1} + \sum_{k=0}^{\infty} \left(\beta\theta\right)^k E_t \left[\pi_{H,t+k}\right] + \left(1 - \beta\theta\right) \sum_{k=0}^{\infty} \left(\beta\theta\right)^k E_t \left(\widehat{mc}_{t+k}\right)$$
(24)

where $\widehat{mc}_t = mc_t - mc$ is the log deviation of real marginal cost, from its steady state value $mc = -ln\left(\frac{\epsilon}{\epsilon-1}\right)$. After some manipulations, the above equation can be presented in a more compact form as follows,

$$\tilde{p}_{H,t} - p_{H,t-1} = \beta \theta E_t \left(\tilde{p}_{H,t+1} - p_{H,t} \right) + \pi_{H,t} + (1 - \beta \theta) \,\widehat{mc}_{t+k} \tag{25}$$

By substituting $\widehat{mc}_t = mc_t^n - p_{H,t} + mc$ in the above equation, I obtain a version of the price setting rule in terms of nominal marginal costs,

$$\tilde{p}_{H,t} = mc + (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \left(\widehat{mc}_{t+k}\right)$$
(26)

In line with the Calvo price setting mechanism followed here, the dynamics of the domestic price index is given by,

$$P_{H,t} \equiv \left[\theta P_{H,t-1}^{1-\epsilon} + (1-\theta) \tilde{P}_{H,t}^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$$
(27)

which can then be loglinearized around the zero inflation steady state to yield,

$$\widehat{\pi}_{H,t} = (1-\theta)\left(\widetilde{p}_{H,t} - p_{H,t-1}\right) \tag{28}$$

Now, by substituting the above, in the equation (24) I obtain,

$$\frac{\widehat{\pi}_{H,t}}{(1-\theta)} = \beta \theta E_t \frac{\widehat{\pi}_{H,t+1}}{(1-\theta)} + \widehat{\pi}_{H,t} + (1-\beta\theta) \,\widehat{mc}_t$$

which can be re-arranged in a more compact form as follows, deriving the Phillips Curve relationship.

$$\widehat{\pi}_{H,t} = \beta E_t \left[\widehat{\pi}_{H,t+1} \right] + \kappa \widehat{mc}_t \tag{29}$$

where, $\kappa \equiv (1 - \beta \theta) (1 - \theta) / \theta$ is the slope coefficient in the Phillips Curve.

3.5 Domestic Market Clearing and Aggregate Production Function

Market clearing condition for the domestically produced goods in terms of real variables, detrended by Z_t is given by,

$$y_t = c_{H,t} + c_{H,t}^* + g_{H,t}y_t + g_{H,t}^*y_t^*$$
(30)

where $y_t = Y_t/Z_t$, $g_{H,t} = G_{H,t}/Y_t$ and hence $g_{H,t}y_t = (G_{H,t}/Y_t)(Y_t/Z_t) = G_{H,t}/Z_t$. Variables with a superscript '*' denotes the corresponding foreign variables, as usual. In the case of SOEs, however, it is reasonable to assume that the foreign government's demand for the domestically produced goods is negligible. This is equally applicable for a small developing country like Sri Lanka and accordingly, I impose, $g_{H,t}^* = 0$, to obtain,

$$y_t = c_{H,t} + c_{H,t}^* + g_{H,t} y_t (31)$$

Let the relative size of the domestic economy (compared to the world economy) be ϑ , $(\vartheta <<)$. Then I can define the share of the domestic economy in the world economy as $\alpha^* = \vartheta \alpha$, which is obviously a very small fraction.

By combining (12) and (30) with some algebraic simplifications, I obtain,

$$y_t = (1 - \alpha) \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} c_t + (\alpha \vartheta) \left(\frac{P_{H,t}/\varepsilon_t}{P_t^*}\right)^{-\eta} c_t^* + g_{H,t} y_t$$

Regulations for government purchases in many countries are mostly in favour of domestically produced goods and it is not different in Sri Lanka as well. Accordingly, I safely set $g_{H,t} = g_t$ (i.e. government purchases are fully allocated to domestically produced goods, as in Gali and Monacelli (2008)) and hence the above equation can be rearranged as follows,

$$y_t = (1 - \alpha) \left(S_t q_t \right)^{-\eta} c_t + \alpha \,\vartheta q_t^{-\eta} c_t^* + g_t y_t \tag{32}$$

The aggregate production function for the domestic economy is obtained by integrating the continuum of individual goods, as follows;

$$y_t = N_t \left[\int_0^1 \left(\frac{P_{H,t}\left(i\right)}{P_{H,t}} \right)^{-\epsilon} di \right]^{-1} = N_t \delta_t^{-1}$$
(33)

where, $\delta_t = \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon} di$, is a measure of relative price dispersion, which is equal to unity in a fully flexible price setting environment.

3.6 The Rest of the World

I assume complete international financial markets and perfect capital mobility which assure Uncovered Interest Rate parity (UIP). Hence, the expected nominal return from the-risk free domestic country bonds must be indifferent to the expected returns from foreign bonds in domestic currency terms. This perfect risk-sharing imply,

$$E_t Q_{t+1} = E_t \left(Q_{t+1}^* \varepsilon_{t+1} \right)$$

which is equivalent to;

$$\left(\frac{c_{t+1}}{c_t}\right)^{\sigma} \pi_{t+1} = \left(\frac{c_{t+1}^*}{c_t^*}\right)^{\sigma} \pi_{t+1}^* e_{t+1}$$
(34)

After substituting for the terms and re-arranging produces,

$$\left(\frac{c_{t+1}}{c_{t+1}^*}\right)^{\sigma} \frac{P_{t+1}}{\varepsilon_{t+1}P_{t+1}^*} = \left(\frac{c_t}{c_t^*}\right)^{\sigma} \frac{P_t}{\varepsilon_t P_t^*}$$
(35)

Above equation relates the domestic consumption growth with that of abroad. To analyse the implications of the level of consumption at home and abroad, I make the assumption that in the initial period, (t = 0), real interest rate is unity (*i.e.* $S_0 = 1$), as in Del Negro and Schorfheide (2008). Further, I make $\vartheta = C_0/C_0^*$, enabling me to deduce:

$$c_t = \vartheta c_t^* S_t^{1/\sigma} \tag{36}$$

Now, I can express market clearing condition combining (31) with the above equation, (35) as follows,

$$y_t = \vartheta c_t^* q_t^{-\eta} \left[(1 - \alpha) S_t^{1/\sigma - \eta} + \alpha \right] + g_t y_t$$

or equivalently;

$$y_t = \frac{1}{(1-g_t)} \vartheta c_t^* q_t^{-\eta} \left[(1-\alpha) S_t^{1/\sigma-\eta} + \alpha \right]$$
(37)

All the state contingent securities are in zero net supply and therefore I obtain the global resources constraint, which essentially says that what is produced domestically and abroad should be equal to the summation of consumption and government expenditure in total, worldwide.

$$c_t + g_t y_t + \frac{\varepsilon_t P_t^*}{Z_t^*} \left(c_t^* + g_t^* y_t^* \right) = \frac{P_{H,t}}{P_t} y_t + \frac{\varepsilon_t P_t^*}{P_t} y_t^*$$

Combining equations (10) and (12) with the above result, I deduce that,

$$(c_t + g_t y_t) + S_t (c_t^* + g_t^* y_t^*) = q_t S_t y_t + S_t y_t^*$$
(38)

In the world market, the share of the domestic economy is negligibly small and accordingly the above relationship can be reduced to¹⁸,

$$c_t^* + g_t^* y_t^* = y_t^*$$

which can be rearranged as,

$$c_t^* = y_t^* \left(1 - g_t^* \right) \tag{39}$$

¹⁸This result s attributable to the fact that both c_t and $y_t \to 0$ when $\vartheta \to 0$, as revealed from (33) and (34) above

3.7 The Government

The consolidated government in the model acts both as the monetary and fiscal authorities. It issues nominally risk-free bonds B_t , receives tax revenue amounting to T_t , and makes expenditure amounting to G_t . Hence, the period by period government budget constraint is given by¹⁹,

$$\frac{B_t}{P_{H,t}} = R_{t-1} \frac{B_{t-1}}{P_{H,t}} + G_t - T_t \tag{40}$$

Since I am dealing with a growing economy, the nominal variables are deflated by the price level and the level of output in line with Bhattarai et al. (2012). Thus with the de-trended variables, I get,

$$\frac{B_t}{P_{H,t}Y_t} = R_{t-1}\frac{B_{t-1}}{P_{H,t}Y_t} + \frac{G_t}{Y_t} - \frac{T_t}{Y_t}$$
(41)

As per the equation (3) above, the government imposes distortionary taxes on the labour income such that;

$$T_t = \tau_t^D \left(W_t N_t \right) / P_{H,t} \tag{42}$$

Now, I define de-trended fiscal variables as follows;

$$b_t = \frac{B_t}{P_{H,t}Y_t} , \quad g_t = \frac{G_t}{Y_t} , \quad \tau_t = \frac{T_t}{Y_t}$$

$$(43)$$

Then I obtain the relationship between the distortionary tax rate and the ratio of total tax as a percentage of output, as follows²⁰,

$$\tau_t = \tau_t^D m c_t \delta_t \tag{44}$$

Thus, after simplifying the terms I obtain government budget constraint (equation (40)) in terms of the de-trended variables, as follows,

$$b_t = \frac{R_{t-1}b_{t-1}y_{t-1}}{\pi_{H,t}z_t y_t} + g_t - \tau_t^D m c_t \delta_t$$
(45)

The time varying exogenous government expenditure is given by,

$$g_t = (1 - \rho_g) \,\overline{g} - \rho_g g_{t-1} + \sigma_g^2 \varepsilon_t^g \tag{46}$$

where, $\varepsilon_{t}^{g} \sim i.i.d. N(0, 1)$.

²⁰Note that,
$$\tau_t = \frac{T_t/P_{H,t}}{Y_t} = \frac{\tau_t^D(W_tN_t)/P_{H,t}}{(Z_tN_t/\delta_t)} = \frac{\tau_t^DW_t\delta_t}{P_{H,t}Z_t} = \tau_t^D\left(\frac{W_t}{P_tZ_t}\right)\frac{P_t}{P_{H,t}}\delta_t = \tau_t^Dw_t\left(\frac{1}{q_tS_t}\right)\delta_t = \tau_t^Dmc_t\delta_t$$

 $^{^{19}}$ Here I start with a budget constraint equation analogous to that of Bhattarai et al. (2012), where all variables are given in real terms.

3.8 Steady State

I denote the time invariant steady state (s.s.) of the variables with their usual symbols, however, excluding their time subscripts.

As per Gali and Monacelli (2005), I let domestic and foreign inflation rates to be zero at the s.s. and accordingly the corresponding gross inflation rates are unity (i.e. $\pi = \pi^* = 1$). I further postulate that the initial value of the real interest rate equals to its s.s. value which is unity (i.e. $S_0 = S^* = 1$). This constant real interest rate suggests that the depreciation of nominal exchange rate in the s.s., is $e = \pi/\pi^*$ is also unity²¹. Domestic nominal interest rate at the s.s. is given by the consumption Euler equation as follows: $R = 1/\beta$. Foreign nominal interest rate at s.s. is then given by the uncovered interest rate parity condition as $R^* = R/e$. TOT at s.s. is unity, since, $q = \left[\frac{1}{1-\alpha} \left(S^{\eta-1} - \alpha\right)\right]^{\frac{1}{1-\alpha}} = 1$. It is revealed that the inflation for the domestic good equals to domestic inflation at the s.s. (i.e. $\pi_H = \pi$). Domestic goods market clearing condition implies that, $y = \frac{1}{1-g} \vartheta c^*$. Perfect risk sharing condition suggests that, $c = \vartheta c^*$. The SOE assumption requires, $y^* = c^* + g^*y^*$ or equivalently, $c^* = y^* (1 - g^*)$ in the s.s. and hence I deduce, $y = \left(\frac{1-g^*}{1-g}\right) \vartheta y^*$. Domestic good production implies that N = y and from the marginal cost equation²², I get, $y = \left[\left(\frac{\epsilon-1}{\epsilon}\right) \frac{(1-\tau)}{(1-\tau)^{\sigma}}\right]^{\frac{1}{\nu+\sigma}}$.

3.9 Log Linearised System of Equations of the model

All "hat" variables, \hat{x}_t , are measured in percentage deviations from their respective steady state values (i.e. the log linear value, $\hat{x}_t = \frac{x_t - x}{x}$). However, the set of three fiscal variables b_t, τ_t and g_t is an exception for which the linear deviation from their corresponding s.s. values is referred to as the "hat" variables (i.e. $\hat{x}_t = x_t - x$). This is for the fact that fiscal variables are already detrended by expressing them as GDP ratios. As usual, Δ represents the first difference operator, such that $\Delta x_t = x_t - x_{t-1}$.

Following Del Negro and Schorfheide (2008), I impose $\varphi = 0$, $\eta = 1$ and $1/\sigma = \mu$ to obtain a version of the model compatible with Lubik and Schorfheide (2007).

With direct analogy to the closed-economy counterpart, the log-linearised (linearised) system of equations contains a forward looking IS-equation (open economy), Phillips curve and both monetary and fiscal policy rules. Exchange rate is introduced through the CPI

²¹This is from the relationship, $S_t = \varepsilon_t P_t^* / P_t$,

²²marginal cost, $mc_t = \left(\frac{1}{1-\tau_t}\right) y_t^{\varphi} q_t^{-1} S_t^{-1} c_t^{\sigma}$, equals to $mc = \left(\frac{\epsilon-1}{\epsilon}\right)$, in the s.s.

definition and under the PPP assumption. In line with Lubik and Schorfheide (2007), the dynamics of the SOE is determined by the following equations:

The open economy IS-equation derived from the consumption Euler equation is as follows;

$$\widehat{y}_{t} = E_{t}\widehat{y}_{t+1} - (\mu + \lambda)\left(\widehat{R}_{t} - E_{t}\widehat{\pi}_{t+1} - \rho_{z}\widehat{z}_{t}\right) + \alpha\left(\mu + \lambda\right)E_{t}\Delta\widehat{q}_{t+1} + \frac{\lambda}{\mu}E_{t}\Delta\widehat{y}^{*}_{t+1} - \frac{1}{(1-g)}\Delta\widehat{g}_{t+1} - \frac{1}{(1-g^{*})}\frac{\lambda}{\mu}E_{t}\Delta\widehat{g}^{*}_{t+1} \quad (47)$$

where, $\lambda = \alpha (2 - \alpha) (1 - \mu)$ and other symbols have their usual meanings as given above. In the absence of government, above equation reduces to the IS equation given in Lubik and Schorfheide (2007).

Optimal price setting behaviour of the domestic firms leads to the open economy Phillips curve with fiscal variables as follows²³:

$$\widehat{\pi}_t = \beta E_t \widehat{\pi}_{t+1} + \alpha \beta E_t \Delta \widehat{q}_{t+1} - \alpha \Delta \widehat{q}_t + \frac{\kappa}{\mu + \lambda} \left(\widehat{y}_t - \overline{y}_t \right)$$
(48)

where \bar{y}_t is the potential output in the absence of nominal rigidities, which is given by,

$$\bar{y}_t = -\frac{\lambda}{\mu} \left[\hat{y}_t^* - \frac{1}{(1-g^*)} \hat{g}_t^* \right] - \frac{1}{(1-g)} \hat{g}_t + \frac{\tau}{1-\tau} \hat{\tau}_t$$
(49)

Nominal exchange rate is introduced via the definition of the CPI, assuming PPP holds. The equations (9) and (14) are used to derive the following;

$$\widehat{\pi}_t = \Delta \varepsilon_t + (1 - \alpha) \,\Delta q_t + \widehat{\pi}_t^* \tag{50}$$

where $\hat{\pi}_t^*$ is a world inflation shock. This is treated as an unobservable as per Lubik and Schorfheide $(2007)^{24}$.

With the prior knowledge of forward looking nature of the monetary policy rule for Sri Lanka suggested by previous studies such as Perera and Jayawickrema (2013), Karunaratne and Pathberiya (2014) and the first chapter of the present thesis, I use the following monetary policy reaction function, in line with Lubik and Schorfheide (2007);

$$\widehat{R}_t = \rho_R \widehat{R}_{t-1} + (1 - \rho_R) \left[\psi_1 \widehat{\pi}_t + \psi_2 \widehat{y}_t + \psi_3 \Delta \widehat{\varepsilon}_t \right] + \varepsilon_t^R$$
(51)

where, $\psi_1, \psi_2, \psi_3 \geq 0$. Interest rate persistence is introduced to the picture by the smoothing parameter ρ_R where, $0 < \rho_R < 1$. The non-systematic portion of the monetary policy 2^{3} Note that, $y_t = \frac{Y_t}{Z_t} = \frac{N_t}{\delta_t}$, and hence, $\hat{y}_t = \hat{Y}_t - \hat{Z}_t = \hat{N}_t - \hat{\delta}_t$. For the dynamics near the zero inflation steady state, it can be shown that $\hat{\delta}_t = 0$. Accordingly, I get $\hat{y}_t = \hat{N}_t$ which is used in deriving the Phillips curve relationship.

²⁴An alternative view is given in Lubik and Schorfheide (2006), where $\hat{\pi}_t^*$ captures misspecification or deviation from PPP.

is represented by the exogenous policy shock, ε_t^R . I have also included a exchange rate coefficient ψ_3 in the rule to see whether our results are in agreement with the previous studies in Sri Lanka where ψ_3 is suggested to be closer to zero.

Log linearisation of the government budget constraint (eq (44) above) produces the following equation,

$$\widehat{b}_t = \beta^{-1}\widehat{b}_{t-1} + \beta^{-1}b\left(\widehat{R}_{t-1} - \widehat{\pi}_t - \widehat{y}_t + \widehat{y}_{t+1} - \widehat{z}_t - \alpha\Delta\widehat{q}_t\right) + \widehat{g}_t - mc\,\widehat{\tau}_t - \tau\,mc\,\widehat{mc}_t \qquad (52)$$

where, $mc = \left(\frac{\epsilon-1}{\epsilon}\right)$, and the log deviation of the marginal cost from its s.s. value (\widehat{mc}_t) is given by,

$$\widehat{mc}_t = \frac{1}{\mu + \lambda} \left(\widehat{y}_t - \overline{y}_t \right) \tag{53}$$

A fiscal rule which is analogues to the monetary rule above is used here, following Bhattarai et al. (2012),

$$\widehat{\tau}_t = \rho_\tau \widehat{\tau}_{t-1} + (1 - \rho_\tau) \left[\psi_b \left(\widehat{b}_{t-1} - \widehat{\widetilde{b}}_{t-1} \right) + \psi_y \left(\widehat{y}_t - \overline{y}_t \right) + \psi_g \widehat{g}_t \right] + \varepsilon_t^\tau$$
(54)

where, \tilde{b} is the time-varying debt/GDP target. $\hat{\tau}_t = \tau_t - \tau$, $\hat{b}_t = b_t - b$, $\hat{b}_t = \tilde{b}_t - \tilde{b}$ and $\hat{g}_t = g_t - g$. The term ε_t^{τ} explains possible non systematic fiscal shocks to the system.

Following Lubik and Schorfheide (2007), I add a law of motion equation for the growth rate of TOT,

$$\Delta \widehat{q}_t = \rho_q \Delta \widehat{q}_{t-1} + \varepsilon_t^q \tag{55}$$

Since the firms have a small market power, the price of the internationally traded goods are not fully exogenous to the domestic economy, even though the size of the economy is negligibly small compared to the rest of the world. Thus, as per Lubik and Schorfheide (2007), I determine TOT endogenously by making it as the relative price that clears international goods markets:

$$(\mu + \lambda) \,\Delta \widehat{q}_t = \Delta \widehat{y}_t^* - \Delta \widehat{y}_t \tag{56}$$

I get the relationship between distortionary tax rate τ_t^D and the total tax/GDP ratio τ_t^D as follows,

$$\widehat{\tau}_t = \tau \left(\widehat{\tau}_t^D + \widehat{mc}_t \right)$$

By substituting for mc_t , this can be expressed equivalently as,

$$\widehat{\tau}_t = \tau \left[\widehat{\tau}_t^D + \frac{1}{\mu + \lambda} \left(\widehat{y}_t - \overline{y}_t \right) \right]$$
(57)

Finally, the AR(1) processes for the exogenous shocks are as follows;

$$\widehat{z}_t = \rho_z \widehat{z}_{t-1} + \varepsilon_t^z \tag{58}$$

$$\widehat{q}_t = \rho_q \widehat{q}_{t-1} + \varepsilon_t^q \tag{59}$$

$$\widehat{y}_t^* = \rho_{y^*} \widehat{y}_{t-1}^* + \varepsilon_t^{y^*} \tag{60}$$

$$\widehat{\pi}_t^* = \rho_{\pi^*} \widehat{\pi}_{t-1}^* + \varepsilon_t^{\pi^*} \tag{61}$$

$$\widehat{g}_t = \rho_g \widehat{g}_{t-1} + \varepsilon_t^g \tag{62}$$

$$\widehat{g}_t^* = \rho_{g^*} \widehat{g}_{t-1}^* + \varepsilon_t^{g^*} \tag{63}$$

$$\tilde{b}_t = \rho_b \tilde{b}_{t-1} + \varepsilon_t^b \tag{64}$$

The remaining two shocks in the model ε_t^R and ε_τ^{τ} are determined endogenously.

4 EMPIRICAL ANALYSIS

I proceed with a brief discussion on the Bayesian approach which is employed in estimating the model. Then I continue with a description of the data used in the empirical work followed by a discussion on selection of priors and their distributions for the analysis. Finally diagnostics and interpretation of results.

4.1 The Bayesian Approach

There is an increasing trend in using Bayesian methodology in empirical macroeconomics and it is a powerful tool in estimating DSGE models and making inferences thereof. Application of Bayesian methods in DSGE models became very popular during the last two decades owing to several attractive features of them. They possess three main desirable attributes in this approach, as highlighted in An and Schorfheide (2007) and Lubik and Schorfheide (2006) : Firstly, the Bayesian approach is a system-based method which fits the solved DSGE model to a vector of aggregate time series (in contrast to other methods such as GMM, this does not depend on specific equilibrium relations such as Euler equations for real consumption). Secondly, the likelihood function generated by the DSGE model is used in estimation²⁵.

²⁵In the previous studies, Rotemberg and Woodford (1997) and Christiano et al. (2005), for instance, the discrepancy between DSGE model impulse response functions and identified VAR impulse responses is used for the purpose.

Thirdly, additional information can be incorporated in to the parameter estimation by using prior distributions.

Following Lubik and Schorfheide (2006), my DSGE model can be represented as a linear rational expectations (LRE) system as follows,

$$\Gamma_0(\theta) s_t = \Gamma_1(\theta) s_{t-1} + \Gamma_{\varepsilon}(\theta) \varepsilon_t + \Gamma_{\eta}(\theta) \eta_t$$
(65)

where, s_t denotes the vector of model variables and the vector ε_t contains the innovations of the exogenous processes and η_t is composed of rational expectations forecast errors²⁶. The θ matrix contains the structural parameters of the model while Γ matrices are the corresponding coefficient matrices. The solution to the above LRE system is in the form of;

$$s_t = \Phi_1(\theta) \, s_{t-1} + \Phi_\varepsilon(\theta) \, \varepsilon_t \tag{66}$$

Model variables s_t can then be related to a set of observables y_t , through a measurement equation,

$$Y_t = A\left(\theta\right) + Bs_t \tag{67}$$

In this model, the vector of observables Y_t , contains eight variables namely, real output growth, inflation, nominal interest rate, exchange rate change, terms of trade change, government expenditure change, debt deviation from the target and tax revenue change. The vector $A(\theta)$ captures the mean of Y_t , that is related to the corresponding structural parameters while B is the coefficient matrix of s_t which is independent of θ . I use software package $Dynare^{27}$ to estimate the vector of parameters θ employing Bayesian methods and then to solve the model.

Conceptually, Bayesian method lie somewhere in between calibration and maximum likelihood estimation. Specifying a prior is related to the practice of calibrating models while maximum likelihood method is connected through estimating the model with data. Priors can effectively be treated as weights on the likelihood function, used to give more emphasis

²⁶ for example, one may define $\eta_t^x = \hat{x}_t - E_{t-1}[\hat{x}_t]$ and and absorb $E_t[\hat{x}_{t+1}]$ to characterize the model.

 $^{^{27}}$ Dynare is a powerful and highly customizable software platform with an intuitive front-end interface, for handling a wide class of economic models, in particular DSGE and overlapping generations (OLG) models. Part of Dynare is programmed in C++, while the rest is written using the MATLAB programming language. More information and additional resources of the software, including the Dynare User Guide (Griffoli (2013)) and Dynare Manual (Adjemian et al. (2014)), working papers etc. can be found at http://www.dynare.org/. Other useful sources of information include the Dynare wiki and various Dynare forums.

on the desired parts of the parameter subspace. Bayes' theorem links the prior with the likelihood function, forming the posterior density.

Let the prior of a particular model \mathcal{M} with a given set of parameters $\theta_{\mathcal{M}}$ be represented by a probability density function (pdf) as follows,

$$p\left(\theta_{\mathcal{M}}|\mathcal{M}\right) \tag{68}$$

Then I consider a likelihood function which describes the density of observed data for a given model with its parameters as given below,

$$\mathfrak{L}(\theta_{\mathcal{M}}|Y_T, \mathcal{M}) \equiv p(Y_T|\theta_{\mathcal{M}}, \mathcal{M})$$
(69)

where Y_t is the vector of observables from the initial time period to the period T. I consider a case where likelihood function is of recursive nature. Hence, it can be represented as,

$$p(Y_T|\theta_{\mathcal{M}}, \mathcal{M}) = p(y_0|\theta_{\mathcal{M}}, \mathcal{M}) \prod_{t=1}^T p(y_t|Y_{t-1}, \theta_{\mathcal{M}}, \mathcal{M})$$
(70)

I am interested in conditional probability distribution of $\theta_{\mathcal{M}}$, for a given set of observed data Y_t , which is the posterior density $p(\theta_{\mathcal{M}}|Y_t, \mathcal{M})$. Bayes theorem connects the likelihood function $p(Y_T|\theta_{\mathcal{M}}, \mathcal{M})$ with the prior density $p(\theta_{\mathcal{M}}|\mathcal{M})$ forming the posterior density as follows,

$$p\left(\theta_{\mathcal{M}}|Y_{t},\mathcal{M}\right) = \frac{p\left(Y_{T}|\theta_{\mathcal{M}},\mathcal{M}\right)p\left(\theta_{\mathcal{M}}|\mathcal{M}\right)}{p\left(Y_{T}|\mathcal{M}\right)}$$
(71)

The posterior density is proportional to the product of likelihood function and prior density, since the marginal density, $p(Y_T|\mathcal{M})$ is constant for a given set of measurements. Therefore it gives,

$$p(\theta_{\mathcal{M}}|Y_T, \mathcal{M}) \propto p(Y_T|\theta_{\mathcal{M}}, \mathcal{M}) p(\theta_{\mathcal{M}}|\mathcal{M}) \equiv \mathcal{K}(\theta_{\mathcal{M}}|Y_T, \mathcal{M})$$
(72)

The prior density, incorporates researcher's knowledge, experience and expectations on the parameters to be estimated. Dynare employs the Kalman filter, a recursive numerical optimisation algorithm to estimate the likelihood function. The explicit form of the posterior distribution, $p(\theta|y)$, is then obtained by using a sampling-like or Monte Carlo method for instance, Metropolis-Hastings algorithm²⁸. Particularly, Markov Chain Monte Carlo (MCMC) simulations based Metropolis-Hastings algorithm is used for this posterior kernel simulation.

 $^{^{28}}$ A detailed discussion of these can be found in, Koop et al. (2007), Griffoli (2013) etc.

4.2 Data Description

I use quarterly data from 1996:Q1 to 2014:Q2 for Sri Lanka, in this analysis²⁹. In addition to the five observation data series which are real output growth $(\Delta \hat{y}_t + \hat{z}_t)$, inflation $(\hat{\pi}_t)$, nominal interest rate (\hat{R}_t) , exchange rate change $(\Delta \hat{\varepsilon}_t)$ and terms of trade change $(\Delta \hat{q}_t)$, as in Lubik and Schorfheide (2007), I use three additional fiscal variables³⁰; government expenditure change $(\Delta \hat{q}_t)$, debt deviation from the target $(\hat{b}_t - \bar{b}_t)$ and tax revenue change $(\Delta \hat{\tau}_t)$, that adds up to a total of eight observables altogether.

These data series are obtained from CBSL and seasonally adjusted. Output growth rate is computed as the log difference of real GDP and scaled by 100, converting them in to quarter-to-quarter percentages. The log difference of the Colombo Consumer Price Index (CCPI) is used as the inflation rate and multiplied by a factor of 400 to make it annualised. The three months treasury bill rate (average) is taken as the interest rate in the model³¹. Nominal exchange rate data are converted by taking the log difference (scaled by 100) to obtain depreciation rates. Similarly, TOT data series is converted in log differences (scaled by 100) to obtain the corresponding percentage change in gross terms.

Tax revenue and government expenditure data series are taken as GDP ratios. Firstdifference of these two series are then calculated obtaining the tax revenue change and government expenditure change respectively. I treat HP trend of debt (as a percentage of GDP), as the target value for the debt series, due to unavailability of any such target prior to 2003. Accordingly, I calculate the deviation of debt from the target, taking the difference of the two series, debt and HP trend of debt.

4.3 Choice of Priors

I choose priors for our estimation, based on several considerations. The priors essentially reflect researcher's beliefs and the anticipated magnitudes of the structural parameters. Most of the parameters, however, are selected based on some observations and simple calculations.

²⁹In some studies such as Canova and Ferroni (2011) and Röhe (2012), the authors decide to stop at 2006 to avoid complications stemming from the GFC. In this study, however, I use data for the above full period due to: 1) Sri Lanka was not seriously affected by the GFC, 2) unavailability of quarterly data for most of the economic variables before 1996 would badly affect the data span, if chop down from 2006 and 3) to avoid possible technical issues which could arise due to too-short data series.

 $^{^{30}}$ These are expressed as a percentages of GDP.)

³¹Annualised inflation figures correspond to $4\hat{\pi}_t$ since this is a quarterly model. The three-month treasury bill rate is taken as the interest rate which corresponds to $4\hat{R}_t$ in the model.

Size restrictions on priors such as non-negativity are imposed by selecting the appropriate distributions for the priors or by truncating the distribution.

Information and characteristics of the Sri Lankan economy play a key role in deciding on the priors. Particularly, the degree of openness, being a commodity producing SOE, level of debt and effective tax rate, etc. are treated as important considerations. Widely recognised SOE literature such as Gali and Monacelli (2005), Lubik and Schorfheide (2007), Del Negro and Schorfheide (2008) and Justiniano and Preston (2010), and the available limited number of GSDE studies in Sri Lanka, Anand et al. (2011), Ehelepola (2014) and Karunaratne and Pathberiya (2014) are also used as guides in deciding on priors. Further, scarcity and unavailability of certain information in Sri Lanka, made me to refer similar studies in other countries, for instance, Liu (2006), Nimark (2009), Çebi (2012) and Ahmed et al. (2013) etc. as steers.

Following Lubik and Schorfheide (2007), the model is parametrised in terms of the steady state real interest rate, R, which is related to the subjective discount factor, β , through the relationship, $\beta = exp \left[-R/400\right]$. Thus, I use 5.88 as the prior for R, which is in good agreement with $\beta = 0.9854$, taken from Ehelepola (2014), and associate it with a large standard deviation of unity. The prior for the degree of openness, α , is tightly centered at 0.35, with small standard deviation of 0.03. These values are assigned considering the average import share of Sri Lanka over the sample period of 1996:Q1 to 2014:Q2, which is supported by Karunaratne and Pathberiya (2014) and other similar emerging market studies. A prior mean of 0.5 each is assigned for both the inverse risk aversion parameter (which is the inter temporal substitution of elasticity) (μ) and the slope coefficient in the Phillips curve (κ), but allow them to vary widely with fairly large standard deviations of 0.25 and 0.2 respectively³².

The priors for three parameters in the forward looking monetary rule, ψ_1, ψ_2 and ψ_3 are influenced by the corresponding values associated with the Taylor rule estimated for Sri Lanka by Perera and Jayawickrema (2013) and authors own work, yet to be published. In the light of these results, I use little tight priors for the parameters to be estimated for the

³²Relative risk aversion parameter ($\sigma = 1/\mu$), is given a value of 2 in Ehelepola (2014) while in a field experiments based study, Cardenas and Carpenter (2008), report that σ in developing countries lies between 0.05 to 2.57. Following Lubik and Schorfheide (2007), I impose $0 < \mu < 1$ to avoid singularity at $\mu = 1$, since the world output shock vanishes from the IS curve otherwise.

monetary policy rule. Accordingly, the three values 1.15, 0.55 and 0.10 are used as the priors respectively, with associated standard deviations of 0.05 each. The priors for the analogous Taylor type contemporaneous fiscal rule parameters, ψ_b, ψ_g and ψ_y are influenced by the author's own estimates, in the absence of any previous similar studies in Sri Lanka to gauge them. The three parameters are assigned with fairly small/moderate priors of 0.20, 0.50 and 0.50 respectively, however, with somewhat relaxed standard deviations of 0.05, 0.15 and 0.15 respectively, letting the data to influence more in determining suitable values for the parameters while imposing less importance to the level of debt.

Prior values for the persistence parameters are motivated by the previous Sri Lankan studies, similar studies in other countries and simple AR(1) analysis. Interest rate persistence parameter in the monetary rule ρ_R is centred at 0.80 with a standard deviation of 0.15. Priors set for the persistence parameters of the fiscal variables ρ_b , ρ_{τ} and ρ_g are influenced by AR(1) analysis, however, they are assigned with slightly large standard deviations. ρ_{g^*} is assigned with 0.80 with a standard deviation of 0.15, based on other similar studies. It is pretty standard to use US economy as a proxy to world economy, in SOE studies and accordingly I exploit Lubik and Schorfheide (2007) to obtain priors for ρ_{y^*} and ρ_{π^*} . Average values of the sample period are used as the corresponding prior means for the variables, R, τ, τ^d, g, g^* and b, in the steady state. They are however, tightly centred around their respective mean values³³.

Finally, priors for the volatility of the shock processes are mainly motivated by the approach followed in Lubik and Schorfheide (2007). It is a common practise to set a same standard deviation for all of the volatility priors and accordingly I set it at 4.00, except for debt³⁴. I follow previous literature in deciding on the appropriate functional forms for the prior distributions. The standard practise³⁵, is to use a Beta distribution for parameters that takes values in the interval [0, 1), Normal or Gamma distributions for parameters with positive real number values (\Re^+) and a Inverse-Gamma distribution for precision of shock processes. Details of the prior distributions of the structural parameters and the standard deviations of the shock processes are given in the Table 1 given below.

 $^{^{33}}$ Initially I assigned s.s. values of these variables with their corresponding the sample period averages, without estimating them (i.e. calibration). However, estimating them, allowing data to influence on the s.s. values seems to be more appropriate. 34 This is to constrain estimated value of volatility of debt being excessively large.

³⁵As reported by many, including Liu (2006), Röhe (2012), Karunaratne and Pathberiya (2014).

Parameter	Domain	Density	P(1)	P(2)
ψ_1	\Re^+	Gamma	1.15	0.05
ψ_2	\Re^+	Gamma	0.55	0.05
ψ_3	\Re^+	Gamma	0.10	0.05
ψ_b	\Re^+	Gamma	0.20	0.05
ψ_y	\mathfrak{R}^+	Gamma	0.50	0.15
ψ_g	\Re^+	Gamma	0.50	0.15
g	\Re^+	Gamma	0.24	0.05
g^*	\Re^+	Gamma	0.33	0.05
b	\Re^+	Gamma	0.90	0.05
$ au^d$	\Re^+	Gamma	0.14	0.03
au	\mathfrak{R}^+	Gamma	0.14	0.05
R	\Re^+	Gamma	5.88	1.00
κ	\Re^+	Gamma	0.50	0.25
α	[0, 1)	Beta	0.35	0.03
μ	[0, 1)	Beta	0.50	0.20
ρ_R	[0, 1)	Beta	0.80	0.15
$ ho_q$	[0, 1)	Beta	0.50	0.20
ρ_z	[0, 1)	Beta	0.50	0.25
$ ho_{y^*}$	[0, 1)	Beta	0.93	0.05
$ ho_{\pi^*}$	[0, 1)	Beta	0.80	0.15
$ ho_b$	[0, 1)	Beta	0.80	0.10
ρ_{τ}	[0, 1)	Beta	0.70	0.20
$ ho_g$	[0, 1)	Beta	0.80	0.10
$ ho_{g^*}$	[0, 1)	Beta	0.80	0.10
σ_R	\Re^+	InvGamma	0.50	4.00
σ_q	\Re^+	InvGamma	3.00	4.00
σ_z	\mathfrak{R}^+	InvGamma	1.50	4.00
σ_{y^*}	\mathfrak{R}^+	InvGamma	1.50	4.00
σ_{π^*}	\mathfrak{R}^+	InvGamma	1.00	4.00
σ_b	\mathfrak{R}^+	InvGamma	4.00	0.50
$\sigma_{ au}$	\mathfrak{R}^+	InvGamma	1.00	4.00
σ_{g}	\mathfrak{R}^+	InvGamma	1.00	4.00
σ_{g^*}	\Re^+	InvGamma	1.50	4.00

Table 1: Prior Distributions

Notes: (1). P(1) and P(2) indicate the mean and standard deviation for Beta, Gamma and Normal distributions; s and ν for the Inverse Gamma distributions, where $p(\sigma|\nu, s) \propto \sigma^{-\nu-1} e^{-\nu s^2/2\sigma^2}$. (2). Persistence parameter and the volatility of the shock process of the debt target (\tilde{b}) , are shown as ρ_b and σ_b here.

Parameter	Prior Mean	Posterior Mean	90 percent HPD*	
			Interval	
ψ_1	1.150	1.176	1.093	1.259
ψ_2	0.550	0.545	0.464	0.626
ψ_3	0.100	0.049	0.014	0.082
ψ_b	0.200	0.315	0.218	0.409
ψ_y	0.500	0.212	0.120	0.301
ψ_g	0.500	0.266	0.151	0.381
g	0.240	0.205	0.137	0.272
g^*	0.330	0.322	0.244	0.399
b	0.900	0.808	0.733	0.882
$ au^d$	0.140	0.105	0.074	0.136
au	0.140	0.103	0.071	0.135
R	5.880	5.887	4.233	7.490
α	0.350	0.291	0.249	0.333
κ	0.500	0.021	0.013	0.028
μ	0.500	0.087	0.055	0.117
$ ho_R$	0.800	0.795	0.726	0.867
$ ho_q$	0.500	0.524	0.402	0.650
ρ_z	0.500	0.718	0.638	0.799
$ ho_{y^*}$	0.930	0.966	0.952	0.981
$ ho_{\pi^*}$	0.800	0.798	0.695	0.908
$ ho_b$	0.800	0.820	0.768	0.874
$ ho_{ au}$	0.700	0.834	0.778	0.893
$ ho_g$	0.800	0.844	0.801	0.888
$ ho_{g^*}$	0.800	0.773	0.700	0.846
σ_R	0.500	0.233	0.147	0.316
σ_q	3.000	2.001	1.720	2.274
σ_z	1.500	0.821	0.679	0.961
σ_{y^*}	1.500	0.387	0.290	0.411
σ_{π^*}	1.000	4.004	3.447	4.544
σ_b	4.000	9.781	8.654	11.131
$\sigma_{ au}$	1.000	0.420	0.343	0.493
σ_{g}	1.000	0.757	0.638	0.875
σ_{g^*}	1.500	0.648	0.429	0.936

Table 2: Posterior estimates using 1,000,000 Markov Chain draws

Notes: *HPD: Highest Posterior Density

4.4 Estimation Results, Diagnostics and Interpretation

I use 1,000,000 Metropolis Hastings simulations in the analysis and Dynare runs various diagnostics checks described below to check the robustness of the estimates. Table 1 summarizes details of the priors used, while Table 2 sums up estimation results, including posterior mean and 95 per cent Highest Posterior Density Interval (HPDI), apart from prior mean, for all the parameters and the shocks.

The results are in general consistent with the previous literature and there are several noteworthy outcomes. I find that the Central Bank of Sri Lanka conducts moderately anti-inflationary monetary policy (ψ_1 =1.18) and reveals considerably large interests on output (ψ_2 =0.54), however, negligibly small concerns for exchange rate movements (ψ_3 =0.05). These observations are supported by the previous studies in Sri Lanka such as Perera and Jayawickrema (2013), Ehelepola (2014) and Karunaratne and Pathberiya (2014). A high degree of interest rate persistence is evident since the smoothing parameter (ρ_R) is estimated to be 0.80. Further, I find that the fiscal policy rule reacts to the debt level and government expenditure to a smaller extent (ψ_b =0.32, ψ_g =0.27) while stabilising output weakly (ψ_y =0.21). Estimated figures reflect that there is strong persistence for fiscal variables as well (ρ_b =0.77, ρ_τ =0.79 and ρ_g =0.80). The fact that posterior distributions of these policy parameters are more sharper and symmetric around their means, compared to priors, suggest that data are informative.

Posterior means obtained for the structural parameters lie within the plausible ranges. Estimate for the steady state real interest rate (R) is much closer to its prior, which is in good agreement with the implied discount factor (β) for Sri Lanka³⁶, as in Ehelepola (2014). The openness parameter of the economy (α), is estimated to be slightly smaller than the actual import share of Sri Lanka³⁷. This is still a justifiable estimate, as per the argument in Lubik and Schorfheide (2007). They estimate α to be 0.11 for Canada while the actual value of the import share is 0.20 and argue that α should not necessarily be equal to import share, based on widely accepted literature, including Lubik and Schorfheide (2006) and Justiniano and Preston (2010). More specifically they explain that α is selected so as to reconcile the volatility in TOT and of CPI inflation in the PPP equation (eq.49) and to obey the cross

³⁶Note that $\beta = \exp[-R/400] = 0.9854$

 $^{^{37}}$ The import share of Sri Lanka for the period considered is 0.35 whereas the estimated value for α is 0.29

co-efficient restrictions embedded in IS and Phillips curve equations. They further state that this argument is valid for the slope parameter in the Phillips curve (κ) and inter temporal substitution elasticity (μ) as well.

I allot the prior for the slope parameter in the Phillips curve (κ), centered at 0.50 fairly loosely, letting data to have more weight in determining its posterior value. The estimated value seems to be acceptable since it is compatible with the implied value for the Calvo price stickiness parameter for Sri Lanka³⁸. Estimated value for the inverse risk aversion parameter or the intertemporal substitution of elasticity (μ) is 0.09 which is considerably smaller than the prior. Given the above argument in Lubik and Schorfheide (2007) justifying a lower value for μ , the estimated posterior value seems to be tolerable.

Finally, I consider persistence and volatility parameters of structural shocks. The autoregressive parameters are estimated to be lie in between 0.52, for the TOT shock (ρ_q) and 0.97, for the foreign output shock (ρ_{y^*}). The prior, posterior plots of the autoregressive parameter depict that posterior distributions are sharply peaked at their respective posterior values having a fine symmetric shape of a normal distribution. Turning to the standard deviations of the shock process, it shows that the interest rate shock is the least volatile shock (σ_R =0.23) while the debt target shock is the most volatile one (σ_b =9.8). Further, the posterior distribution of the debt target shock parameter is wide with a somewhat asymmetric shape, suggesting substantial parameter uncertainty and possible poorly identification. For all the other shocks, posterior distribution are sharply peaked suggesting fairly strong identification.

Historical and smoothed variable plots of the observable data are shown in the figure F1. Actual observed data are denoted by the dotted black line while the estimate of the smoothed variable (best guess for the observed variable given all observations), derived from the Kalman smoother at the posterior mean (Bayesian estimation), is shown by the red continuous line. The two lines overlap with each other, in the absence of measurement errors which is the case in this model.

The figure F2 demonstrates the smoothed estimated shocks which are reconstructed with ³⁸The slope coefficient in the Phillips curve, $\kappa \equiv (1 - \beta\theta)(1 - \theta)/\theta$. Ehelepola (2014) use a value of 0.75 for Calvo price stickiness parameter for Sri Lanka. The estimated value for κ =0.13, however, implies a little larger value of 0.86 for θ , Calvo price stickiness parameter. In the DSGE literature, it is common to use a value of 0.8 for this parameter and accordingly the estimated value seems to be tolerable.

the values of unobserved shocks over the sample, using all the information embedded in the observed data. Kalman smoother is used in this computation. The assumption of the model that the shocks are of zero mean implies that they should be centered around zero. If the smoothed shocks are systematically away from zero, however, suggest that an issue in the model; either missing constants or a mismatch between the meaning of the variables in the model and in data used. Clearly, all of the plots of smoothed estimated shocks in figure F2 are centered around zero, suggesting that the model is free from any such problems.

4.4.1 Brooks and Gelman Convergence Statistics

This test is based on Brooks and Gelman (1998), for monitoring the convergence of iterative simulations by comparing between and within variances of multiple chains, aiming at obtaining a set of tests for convergence³⁹.

Dynare presents Monte Carlo Markov Chains (MCMC) univariate diagnostics which provides the key source of feedback to build up confidence on the fitted model, or to locate a possible problems with results. Dynare runs several Metropolis-Hastings simulations each time starting from a different initial value. As per Griffoli (2013) and Pfeifer (2014), if the results from one chain are sensible, and the optimizer did not get stuck in an odd area of the parameter subspace, two things should happen: First, results within any of the different iterations of Metropolis-Hastings simulation should be compatible. Second, results among the different chains should not be far away. Accordingly, the red and blue lines in the charts, which are specific measures of the parameter vectors both within and among chains, expected to be approximately flat (though there exists some fluctuations) and they converge⁴⁰. Dynare further reports multivariate statistics which is essentially a summary that takes together all univariate diagnostics⁴¹.

Figures F4 and F5 depict MCMC univariate diagnostics and multivariate convergence diagnostics for the model. They clearly show that both lines in each of the above three

³⁹Brooks and Gelman (1998) review methods of inference from simulations to build up convergence-monitoring summaries applicable for the purposes. Based on the comparison of inferences from single sequences and from the mixture of sequences, the authors advocate a series of tests for mixing. They further discuss simultaneous convergence of several parameters assessments using analogues tests.

⁴⁰Dynare reports three measures: "interval", being constructed from an 80 per cent confidence interval around the parameter mean, "m2", being a measure of the variance and "m3", based on third moments. In each case, Dynare reports both the within and the between chains measures.

⁴¹Which is an aggregate measure calculated by the eigenvalues of the variance-covariance matrix of individual parameters.

measures, are relatively constant and converge to each other in almost all graphs. Moreover, the figures reveal that the diagnostics of overall convergence is achieved both within and between chains, for all three moments considered. Non of the parameters show convergence issues notwithstanding the fact that for some of them this evidence is stronger than the others.

4.4.2 Prior and Posterior Plots

Prior-posterior plots presented in Figure F6, provides a lot of insights about the estimation results. The grey line and the black line denote the distribution of prior density and the posterior density respectively while the green vertical line indicates the posterior mode. Approximately identical prior and posterior distributions suggest either the prior is very accurate such that it provide all the information data carries, or the parameter under consideration is only weakly identified and the data does not provide much information to update the prior⁴². Further, the mode calculated from the numerical optimization of the posterior kernel (green vertical line) should not deviated much from the mode of the posterior distribution (black curve) and finally, the shape of the posterior distribution should have nearly normal shape.

Figure F6 reveals that the shape of the posterior distributions in most of the cases are of normal shape, matching with asymptotic properties of Bayesian estimation (except σ_b curve, which is considerably skewed). The above said two modes occur very closer to each other in almost all the graphs. More importantly, the prior and posterior distributions are clearly distinct in majority of the cases suggesting that observed data do provide additional information in estimations and they are not solely driven by priors. In few cases, however, posterior distributions are nearly identical to their respective priors. This could be due to the fact that priors already provide full information embedded in data, particularly for the s.s. parameters and monetary policy parameters.

4.4.3 Impulse Response Functions (IRFs)

I further analyse the model dynamics by using the IRFs of the variables, in response to temporary shocks of one standard deviation. As revealed from the figures F6 to F15, the

 $^{^{42}}$ For more details Canova (2007)

variables return back to their corresponding steady state values gradually for all shocks, ensuring stability of the model⁴³. Analysing all the variables dynamics for each of the shocks is, however, not the objective of the study and accordingly the discussion is limited to a technology shock, a monetary (interest rate) shock and a government purchase shock only.

IRFs of the Technology Shock

As expected, a positive technology shock of one standard deviation creates expansionary consequences across the economy as evident from the IRFs. Output and its components rise immediately after the shock, but returns back to their s.s. values over the time. Similar to the corresponding IRFs of Lubik and Schorfheide (2007), decline in production costs owing to increased productivity which in turn lower the prices of domestically produced goods is attributable for the downward trend in inflation after the first period. This lowering of inflation, however, is dampen away soon due to the newly enhanced demand and possible upward wage revisions. In line with the initial rise in inflation, central bank increases nominal interest rate in the first few periods, but starts to ease it, as the inflation tend to decline gradually. In response to the rise in interest rate, exchange rate appreciates but returns back to the s.s. level in about 10 quarters. With the increased output, tax revenue tend to raise, thereby increasing the tax, but, decline gradually towards the s.s level, after that.

IRFs of the Monetary Shock (interest rate)

Contractionary monetary policy rises nominal interest rate, thereby lowering inflation and output while appreciating currency, as depicted in the IRF plots. They return back to their corresponding s.s. levels in approximately 15 quarters. With the lowered output, tax revenue also tend to go down, but recover in about 15 quarters.

 $^{^{43}}$ In these graphs, the grey shaded areas provide Highest Posterior Density Intervals (HPDI). These Bayesian IRF plots are generated by the *bayesian irf* option of the estimation-command. If one wants to compute classical IRFs after estimation, *stoch simul* command should be used after estimation as the latter will set the parameters to the posterior mode or mean, depending on whether you use maximum likelihood or Bayesian estimation (Pfeifer (2014)). The main difference between these two is that the *stoch simul-IRFs* are computed at the calibrated parameter combination, while the Bayesian IRFs are the mean impulse responses (not to be confused with the IRFs at the mean).

IRFs of the Government Purchase Shock

Government purchase shock raises aggregate output and the higher demand created fuel up inflation. Monetary authority responds to inflation by raising interest rate initially, but begin to relaxes soon, as the inflation drops down to its s.s. level gradually. Raised interest rate appreciates exchange rate. Escalation of government expenditure inclined to increase total tax, but drops back as the effect of the shock fades out. The level of debt (as a percentage of output) drops as the output grows rapidly, however, it gradually level off in about 20 quarters.

5 SUMMARY AND CONCLUDING REMARKS

DSGE models became the corner stone in the contemporary macroeconometric modelling, receiving high recognition from both policy makers and academia, all over the world. Owing to the advancement of computer technology together with improved econometric techniques, Bayesian methods have emerged as the most rewarding method in estimating DSGE models and accordingly it has been widely adopted in the recent years. Over the last two decades many different variants of the DSGE models developed to serve different purposes and SOE tradition is an important one among them. Most of these SOE-DSGE models, however, abstract from fiscal policy, despite the major role of it, in stabilizing the economic fluctuations. The present study where I use explicit monetary and fiscal policy rules in a SOE-DSGE environment is an attempt to fill this gap, while modelling business cycle fluctuations of Sri Lanka.

This study formulates a SOE New-Keynesian DSGE model for Sri Lanka, and estimates it using Bayesian techniques, with quarterly data for the period 1996:1 to 2014:2. The diagnostics, particularly the quality of the numerical posterior kernel maximization and convergence of the Metropolis-Hastings algorithm, suggests that the estimates are robust in vast majority of the parameters. Plots of the prior and posteriors density distributions indicate that data are fairly informative and model fits with the observed data quite well. Estimated results, appears to be satisfactory, in general, and consistent with the limited previous literature available in Sri Lanka. Further, the estimates seem sensible from an economic perspective, in most cases. Among them, some of the findings are notably important in policy analysis: firstly, the central bank of Sri Lanka responds to inflation fairly strong manner while stabilizing output reasonably well, however, without much concern on the exchange rate dynamics. Secondly, the existence of fairly strong interest rate persistence, in the monetary policy rule. Thirdly, the fiscal authority responds to the debt level and government expenditure to a moderately small extent, while stabilizing output, to a smaller extent.

Withstanding to the fact that estimates are in overall satisfactory, estimates do contain few concerns, similar to any other empirical study. The posterior distribution of debt volatility does not seem to be symmetric enough, suggesting poor quality of estimation of the parameter. The slightly large value of 0.86 for the price stickiness parameter implies a price fixity for nearly 7 quarterly, which seems to be little larger for Sri Lanka. The estimated import share, 0.29 is also smaller than the expectations sine the average of the sample period of the same is observed to be 0.35. The debt coefficient in the fiscal rule, 0.32 is also somewhat larger than what is expected. Single equation fiscal policy rule estimations suggest a very small coefficient of below 0.1 for the same. This discrepancy could partly be due to the fact that current study determines the debt coefficient in a multi equation DSGE environment with a built in government budget constraint, in contrast to the single equation case. It could also be partly due to the filtering of debt data to make the series stationary⁴⁴. In few cases , it is also noted that posterior values are highly influence by the priors, even though data being informative in many instance. This could, however, be due to the fact that priors already carry the information embedded in the data.

The model can be strengthen further, by incorporating additional realistic features such as imperfect exchange rate pass-through, capital accumulation and investment rigidities, money and labour market dynamics. Additionally, one can extend the model to analyse welfare implications of different policy regimes or to perform optimal policy analysis, approximating the utility functions up to second order accuracy.

 $^{^{44}}$ The difference in the (1) sample periods used and (2)the time varying target value used may also explain this difference, to some extent.

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A Appendix 1: Abbreviations Used

Abbreviation	Description	
CBSL	Central Bank of Sri Lanka	
CPI	Consumer Price Index	
CCPI	Colombo Consumer Price Index	
DSGE	Dynamic Stochastic General Equilibrium	
FEVD	Forecast Error Variance Decomposition	
GDP	Gross Domestic Product	
GFC	Global Financial Crisis	
GMM	Generalised Method of Moments	
HPDI	Highest Posterior Density Interval	
IRF	Impulse Response Function	
LOP	Law of One Price	
LRE	Linear Rational Expectations	
MCMC	Markov Chain Monte Carlo	
MH	Metropolis Hastings	
ML	Maximum Likelihood	
NK	New Keynesian	
pdf	probability density function	
PPP	Purchasing Power Parity	
RBC	Real Business Cycle	
SOE	Small Open Economy	
S.S.	steady state	
TOT	Terms of Trade	
UIP	Uncovered Interest rate Parity	
VD	Variance Decomposition	

 Table 4: Abbreviations Used

B Appendix 3: Graphical Representations

More technical details of the graphs plotted using Dynare can be found at Pfeifer (2014).

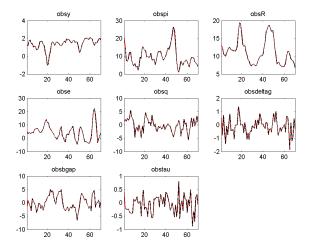
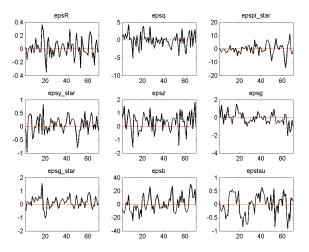


Figure F1: Historical and Smoothed Variables

Note: The dotted black line depicts the actually observed data, while the red line describes the estimate of the smoothed variable (i.e. best guess for the observed variables, given all observations). It is derived from the Kalman smoother at the posterior mode or posterior mean (of the Bayesian estimation). If the measurement error is zero, then the two series overlap with each other.

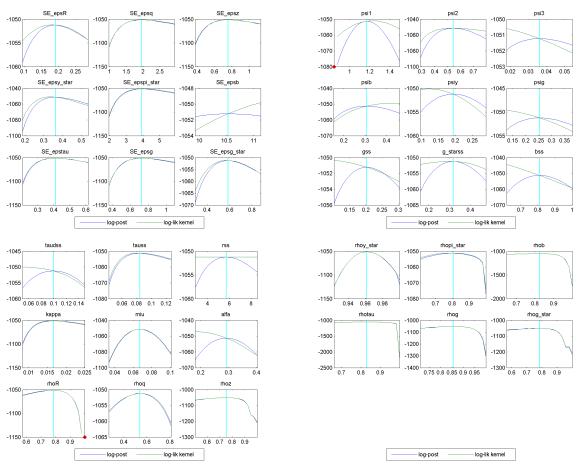




Note: Smooth shocks are a reconstruction of the values of unobserved shocks over the sample, using all the information contained in the sample of observation. It is computed via the Kalman smoother⁴⁵

⁴⁵The model assumes that the shocks are of zero mean. Hence, if one obtain smoothed shocks that are systematically away from zero, there is indeed a problem in the model (either missing constants or a mismatch between the meaning of the variables

Figure F3: Mode check plots



Note: These figures assist in finding whether the mode-computation hit the (local) mode. A range of parameter values in the vicinity of the estimated mode (horizontal magenta line) is given in the x-axis while the log-likelihood kernel shifted up or down by the prior value at the posterior mode (green line) and of the posterior likelihood function (blue line) is indicated by the y-axis. Thus, the differences between the likelihood kernel and the posterior likelihood indicate the role of the prior in influencing the curvature of the likelihood function. The estimated mode is expected to be observed at the maximum of the posterior likelihood.

in the model and in the data).

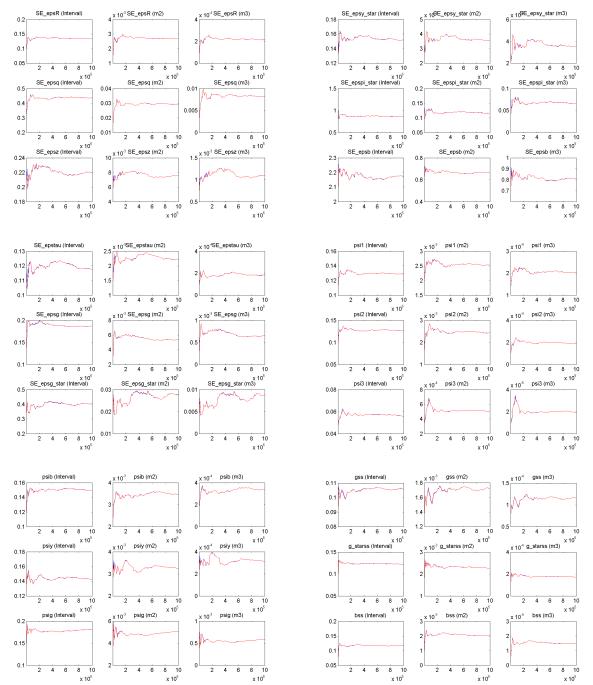
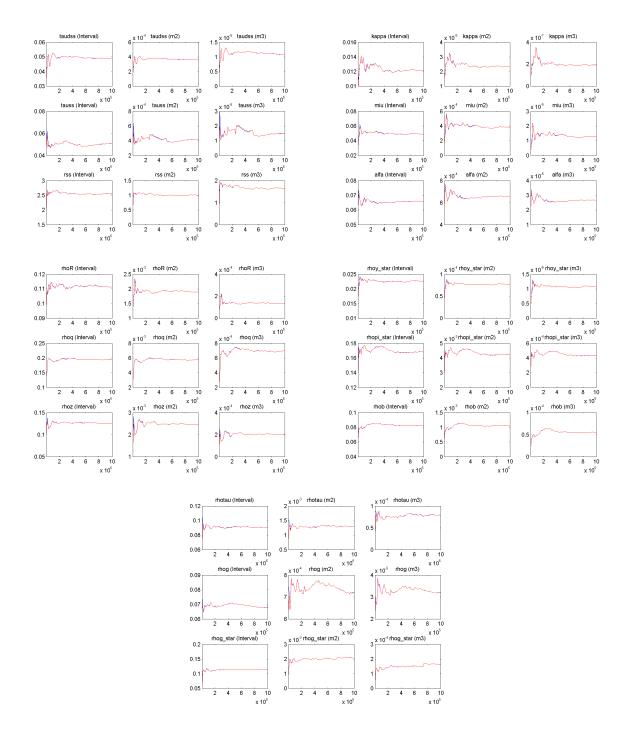
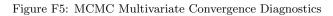
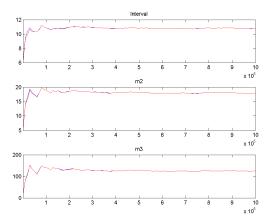


Figure F4: MCMC Univariate Convergence Diagnostics



Note: The red and blue lines that corresponds to two different Metropolis-Hastings simulation chains, should ideally be not far away from each other and further, they should be approximately flat.





Note: These graphs are analogous to the univariate graphs above, except the fact that the statistics now being based on the range of the posterior likelihood function, instead of the individual parameters.

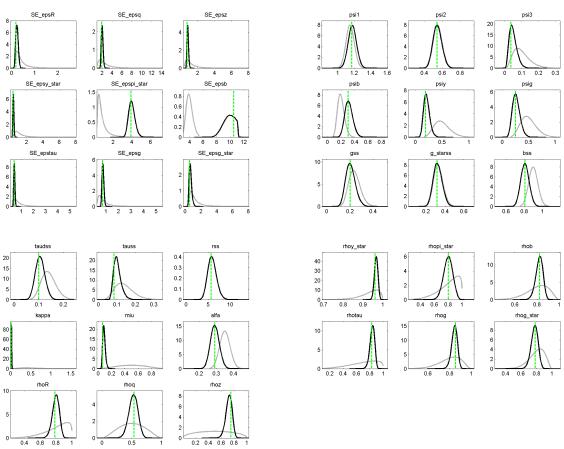


Figure F6: Priors and Posteriors

Note: The x-axis shows the main range of the prior distribution, while the y-axis displays the corresponding density. The grey curve displays the prior density and the black curve shows the density of the posterior distribution. The posterior mode is indicated by the green horizontal line.

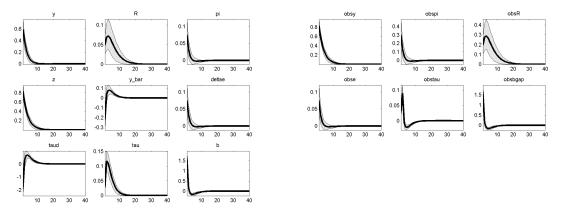


Figure F7: Impulse Response Functions of the Variables to a Technology Shock of One Standard-Deviation

Note: Bayesian IRF plot generated by the bayesian irf-option of the estimation-command. The highest posterior density intervals (Highest Posterior Density Interval (HPDI)) is shown by the gray shaded areas.

Figure F8: Impulse Response Functions of the Variables to a Monetary (interest rate) Shock of One

Standard-Deviation

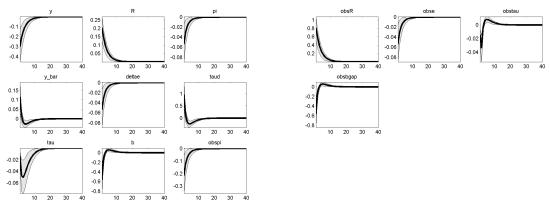
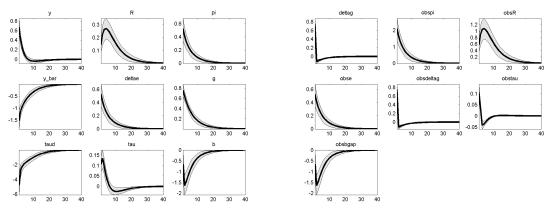


Figure F9: Impulse Response Functions of the Variables to a Government Purchase Shock of One

Standard-Deviation



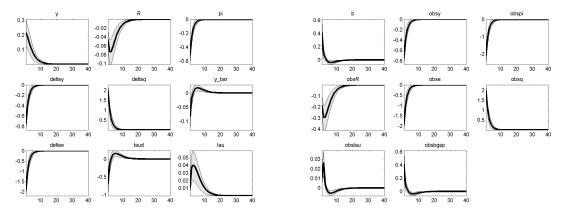


Figure F10: Impulse Response Functions of the Variables to a TOT Shock of One Standard-Deviation

Figure F11: Impulse Response Functions of the Variables to a World Output Shock of One Standard-Deviation

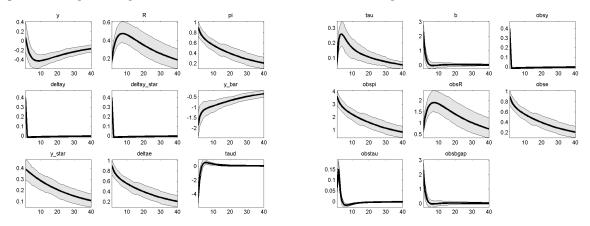


Figure F12: Impulse Response Functions of the Variables to a World Inflation Shock of One Standard-Deviation

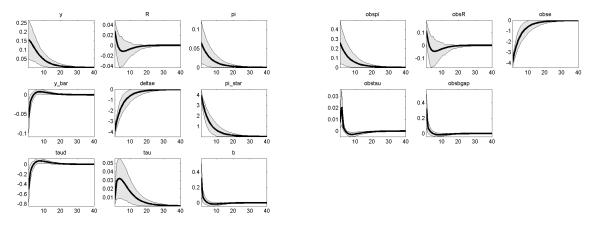


Figure F13: Impulse Response Functions of the Variables to a World Government Purchase Shock of One Standard-Deviation

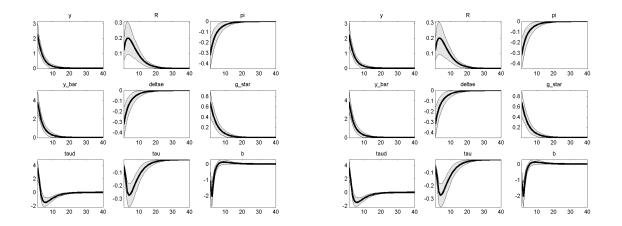


Figure F14: Impulse Response Functions of the Variables to a Government Debt Shock of One Standard-Deviation

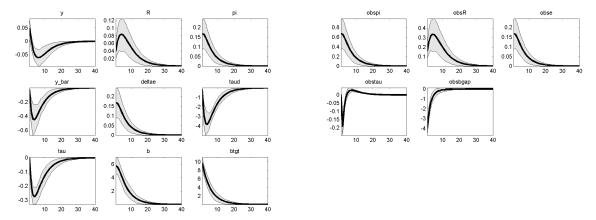


Figure F15: Impulse Response Functions of the Variables to a Total Tax Shock of One Standard-Deviation

