Bayesian New Keynesian Small Open Economy
Dynamic Stochastic General Equilibrium Model for
Sri Lankan Economy

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Abstract
In this paper we present a variant of the Small Open Economy Dynamic Stochastic General Equilibrium Model (SOE-DSGE) and empirically validate it using quarterly data for Sri Lanka (1999Q1-2013Q4). The SOE-DSGE model incorporates low exchange rate pass-through dynamics and thereby departs in a significant manner from the canonical closed economy New Keynesian DSGE model which has become the work-horse of monetary policy design by modern central banks. The empirical validation of the SOE-DSGE model in the paper using Bayesian estimation techniques provides insights that would enable the Central Bank of Sri Lanka to advance its reform agenda to transition from the current monetary targeting framework to a flexible inflation targeting framework in the medium-term. The model presented herein can be extended evaluate the welfare implications of alternative monetary policy regimes and make out-of-sample inflation forecasts to get a handle on the type of policy adjustments that should be undertaken to design a monetary policy stance to achieve a predetermined stabilization target.

JEL Classifications C15, C51, E52, F37, F47.

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1. Introduction

During the past three decades dramatic changes in the global financial landscape and the science and art of macroeconomic theory and macroeconometrics have begun to influence the design and implementation of monetary policy in both advance and developing countries. The revolutionary changes in macroeconomic theory and macroeconometric modeling that has thrown open the gauntlet to monetary policymakers and central bankers to spruce up their act of monetary policy design to meet the world best practice standards in order to deliver more effective countercyclical stabilization policy outcomes.

The changes in the macroeconomic theory and modeling that guided monetary policy making by central banks have changed from the Cowles Commission type of models that predicted outcomes of monetary policy based fixed coefficient econometric models. The Lucas (1976) critique highlighted that fixed coefficients in these models do not change with expected policy changes and therefore they were structurally invariant, and policy predictions based on such models would be misleading. The rational expectations theorists responded to the Lucas critique by asserting that the Keynesian type of stabilization policies based on fixed coefficients models that bastardized in Hicks IS-LM type models were fundamentally flawed. The stagflation that occurred during the oil crises in the 1970s gave credence to the claims of the rational expectations theorists that Keynesian type of stabilization policy could deliver an optimal inflation rate based on a tradeoff between inflation and unemployment was a dodo based on ad hocery. Around this time Real Business Cycle (RBC) theorists advanced what was regarded as a heretical proposition that the business cycle was caused solely by technological shocks and the Keynesian type of stabilization policies were misplaced and if implemented would contribute to further instability. In the seminal paper, Time to build and aggregate fluctuations, Kydland and Prescott (1982) argued that RBC models were based on firm micro-foundations, where rational agents engage inter-temporal and intra-temporal optimization decisions subject to inter-temporal budget constraints. The RBC models whilst appealing to academia because of they were based on theoretically sound micro-foundations by-passed monetary policymakers and central bankers as the RBC theories accorded no role monetary policy countercyclical stabilization. However, this unrealistic hiatus between the new perceptions in macroeconomic theory and the practice of monetary policy led New Keynesian economists to highlight the importance nominal and real rigidities that accorded a significant role to monetary policy aimed at stabilizing economic fluctuations, in the short-run. The models resulting from this New Neoclassical synthesis, have been christened New Keynesians and
highlights that changes in the monetary policy stance have non-trivial effects on the business cycle and real variables. The main features of these new generation models have been surveyed in Gali and Gertler (1999) and others. These models provide the theoretical foundations of the Dynamic Stochastic General Equilibrium (DSGE) models advocated in this paper to upgrade the design and practice of monetary policy in Sri Lanka to world best practice standards.

The canonical DSGE model that has become the workhorse model for the design of monetary policy to implement counter cyclical stabilization policies are characterized by two major departures from the Keynesian economists in the form of imperfect competition and staggered price and wage setting. The canonical New Keynesian (NK) model comprises of three equations where the structural coefficients or deep parameters are immune to the ‘Lucas critique’ as the parameters are structurally invariant to policy changes and these deep parameters are derived from first order conditions (FOCs) from solution of the inter-temporal optimization problems facing representative agents (households, firms, government) operating in the economy. The equations are: i). A dynamic inter-temporal optimizing IS-curve or the consumption Euler equation. ii). A New Keynesian Phillips Curve (NKPC), wherein the inflationary expectations are determined by past inflation or inertia, the output gap and exogenous supply shocks. iii). The model is closed using a monetary policy reaction function or a Taylor type interest rate smoothing rule, which although ad hoc can be regarded exhibiting behaviour that is consistent with micro-founded behaviour [Taylor (1993)].

Currently the CBSL conducts monetary policy using a monetary targeting framework. The ultimate goal of monetary policy is price and economic stability and it is achieved by manipulating the level of reserve or high powered money which is linked to broad money aggregate (M2b). The main policy instrument is the short-term nominal interest rate or the policy rate (i.e. interest rates on overnight repurchase and reverse repurchase agreements and OMO). The CBSL sets out a program which maps out the desired path of money supply or liquidity growth taking into account the macroeconomic shocks that buffet the economy. The desired path of money growth is achieved by meeting quarterly reserve money targets by conducting open market operations (OMO) within an interest rate corridor formed by the policy rates (repo and reverse repo rates) in order to meet the prescribed reserve money target. However, number of recent studies have proposed an inflation targeting framework for the CBSL [for example see Perera and Wickramanyaka (2013) and Anand, Ding and Peiris (2011)] in line with the trend set by most developed and emerging market countries with moving to
such framework. The CBSL has also announced in its medium-term policy framework published in 2011 [CBSL (2011)] that the CBSL would target mid-single digit level of inflation in the medium-term.

To the best of our knowledge, the only New Keynesian type empirical DSGE study conducted for Sri Lanka is Anand, Ding and Peiris (2011). They developed a practical model-based forecasting and policy analysis system (FPAS) to support a transition to an inflation forecast targeting regime in Sri Lanka. The model features a small open economy including forward-looking aggregate supply and demand with micro-foundations and with stylized (realistic) lags in different monetary transmission channels. Output developments in the rest of the world feed directly into the Sri Lankan economy as they influence foreign demand for Sri Lankan products. Changes in foreign inflation and interest rates affect the exchange rate and, subsequently, demand and inflation in the Sri Lankan economy.

However, an important element of Sri Lankan economy is low pass-through of exchange rate movements into prices. Some facets of exchange rate pass-pass through in Sri Lanka have been analysed by Wimalasuriya (2007) and Duma (2008) using VAR (Vector Autoregression) and linear regression modeling. They found that of external shocks such as the exchange rate, oil and import prices had only a 10 percent pass-through to consumer prices during the first month and gradually rose to a maximum of about 40 percent in four months.

The SOE-DSGE modeling framework used in this paper [Monacelli (2003,2005)] focuses on the implications of low-pass through due to the deviations from law-of-one–price (LOP) for the design of optimal monetary policy design and practice. Incomplete pass-through is a crucial feature in the New Keynesian sticky price DSGE models when applied to analyse monetary policy in a SOE economy as it yields fundamentally different outcomes from the closed economy model. Incomplete pass-through which allows for deviations from the LOP generates endogenously a short-run tradeoff between stabilization inflation and output gap. The model is also incorporated with price rigidity. Bayesian techniques will be deployed to estimate the deep structural parameters of the SOE-DSGE model proposed to analyse and overcome the

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1 Ehelepola (2014) has also presented a version of calibrated DSGE model for Optimal Monetary and Fiscal Policy analysis for Sri Lanka in the 7th International Research conference of the CBSL.

importance of market frictions and price adjustments required to deliver best practice monetary policy outcomes.

The main objective of this study is to estimate a small open economy New Keynesian DSGE model for the Sri Lankan economy using Bayesian techniques. This study can be considered as the first step towards formulating a coherent policy simulation framework for implementing monetary policy in Sri Lanka in a transparent manner based on sound micro-foundations. We investigate whether this model with few nominal rigidities can provide a reasonable explanation of the Sri Lankan economy. In particular we are interested in monetary policy transmission mechanism or channels through which innovations to policy variables such as the policy rate affect macroeconomic variables such as output and inflation.

The rest of the paper is organized as follows: Section 2 details the structure of the version of the benchmark SOE-DSGE model required to complement monetary policy design in Sri Lanka, Section 3 outlines the Bayesian techniques that are deployed to estimate the proposed SOE-DSGE model. Section 4 reports the data and reviews some of the empirical results from validation of the SOE-DSGE model using Bayesian techniques. Section 5 presents some concluding observations.

2. The Model
The SOE-DSGE model postulates that the world economy is a continuum of SOEs contained in the unit interval. Each SOE economy has zero impact on the world economy. Although each SOE is buffeted by different imperfectly correlated productivity shocks they share the same structure in terms of technology, preferences and markets. The SOE-DSGE model nests the canonical close economy DSGE model that has become the work-horse of monetary policy design by central banks in advance countries and a large number or emerging market economies. This canonical version of the DSGE model is enunciated in seminal papers of leading researchers in the field such as Gali and Monacelli (2005) and Smets and Wouters (2003).

In this section we attempt to briefly derive structural equations of SOE-DSGE model using four interrelated blocks: the demand bloc, open economy block, a supply block and monetary policy reaction function block in a manner analogous to that outlined by (Sbordone et al. (2010). Our algebraic exposition of the SOE-DSGE modeling is indebted to the path-breaking contributions of leading exponents in the DSGE modeling field such as Gali and Monacelli
In particular, our model emulates the modeling of Monacelli (2005), which incorporates a low pass-through environment. This class of DSGE modeling has been replicated in country-specific contexts in noteworthy papers for Australia by Jaaskela and Nimark (2011), for New Zealand by Liu (2006), for Pakistan by Haider and Khan (2008) and for Armenia by Mkrtchyan, Dabla-Norris and Stepanyan (2009).

In the SOE-DSGE model agents engage intra-temporal and inter-temporal optimizing decision making when they buffeted by shocks that result in market clearing and convergence towards a macroeconomic equilibrium. The monetary authority manipulates the short-term interest rate to achieve the goals of counter-cyclical stabilization policy.

**Demand Block**

The demand block of the SOE-DSGE model explains how representative infinitely-lived households attempt to maximize the present value of expected utility by smoothing consumption over the infinite horizon or their lifetime subject to an inter-temporal budget constraint. The general solution for the households utility maximization problem yields the dynamic IS-curve or the Euler equation for consumption. In more algebraic terms we contend that the SOE that is inhabited by a representative households that seek to maximize the lifetime utility function:

\[
U = E_0 \sum_{t=0}^{\infty} \beta^t \{U(C_t) - V(N_t)\} 
\]

where:  
\[ U(C_t) = \frac{(C_t)^{1-\sigma}}{1-\sigma} \quad \text{and} \quad V(N_t) = \frac{N_t^{1+\varphi}}{1-\sigma} \]

\( \beta \): rate of time preference,  
\( \sigma \): inverse elasticity of inter-temporal substitution,  
\( \varphi \): inverse elasticity of labour supply.  
\( N_t \): hours of labour.,  
\( C_t \): composite consumption index of foreign and domestically produced goods defined by:

\[
C_t \equiv \left( (1-\alpha)\frac{1}{\eta} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha\frac{1}{\eta} C_{F,t}^{\frac{\eta-1}{\eta}} \right)^{\frac{1}{1-\eta}} 
\]

where: \( \alpha \in [0,1] \): import ratio measuring the degree of openness,  
\( \eta > 0 \): elasticity of substitution between home and foreign goods.

The aggregate consumption index for domestic and for foreign goods, assuming elasticity of substitution between varieties of goods are the same (\( \varepsilon > 0 \)) is given by:
\[ C_{F,t} \equiv \left( \int_0^1 C_{F,t}(i) \, di \right)^{\frac{\epsilon}{\epsilon-1}} \text{ and } C_{H,t} \equiv \left( \int_0^1 C_{H,t}(i) \, di \right)^{\frac{\epsilon}{\epsilon-1}} \] (3)

The households maximize the utility function \( U \) subject to the inter-temporal budget constraint:
\[ \int_0^1 \left\{ P_{H,t}(i)C_{H,t}(i) + \{P_{F,t}(i)C_{F,t}(i)\} \right\} \, di + E_t\{Q_{t,t+1}D_{t+1}\} \leq D_t + W_tN_t \] (4)

where, \( P_{H,t}(i) \) and \( P_{F,t}(i) \): prices of domestic and foreign good \( i \), respectively. \( Q_{t,t+1} \): stochastic discount rate on nominal payoffs, \( D_t \) is the nominal payoff on a portfolio held at \( t-1 \) and \( W_t \) is the nominal wage.

The Eq. (4) can be further simplified the inter-temporal budget constraint given by:
\[ P_tC_t + E_t\{Q_{t,t+1}D_{t-1}\} \leq D_t + W_tN_t \] (5)

The steps required to derive Eq. (5) are outlined in Liu (2006) as follows: The optimal allocation for good \( i \) given by the CES aggregator for good \( C_{F,t} \) and \( C_{H,t} \) in equation (3) enables the derivation of the following demand functions:
\[ C_{H,t}(i) = \left( \frac{P_{H,t}(i)}{P_t} \right)^{-\epsilon} C_{H,t} \text{ and } C_{F,t}(i) = \left( \frac{P_{F,t}(i)}{P_{F,t}} \right)^{-\epsilon} C_{F,t} \]

where \( P_{H,t} \) and \( P_{F,t} \): price index of home and imported goods respectively. Based on the assumption of symmetry across all \( i \) goods, the optimal allocation of expenditure between domestic and imported goods is:
\[ C_{H,t} = (1 - \alpha) \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} \text{ and } C_{F,t} = \alpha \left( \frac{P_{F,t}}{P_t} \right)^{-\eta} C_t \]

The above definitions can be combined to derive overall CPI as:
\[ P_t \equiv (1 - \alpha)P_{H,t}^{-\eta} + \alpha P_{F,t}^{\frac{1}{1-\eta}} \]

Therefore, the total consumption expenditure for the domestic household is \( P_{H,t}C_{H,t} + P_{F,t}C_{F,t} = P_tC_t \), which can be used to express the inter-temporal budget constraint given in Eq. (5).

The solution to the households constrained optimization problem is obtained from the Lagrangian formed by Eq.(1) and Eq. (4) above yields the following FOCs:
\[ (C_t)^{-\sigma} \frac{W_t}{P_t} = N_t^\sigma \] (6)
\[ \beta R_t E_t \left( \frac{C_{t-1}}{C_t} \right)^{-\sigma} = 1 \] (7)
where, $R_t = 1/E_t Q_{t, t-1}$: gross nominal return on a riskless one period bond maturing in $t+1$.

Eq. (6) specifies that the marginal utility of consumption is equal to the marginal value of labour.

Eq. (9) is the Euler equation for inter-temporal consumption.

The log-linear approximations of the solution of above FOCs, where lower letters represents logs, yields:

\[ w_t - p_t = \varphi n_t - \sigma c_t \]  \hspace{1cm} (8)

\[ c_t = E_t c_{t+1} - \frac{1}{\sigma} (r_t - E_t \pi_{t+1}) \]  \hspace{1cm} (9)

**Open Economy Block**

Households in the foreign economy replicate isomorphic optimization behaviour and due to the SOE assumption the domestic economy has no repercussions on the foreign economy, implying that the foreign consumption is restricted solely to foreign produced goods so that: $C_t^* = C_F^*$ and $P_t^* = P_F^*$. Also the FOCs obtained for the domestic economy prevail in the foreign economy with all variables in Eq.(8) and Eq.(9) superscripted by (*)

The open economy block is an important addition to the canonical workhorse New Keynesian DSGE model for the closed economy as exposited by Monacelli (2005), Gali and Monacelli (2005) and Liu (2006) and others. In this section we attempt to describe the open economy dynamics by analysing the linkages between inflation, the real exchange rate (RER) and the terms-of-trade (TOT).

For a SOE like Sri Lanka which is a price-taker in the world market we assume the LOP holds for exports as export prices which are determined in the world market are passed through completely to domestic prices after conversion to domestic prices using the exchange rate. However, in the case of prices of imports there is incomplete exchange rate pass through. This occurs because on the import side, competition in the world market brings import prices equal to marginal costs at the wholesale level, but rigidities arising from monopolistic competition and inefficient distribution by the network of retailing firms cause the domestic import prices to deviate from the world prices violating the tenets of LOP.
The implications of the phenomenon of incomplete exchange rate pass through for imports are analysed further here. We define the terms of trade (TOT) as $s_t = \frac{p_{F,t}}{p_{H,t}}$, the price of foreign goods in terms of a unit of home goods which translates in logs to:

$$s_t = p_{F,t} - p_{H,t} \quad (10)$$

Increase in $s_t$ implies that international competitiveness improves because home goods have become cheaper relative to foreign goods as prices of home goods have decreased.

Log linearizing the overall CPI equation around a steady and combining it with TOT yields;

$$p_t \equiv \{(1 - \alpha) p_{H,t} + \alpha p_{F,t}\} \quad (11)$$

$$p_t = p_{H,t} + \alpha s_t$$

The Eq. (11), when first differenced yields following equalities:

$$\pi_t = (1 - \alpha) \pi_{H,t} + \alpha \pi_{F,t} \quad (12)$$

$$\pi_t = \pi_{H,t} + \alpha \Delta s_t \quad (13)$$

$$\Delta s_t = \pi_{F,t} - \pi_{H,t} \quad (14)$$

This implies that the change in TOT is proportional to the difference between overall inflation rate and home inflation rate and degree of the degree of openness ($\alpha$). Furthermore, using the log nominal exchange rate: $e_t = \log(\epsilon_t)$ (domestic currency in terms of foreign currency or LKR per US$) we can define the real exchange rate ($\zeta_t$) as the rate which makes representative domestic and foreign baskets when measured in the same, implying purchasing power parity and LOP prevails if $\zeta_t$ defined below is unity:

$$\zeta_t = \frac{\epsilon_t p_t^*}{p_t}$$

$$q = \log(\zeta_t) = e_t + p_t^* - p_t \quad (15)$$

$$q = (e_t + p_t^* - p_{F,t}) + (1 - \alpha)s_t$$

$$q = \psi_t + (1 - \alpha)s_t$$

Where: $\psi_t = (e_t + p_t^* - p_{F,t}) \quad (16)$

denotes the deviation of the world price form the domestic currency price of imports, which is a measure of the deviation from law of one price. In this study, this measure is defined as law of one price gap (LOP gap).
International Risk Sharing and UIP Condition

The open economy block also enables us to make a foray into international risk sharing which yields the uncovered interest parity (UIP) condition. Under the assumption of complete international financial markets and perfect capital mobility, the expected nominal return from risk-free bonds, in domestic currency terms, must be the same as the expected domestic currency returns from foreign bonds. This assumption typically yields following log linearized conditions:\(^3\)

\[ c_t = c_t^* - \frac{1}{\sigma} q_t \]
\[ c_t = y_t^* - \frac{1}{\sigma} q_t \] (17)

Which can also be written as:

\[ c_t = y_t^* - \frac{1}{\sigma} ((1 - \alpha)s_t + \psi_t) \] (18)

Under complete international market assumption it is also possible to derive following log-linear condition:

\[ r_t - r_t^* = E_t \Delta e_{t+1} \] (19)

Where \( r_t^* \) is the world real interest rate.

Similarly, the RER can be expressed as:

\[ E_t \Delta q_{t+1} = (r - \pi_{t+1}) - r_t^* \] (20)

The Supply Block - Firms & Production Technology

The supply of the SOE-DSGE model purports to explain the price setting behaviour of firms. At the aggregate level an increase in output or real activity will cause the price level or inflation to increase. Production or supply is through a continuum of identical monopolistically competitive firms. In the continuum the \( j^{th} \) firm produces a differentiated good \( Y_j \) using a linear production function technology given by:

\[ Y_t(j) = A_t N_t(j) \] (21)

where, \( a_t = ln A_t \), and \( a_t = \rho_a a_{t-1} + v_t^a \) is a firm-specific productivity index that follows an AR(1) process.

\(^3\) See Monacelli (2005) and references therein for derivations.
The intermediate goods are produced by monopolistically competitive firms as postulated by Dixit and Stiglitz (1977). The firms set prices to satisfy demand for the goods from the j-firms as defined by:

\[ Y_t(i) = Y_t(P_t(i)/P_i)^{-\theta} \]

where \( P_i \) price of good i and \( \theta \) is elasticity of demand.

Total output or supply can be defined by aggregate output of the j-firms yielding:

\[ Y_t = \left[ \int_0^1 Y_t(j)^{-(1-\phi)} dj \right]^{1/(1-\phi)} \tag{22} \]

The first order linear approximation of the linear production function, under the assumption of symmetric equilibrium across all firms yields the following aggregate production function:

\[ y_t = a_t + n_t \tag{23} \]

and the firm’s total real cost of production as:

\[ TC_t = \frac{W_i Y_t}{P_{H,t} A_t} \]

This yields the same real marginal cost for all of the domestic firms as:

\[ mc_t = w_t - p_{H,t} - a_t \tag{24} \]

**Price Setting Behaviour and Incomplete Pass-through**

In domestic economy, monopolistic firms set prices in a staggered fashion as hypothesized by Calvo (1983). In any given period only a fraction of the domestic firms \((1-H)\), where \( H \in [0,1] \) set prices optimally and remaining fraction of firms \( H \), that fail to set price optimally, are assumed to set their prices by indexing them to the last period’s inflation. The degree of past indexation is assumed to be the same as the probability of setting its prices, implying that the Phillips curve is vertical in the long-run. With this set up it is possible to derive the standard New Keynesian Phillips curve (NKPC) as follows,

\[ \pi_{H,t} = \beta(1-H)E_t \pi_{H,t+1} + H \pi_{H,t-1} + \lambda_H mc_t \tag{25} \]

Where \( \lambda_H = \frac{(1-\beta)(1-H)}{H} \). The Calvo pricing structure yields a familiar NKPC, i.e. the domestic inflation dynamic has both a forward looking component and a backward-looking component. If all firms were able to adjust prices at each and every period, i.e. \( H = 0 \), the inflation process would be purely forward looking and deflationary policy would be completely costless.

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4 See Liu (2006) for the derivation of both Calvo and import price setting behaviour.
In the import sector, we assume LOP holds at wholesale level for imports. However, inefficiency in distribution channels together with monopolistic retailers keep domestic import prices above the marginal cost. As a result, the LOP fails to hold at retail level for domestic imports. In setting the new price for imports, domestic firms are concerned with the future path of inflation as well as the LOP. This non-zero LOP gap represent a wedge between world and domestic import prices. This provides a mechanism for incomplete import pass-through in the short-run, implying that changes in the world import prices have a gradual effect on the domestic economy. This yields the following equation;

\[
\pi_{F,t} = (1 - \theta_F) E_t \pi_{F,t+1} + \theta_F \pi_{F,t-1} + \lambda_F \psi_F
\]

(26)

where \( \lambda_F = \frac{(1 - \beta_F)(1 - \theta_F)}{\theta_F} \) and \( \theta_F \in [0,1] \) is the fraction of importer retailers that cannot re-optimize their prices every period.

The definition of overall inflation, Eq.(12) and Eq.(25) and Eq.(26) provides a road-map of the inflation dynamics in the SOE-DSGE model. These inflation dynamics in the in sticky price models are generated nominal rigidities associated with preference smoothing and price decisions of the firms. If prices were flexible nominal rigidities and inflation costs arising from price adjustment would vanish. The New Keynesian models differ from the frictionless market RBC model because of short-run adjustment costs arising from sticky prices, these adjustment costs are manifest in the NKPC. From the vantage point of a social planner optimal monetary policy is conceptualized as a policy that minimizes the deviations of the marginal costs and LOP gap from their steady state. In this paper rather than analysing the implications of a social planner’s optimization program we explore the dynamics that ensue when the central bank a simple monetary policy reaction function to achieve the goal of a fully flexible price equilibrium.

**Marginal Cost and Inflation Dynamics**

From the above NKPC eq. (25) and Eq. (24) we can derive the real marginal cost of the monopolistic firm (assuming symmetric equilibrium):

\[
mc_t = w_t - p_{H,t} - a_t
\]

\[
mc_t = (w_t - p_t) + \alpha s_t - a_t
\]

\[
mc_t = \sigma c_t + \varphi y_t + \alpha s_t - (1 + \varphi)a_t
\]

\[
mc_t = \varphi y_t - (1 + \varphi)a_t + \sigma y_t^* + s_t + \psi_t
\]

(27)
**Goods Market Equilibrium**

Log-linearised isoelastic demand functions (both domestic and foreign) yield following two equations;

\[
c_{H,t} = -\eta(p_{H,t} - p_t) + c_t = \alpha \eta s_t + c_t \tag{28}
\]

\[
c_{H,t}^* = -\eta(p_{H,t}^* - p_t^*) + c_t^* = \eta(p_{F,t} - \psi_t) - p_{H,t} + c_t^* = \eta(s_t + \psi_t) + c_t^* \tag{29}
\]

The demand for imports is;

\[
c_{F,t} = -\eta(p_{F,t} - p_t) + c_t = -\eta(1 - \alpha)s_t + c_t \tag{30}
\]

Goods market clearing implies \( y_t = (1 - \alpha)c_{H,t}(i) + \alpha c_{H,t}^*(i) \) for all goods \( i \). Once aggregate and substitute above demand functions it is possible to derive the following proportionality relation between domestic output and foreign output, which is affected by the existence of incomplete pass-through.

\[
y_t - y_t^* = \frac{1}{\sigma}(\omega_s s_t + \omega_\psi \psi_{F,t}) \tag{31}
\]

Where, \( \omega_s = 1 + \alpha(2 - \alpha)(\sigma \eta - 1) > 0 \) and \( \omega_\psi = 1 + \alpha(\sigma \eta - 1) > 0 \)

**The Monetary Block and the Policy Reaction Function**

The SOE model is closed by specifying a monetary policy reaction function or policy instrument that is used by the central bank to determine the stance of monetary policy. Rather than prescribing an optimizing program to be implemented by the monetary authority [for example see Woodford (2003)] we prescribe a simple forward looking monetary policy reaction function which can be approximated under sticky prices by:

\[
r_t = \rho_r r_{t-1} + (1 - \rho_r)[\omega_1 \pi_t + \omega_2 ygap_{t+1}] \tag{32}
\]

Where \( ygap_{t+1} = y_{t+1} - ynat_t \) is the per cent deviation of one period ahead output where;

\[
ynat_t = \left(\frac{\omega_s(1 + \varphi)}{\sigma + \varphi \omega_s}\right) a_t + \left(\frac{\sigma(1 - \omega_s)}{\sigma + \varphi \omega_s}\right) y_t^* \]

\( ynat_t \) is the natural level of output i.e. the output that would obtain in the case of both flexible domestic prices and complete pass through.
Assigning a forward looking monetary policy reaction function for Sri Lanka was motivated by Perera, R and Jayawickrema (2013)’s finding that Sri Lanka’s monetary reaction function is best described by such a specification.

3. Bayesian Estimation
We estimate the SOE-DSGE model for Sri Lanka using Bayesian techniques using the Dynare preprocessor as incorporated in Matlab software. The Bayesian estimation requires the definition of prior and posterior distributions. The Kalman Filter, a recursive numerical optimization algorithm, is used to estimate the mode of the likelihood function, that is used to update the priors and obtain posterior functions. The Dynare program uses Metropolis Hastings algorithm based on Markov Chain Monte Carlo (MCMC) simulations to estimate the posterior distribution. Bayesian modeling techniques acknowledge that all models are false and do not assume that they are working with the correct model and attempt to replicate the same model with highest probability given the evidence. Bayesian techniques provide probabilistic statements about unknown parameters and differs from classical procedures which are applied in repeated number of samples to deliver the correct answer in pre-specified percentage terms. Bayesian estimation techniques, because of its theoretical robustness and computational alacrity due to advances high speed desktop computing is gaining widespread popularity and is being now applied in diverse activities that range from space travel, weather forecasting, criminal law, marketing, business, economics to central banking. There are a number factors that favour Bayesian estimation techniques based on numerical optimization over analytical like Maximum Likelihood Estimation (MLE) and Generalised Method of Moments (GMM) in the estimation of DSGE models and they are listed in Dynare User Guide (Grifoli (2007).

1. The Bayesian technique fits and solves the complete DSGE model rather than reduced form equations like the Euler equation as in the case of MLE or GMM methods. The Bayesian estimation technique overcomes the’ tyranny of dimensionality’ that vitiates the estimation of the liklelihood function of DSGE models or BSVARS by shrinkage using priors.

2. Bayesian techniques by using priors as weights in the estimation of the likelihood function in DSGE modeling avoids the likelihood function peaking at strange points thereby avoiding the “dilemma of absurd parameter estimates”.

---

5 Dynare is a free software (available at www.dynare.org) which can be used to estimate Bayesian DSGE models in collaboration with Matlab software. More details are available in Dynare User Guide (Grifoli (2007) and Dynare Manual (Adjemian et al. (2014).
3. Bayesian techniques by inclusion of priors tackles the problem of identification which arises because different values of structural parameters can generate the same joint distribution for observables. More technically, the problem is caused by the posterior distribution being flat over a subspace of posterior values. The weighting of the likelihood with the prior densities adds sufficient curvature in the posterior distribution to facilitate numerical maximization and identification.

4. Bayesian techniques explicitly addresses the issue of model mis-specification by including shocks, which can regarded as measurement errors, in the structural quations. Thereby, it satisfies the Blanchard-Khan condition required for solving the DSGE model.

5. Bayesian techniques can provide through posterior distribution measures to compare the best fit of the models or scenarios to a given set of data through the computation of posterior odds ratios.

Bayesian estimation techniques spans a bridge between calibration and maximum likelihood estimation where calibration is the traditional method of choosing values for parameters based on prior knowledge or beliefs. Statistically the values of parameters are chosen to ensure that “moment matching” that is the priors chosen should match the mean, variance and measures of skewness exhibited by the data. A noteworthy difference between calibration and estimation procedures is that the parameter values chosen by calibration do not necessarily best fit the data. Nonetheless in Bayesian estimation calibration and estimation should not be regarded as mutually exclusive rivals but rather complements.

In the Bayesian technique, all the information about the parameter vector \( \theta \) garnered from the data is embodied in likelihood function. For any given model M, the posterior density of the model parameter \( \theta \) can be specified as follows:

\[
p(\theta|Y^T) = \frac{L(Y^T|\theta,M)p(\theta|M)}{\int L(Y^T|\theta,M)p(\theta|M)d\theta}
\]  

(33)

where, \( p(\theta|M) \) is the prior density function and \( L(Y^T|\theta,M) \) is the likelihood function. The object of Bayesian estimation technique is to find the model M that maximizes the posterior probability function given by \( p(\theta|Y^T) \).

The likelihood function can be computed via the state-space representation on the model together with the measurement equation linking the observed data and the state vector. The economic model described in Section 2 has the following state-space representation:

\[
S_{t+1} = \Gamma_1 S_t + \Gamma_2 \omega_{t-1}
\]

(34)
\[ Y_t = \Lambda S_t + \mu_t \]

Where \( \omega_t \) is a state innovation vector, \( Y_t \) is kx1 vector of observed variables and \( \mu_t \) is measurement error. The matrices \( \Gamma \) are functions of the deep parameters \( P, Q, R \) and \( S \). \( \Lambda \) represents the relationship between observed and state variables, assuming that state innovations and measurement errors are distributed asymptotically normal with zero mean and variance-covariance matrices \( \Xi \) and \( \Upsilon \) respectively. Yielding the likelihood function of the model as:

\[
\ln L(Y^T|\Gamma_1, \Gamma_1, A, \Xi, Y) = \frac{T N}{2} \ln 2\pi + \sum_{t=1}^{T} \left[ \frac{1}{2} \ln |\Omega_t|t-1| + \frac{1}{2} \mu' \Omega^{-1}_t \mu_t \right]
\]  

(35)

Note that the numerator in Eq. (55) is the posterior kernel (or the unnormalized posterior density) given that the denominator, the marginal density that is constant. This demonstrates that the posterior density is proportional to the likelihood function.

\[ p(\theta|Y^T) \propto L(Y^T|\theta, M) p(\theta|M) \]  

(36)

The above fundamental equations enables us to rebuild the posterior moments. Here, the likelihood function can estimated using the Kalman filter and used to simulate posterior kernel using MCMC Metropolis-Hastings algorithm.

The log posterior kernel can be expressed as:

\[ \ln p(\theta|Y^*_t) = \ln L(\theta|Y^*_t) + \ln p(\theta) \]  

(37)

The first term in Eq. (59) the log likelihood can be estimated using the Kalman filter recursion and the second term is the calibrated value of the priors.

In order to find the posterior of the mode Dynare maximizes the posterior kernel with respect to \( \theta \). The likelihood function is non-Gaussian w.r.t \( \theta \) and is a nonlinear complex function of deep parameters. But the posterior distribution can be simulated using MCMC Metropolis-Hastings algorithm. It is a “rejection sampling algorithm” that is implemented step-wise as follows:

In step 1 a candidate parameter \( \theta^* \) is chosen from a Normal distribution, with mean set at \( \theta^{t-1} \).

In step 2 the value of the posterior kernel is computed from the mean of the drawing distribution. In step 3 keep the candidate parameter if the acceptance ratio is greater than unity.

Then do two things: Update the mean of the drawing distribution and note the value of the parameter that is retained. Repeat these steps a large number of times and build a histogram of the retained values. Then the resulting smoothed histogram after a large number of iterations will be the posterior distribution.
The posterior distribution provides a method for determining which of the rival models fit best fits the given data. Suppose we have prior distributions for two competing models \( p(A) \) and \( p(B) \) then using the Bayes’ theorem we can compute the posterior distribution over the models,

\[
p(I|Y_T) = P(I)p(Y_T|I)/\sum_{I=A,B} P(I)p(Y_T|I) \tag{38}
\]

where \( I = A, B \) can be generalized to \( N \) models.

If the posterior odds ratio computed in Eq. (61) below exceeds unity we conclude that model A fits the data better than model B.

\[
p(A|Y_T)/P(B|Y_T) \tag{39}
\]

We are indebted to Liu (2006) and Griffoli (2007) for the exposition of the Metropolis-Hastings algorithm and the Posterior odds ratio.

### 4. Empirics

#### 4.1 Data and Priors

Data from 1999Q1-2013Q4 for Sri Lanka is used to empirically validate the SOE-DSGE model. Quarterly observations on per capita GDP \( (y_t) \), nominal interest rates \( (r_t) \), overall inflation \( (\pi_t) \), imported inflation \( (\pi_{F,t}) \), real exchange rate \( (q_t) \), terms of trade \( (s_t) \), world output \( (y_{t}^*) \) and world real interest rate \( (r_{t}^*) \) are taken mainly from CBSL and various other sources. The United States economy has been considered as proxy to the world economy. All the variables (except inflation and the interest rate) were rescaled as deviations from the mean and could be expressed as percentage deviations from the means. Detailed description about variable transformation, proxies and sources are given in Table A1. The correlations between the variables and their graphical plots are given in Table A2 and Figure A1 respectively.

#### 4.2 Choice of Priors and Shocks for Model Calibrations

The choice of priors used for the calibrations used in the model was based on a number of considerations. i). The choice of structural parameters reflected the researchers belief that they reflected the structure of the commodity exporting economy –Sri Lanka. ii). The priors were also assumed to reflect the structural character of the SOE such as the degree of openness, the commodity export orientation and the fragility of the institutional fabric of a developing economy. iii). The absence of micro-level studies to capture all the to be estimated parameters in Sri Lanka was overcome by sourcing information from other emerging market cross-country
studies as was done in other studies iv). Country-specific information was also sourced from IMF working papers and Central Bank Staff Studies as described below. v). The choice of priors were also reflected the imposition of non-negativity or interval restrictions. For example, beta-distributions were chosen to proxy parameters that were confined to the unit-interval. Gamma and Normal distributions were chosen to represent parameters in $\mathbb{R}^+$. Inverse-gamma was selected to account for precision of the shocks.

Discount rate ($\beta$) was estimated following Ahmed et al. (2012). Return on government paper and change in CPI were used to measure the interest rate and inflation respectively. Lagged inflation was used to calculate the real interest rate. Annual data were used during 1995-2013. Degree of openness ($\alpha$) was approximated using imports to GDP ratio during the period 2001 to 2013. The estimated value of 0.35 is on par with the other emerging market economies.

Prior means for domestic and imported price stickiness parameters ($\theta_H$ and $\theta_F$ respectively) were set 0.5 each. This is supported by the fact that Wimalsuriya (2007) and Duma (2008) finding of exchange rate pass-through into prices takes around two quarters in Sri Lanka.

Monetary policy reaction coefficient parameters are set at 1.5 for inflation coefficient ($\phi_1$) and 0.5 for output gap coefficient ($\phi_2$). Although it is common practice to set $\phi_1$=1.5, we set $\phi_2$=0.5, 25 basis points above the most preferred value in the literature since Perera R and Jayawickrama (2013) found that CBSL’s response for output fluctuations is greater. The prior distributions and standard deviations of shocks used for the calibration of the model are reported in Table A3.

4.3 Convergence Diagnostics

Various convergence diagnostics measures carried out to examine the fit of the SOE-DSGE model to the data are reviewed in the sequel.

Brooks and Gelman Convergence Statistics

Monet Carlo Markov Chains (MCMC) uni-variate diagnostics [Brooks and Gelman (1998)] is a major source of feedback to gain confidence, spot problems with the results. The analysis completed with 500,000 Metropolis Hastings simulations. If the results from one chain are sensible, and the optimizer did not get stuck in an odd area of the parameter subspace, two
thing should happen. 1. Results within any of the many iterations of the MH simulations should be similar. 2. Results between the various chains should be close [for more details see Pfeifer (2014)]. The results are shown in Figure A2. The two lines on the charts represent specific measures of the parameter both within and between chains. For the results to be sensible, these should be relatively constant and should converge. The charts show that this requirement is accomplished in our study.

Multivariate convergence diagnostics are based on the range of the posterior likelihood function instead of the individual parameters. Figure A3 shows the proximity of the two lines indicate convergence. Figure A4 depicts the smoothed shocks. It is a useful illustration to eye-ball the plausibility of the size and frequency of the shocks. The horizontal axis represents the number of periods in the sample. It is important check whether the shocks are centered around zero as expected. In this case the shocks are centered around zero.

Historical and smoothed variable plots are shown in Figure A5. It depicts the observed data and the estimates of the smoothed variable. If the model fit is satisfactory, both lines should overlap. This is true for our model in most cases. All the MCMC diagnostic tests suggest that the Markov chain has converged to its stationary distributions after a large number of iterations.

**Prior and Posterior Distributions**

Prior and posterior distributions of estimated parameters are depicted in Figure A6. The grey lines show the prior density while the black line shows the posterior density. The horizontal line indicates the posterior mode. These graphs provide information on the confidence on the results or indicate that there are problems. First, prior and posterior distribution should not be excessively different. On the other hand the posterior plot should not be exactly the same as prior as shows that data has provided much information to update prior. Second, posterior distributions should be close to normal shape. Third, the mode (calculated from numerical optimization of the posterior model should not be far away from the mode of the posterior distributions. Most of the posterior distributions exhibit the required normal shape, $\rho_{hor}$ appears to be the only exception.

Based on the two independent Markov Chains we computed the posterior mean, median and the 95 percent probability intervals for each of the parameters and the results are reported in Table A5.
The posterior mean ($\eta$) for the inter-temporal elasticity of substitution between home and foreign goods is 0.99. The relatively low value $\eta$ is consistent with the fact that Sri Lanka is a commodity producer and its consumption basket is weighted by domestically produced goods. The inverse elasticity of labour supply ($\varphi$) is approximately unity, and this implies that a 1% increase in the real wage will increase labour supply by the same amount.

On the supply side, the mean estimate of the probability of not changing price in a given quarter, or equivalently the proportion of firms that do not re-optimize their prices in a given quarter is approximately 57 per cent for domestic firms and 44 per cent for import retailers. The Calvo coefficients imply that the average duration of price contracts is around 2.3 quarters for domestic firms and approximately 1.8 quarters for import retailers. These figures indicate that the average degree of nominal price rigidity for an emergent market economy is much lower than that observed for the advanced economies such as US.

The monetary policy reaction function specified for the SOE-DSGE model is a good indicator the performance of monetary policy during the study period with relatively stable inflation. The posterior mean for the degree of interest rate smoothing is estimated to be 0.19 with 1.19 and 0.58 being the weights on inflation and output respectively.

### 4.4 Impulse Response Functions (IRFs)
Bayesian IRFs are calculated to evaluate the response of the economy to various types of shocks considered in our model. Impulse response values can be considered as percentage deviation from steady state values. In this section, we review the dynamics of some important shocks through the IRF graphics (See Figures A7-A11 for graphical representation of each IRF due to different shocks). The size of the shock in each case is one standard deviation of the shock itself in the estimated model, which is reported in Table A4. The grey shaded area in the graphs provides highest posterior density intervals.

**Monetary Policy Shock**
Monetary policy is considered as an interest rate rule. Response of the endogenous variables to a one standard deviation shock to the nominal interest rate is depicted in the in Figure A7. Monetary tightening contracts the output and reduces both domestic and imported inflation, as expected by the monetary authority. Increase in interest rates reduces consumption and appreciates both nominal and real interest rates. Appreciation in exchange rate would make domestic agents substitute out domestically produced good into foreign produced goods. Domestic firms respond to lower demand by reducing nominal wages causing the price of
domestically produced goods to eventually decline. Appreciation in the exchange rate reduces the prices of imported goods. Response of the central bank to reduction in output and inflation is to cut interest rates until returning to the equilibrium. The results of interest rate shock show that monetary policy is effective in the sense that it affects significantly to the main target variables: Inflation and Output. This result is in agreement with the recent findings of Perera, A and Wickramanayake (2013), where they also found that monetary policy is effective to influence target variables. They performed a Structural VAR analysis to come to that conclusion.

**Domestic Inflation Shock**
A domestic inflation shock can be considered as a supply shock. One standard deviation shock to the domestic inflation initially worsens domestic competitiveness by generating a real exchange rate appreciation. The central bank responds to the increase in inflation by monetary tightening. This would appreciate the exchange rate further more. This would reduce output via a decline in export demand. However, systematic loosening of monetary policy would ultimately bring the economy back to steady state after 10 quarters of the shock.

**Imported Inflation Shock**
One standard deviation shock to imported inflation initially improves domestic competitiveness. This induces expenditure switching towards domestic goods. However, overall inflation is higher on impact as higher import prices increases cost of production. Monetary tightening of the central bank due to high inflation causes decrease in consumption and appreciation of real interest rate. Reduction of output due to monetary tightening outweighs the initial increase in demand for domestic goods due to improved competitiveness to reduce the output on impact. The system returns to steady state 10 quarters after the shock.

**Productivity Shock**
One standard deviation temporary productivity shock reduces real marginal cost which in turn reduces prices of domestically produced goods. This increases degree of domestic competitiveness which will see domestic agents substitute out foreign produced goods into domestically produced goods. Inflation falls initially as reduction in production cost lowers the prices. The central bank responds it by loosening monetary policy. This act depreciates exchange rate. Gradual increase in interest rate after fifth quarter helps to restore equilibrium.
In summary the above analysis supports the overview that the empirics on IRFs and exogenous shocks derived from SOE-DSGE modeling capture the business cycle dynamics of the Sri Lankan economy focusing on monetary policy stabilization goals of the CBSL.

5. Conclusion

In this paper we presented a variant of SOE-DSGE model for the Sri Lankan economy and empirically validated it using quarterly data during 1999Q1-2013Q4. The IRF analysis based on Bayesian estimation of the SOE-DSGE model confirms that the monetary targeting policy implemented through the interest channel operating through various transmission channels appear to deliver the ultimate targets of inflation and output as predicted in the period under analysis.

Our study complements the other empirical study on estimated DSGE model for Sri Lanka based on a dated database that advocates that Sri Lanka should transit from the current monetary targeting framework to a flexible inflation targeting framework\(^6\). The SOE-DSGE modeling undertaken in this study provides a policy modelling framework for already announced CBSL medium-term strategy to transition to inflation targeting framework.

The bare-bones SOE-DSGE model that we have presented can be extended to evaluate the welfare implications of alternative policy regimes by particular parameterization of representative agent’s preferences derived from second order approximations of consumer’s utility. Furthermore, the SOE-DSGE model can be used to make out-of-sample inflation forecasts and get a handle on the type of policy adjustments that should be undertaken to design a monetary policy stance to achieve a predetermined stabilization target. Furthermore, the bare-bones SOE-DSGE model provides scope for addition of components to examine the implications, fiscal policy, labour markets, financial markets into the mix as demonstrated by Ramses - DSGE modeling undertaken by Sevriges Riksbank.

\(^6\) Perera, A and Wickramanayake (2013) also recommended a flexible inflation targeting framework for Sri Lanka using an SVAR methodology.
Appendix: Tables and Figures

Table A1: Data Description

<table>
<thead>
<tr>
<th>Variable</th>
<th>Code</th>
<th>Description</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_t$</td>
<td>y</td>
<td>Per capital log real GDP (linear de-trended and seasonally adjusted)</td>
<td>CBSL</td>
</tr>
<tr>
<td>$r_t$</td>
<td>r</td>
<td>Interest Rate: 3 months treasury bill rate (average)</td>
<td>CBSL</td>
</tr>
<tr>
<td>$\pi_t$</td>
<td>$\pi c$</td>
<td>Overall Inflation: Annual growth rate of Colombo Consumers’ Price Index</td>
<td>CBSL</td>
</tr>
<tr>
<td>$\pi F,t$</td>
<td>$\pi f$</td>
<td>Imported inflation: Annual growth rate of unit value of Import Index</td>
<td>dXtime®-EMED Emerging Asia data base</td>
</tr>
<tr>
<td>$q_t$</td>
<td>q</td>
<td>Real exchange rate: Log nominal exchange rate (LKR/USD) multiplied by US-Sri Lanka CPI price ratios</td>
<td>CBSL and dXtime-OECD database</td>
</tr>
<tr>
<td>$s_t$</td>
<td>s</td>
<td>Terms of trade: Ratio of unit value of import index and unit value of export index (log)</td>
<td>dXtime-EMED Emerging Asia data base (code:LK.FTI.M5_3.1)</td>
</tr>
<tr>
<td>$y_t^*$</td>
<td>$y s t$</td>
<td>Foreign output: US seasonalised linear de-trended log real GDP considered as proxy to foreign output</td>
<td>U.S. Department of Commerce: Bureau of Economic Analysis</td>
</tr>
<tr>
<td>$r_t^*$</td>
<td>$r s t$</td>
<td>Foreign Real interest rate: US federal funds rate – US expected inflation (lead) is considered as proxy to foreign real interest rate</td>
<td>US Federal Reserve and dXtime-OECD database</td>
</tr>
</tbody>
</table>

a/ The codes for the unobservable variables are: Consumption- $c$ and Dom. inflation - $\pi h$
b/ dXtime is a data base that is available at :www.econdata.com/database.

Table A2: Contemporary Correlations

<table>
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<tr>
<th></th>
<th>$r^*$</th>
<th>$y^*$</th>
<th>Y</th>
<th>s</th>
<th>$r^*$</th>
<th>q</th>
<th>pif</th>
<th>Pi</th>
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<td>1.00</td>
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<td>-0.02</td>
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<tr>
<td>$y^*$</td>
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<td>0.09</td>
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<td>0.20</td>
<td>0.01</td>
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<td>Y</td>
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<td>0.09</td>
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<td>0.03</td>
<td>0.13</td>
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<tr>
<td>S</td>
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<td>0.11</td>
<td>-0.19</td>
<td>1.00</td>
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<td>-0.18</td>
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<td>Q</td>
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<td>Pi</td>
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Table A3: Prior Distributions
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<th>Variance</th>
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Table A4: Standard Deviation of Shocks

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<th>Shock</th>
<th>Mean Std. Dev</th>
<th>90% HPD Interval</th>
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</thead>
<tbody>
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<td>UIP - eps_q</td>
<td>1.7411</td>
<td>[0.47, 3.19]</td>
</tr>
<tr>
<td>Dom. Inflation - eps_pih</td>
<td>4.3168</td>
<td>[3.61, 5.04]</td>
</tr>
<tr>
<td>Imported Inflation - eps_pif</td>
<td>7.3852</td>
<td>[6.23, 8.48]</td>
</tr>
<tr>
<td>Interest rate - eps_r</td>
<td>2.9709</td>
<td>[2.41, 3.53]</td>
</tr>
<tr>
<td>Productivity - eps_a</td>
<td>2.0059</td>
<td>[1.47, 2.53]</td>
</tr>
<tr>
<td>TOT - eps_s</td>
<td>16.2226</td>
<td>[13.69, 18.54]</td>
</tr>
<tr>
<td>Foreign Output - eps_ystar</td>
<td>0.3624</td>
<td>[0.30, 0.42]</td>
</tr>
<tr>
<td>For. Real Int. Rate - eps_rstar</td>
<td>0.9319</td>
<td>[0.79, 1.07]</td>
</tr>
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</table>

Table A5: Posterior Estimates using 500,000 Markov Chain Draws
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior Mean</th>
<th>Post. mean</th>
<th>Post. 90% Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>1.00</td>
<td>1.32</td>
<td>[1.17, 1.47]</td>
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<tr>
<td>$\eta$</td>
<td>1.00</td>
<td>0.99</td>
<td>[0.51, 1.46]</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>1.00</td>
<td>1.06</td>
<td>[0.56, 1.55]</td>
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<td>$\theta_H$</td>
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<td>0.57</td>
<td>[0.45, 0.69]</td>
</tr>
<tr>
<td>$\theta_F$</td>
<td>0.50</td>
<td>0.44</td>
<td>[0.34, 0.54]</td>
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<tr>
<td>$\phi_1$</td>
<td>1.50</td>
<td>1.19</td>
<td>[1.06, 1.33]</td>
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<tr>
<td>$\phi_2$</td>
<td>0.50</td>
<td>0.58</td>
<td>[0.40, 0.75]</td>
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<tr>
<td>$\rho_r$</td>
<td>0.70</td>
<td>0.19</td>
<td>[0.06, 0.33]</td>
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<tr>
<td>$\rho_a$</td>
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<td>0.97</td>
<td>[0.95, 0.997]</td>
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<tr>
<td>$\rho_r^*$</td>
<td>0.50</td>
<td>0.83</td>
<td>[0.75, 0.93]</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>0.50</td>
<td>0.94</td>
<td>[0.89, 0.98]</td>
</tr>
</tbody>
</table>

Figure A1: Data Plots
Figure A2: MCMC Uni-Variate diagnostics (Brooks and Gelman 1998)

Figure A2 Ctd.: MCMC Uni-Variate Diagnostics (Brooks and Gelman 1998)
Figure A3: MCMC Multivariate Diagnostics (Brooks and Gelman 1998)

Figure A4: Smoothed Shocks
Figure A5: Historical and Smoothed Variables

Figure A6: Prior and Posterior Distributions
Figure A6 Ctd. Prior and Posterior Distributions

Figure A7: Impulse Response Functions for One Standard Deviation Shock in Interest Rates (Monetary Shock)
Figure A8: IRF - One Standard Deviation Shock in Domestic Inflation

Figure A9: IRF - One Standard Deviation Shock in Imported Inflation
Figure A10: IRF – One Standard Deviation Shock in Productivity

Figure A11: IRF - One Standard Deviation Shock in Terms of Trade
References


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